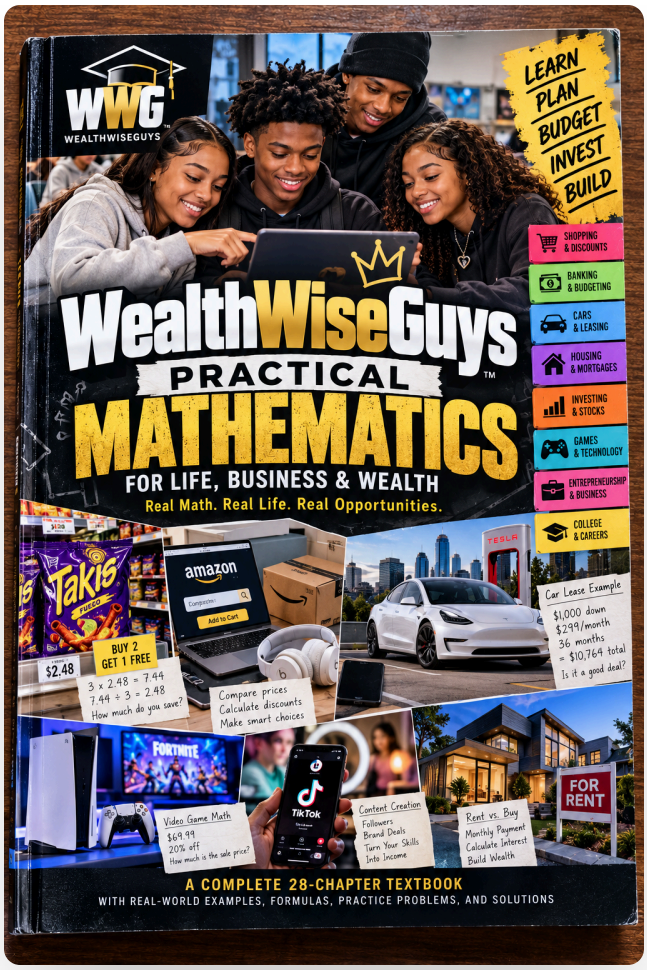


ANTMACK215 × WEALTHWISEGUYS

WealthWiseGuys Practical Mathematics

Digital draft edition. Not yet reviewed for publisher printing.

Student Textbook · Grades 7–12 · Also for adult learners

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CHAPTER 1

Whole Numbers: How to Dissect and Solve Word Problems

Every money decision starts with whole numbers: ticket counts, hours worked, items on a receipt. This chapter builds number sense, then gives you a repeatable blueprint for taking any word problem apart.

1.1

Reading, Writing, and Rounding Whole Numbers

LEARNING OBJECTIVES

After this section you will be able to:

1. Identify the place value of any digit in a whole number.
2. Write whole numbers in words and in numerals.
3. Round whole numbers to a stated place.

REAL-LIFE SITUATION

Sold-out arena

A Philadelphia arena reports that 48,627 fans attended a playoff game. A news anchor says "about 49,000 fans." A ticket app says "over 48,600." Both are rounding the same number to different places.

Ten thousands	Thousands	Hundreds	Tens	Ones
4	8	6	2	7

Figure 1.1 Place-value chart for 48,627

The concept

Our number system is base ten. Each place is worth ten times the place to its right. Digits are grouped in threes, called periods: units, thousands, millions, billions. Commas separate the periods.

To read a number, read each period as a three-digit number and say the period name, except the units period. The word "and" is saved for the decimal point, so we never say it inside a whole number.

To round, find the place you are rounding to and look at the digit immediately to its right. If that digit is 5 or more, add 1 to the rounding digit. If it is 4 or less, leave the rounding digit alone. Every digit to the right becomes zero.

Reading, Writing, and Rounding Whole Numbers: solution map

1. Identify the question

   `Write 3,206,450 in words.`

2. Calculation 1

   `Split into periods: 3 | 206 | 450.`

3. Calculation 2

   `Millions period: three million.`

4. Calculation 3

   `Thousands period: two hundred six thousand.`

5. Calculation 4

   `Units period: four hundred fifty.`

6. Interpret and check

   `Three million, two hundred six thousand, four hundred fifty`

Figure 1.1.V Reading, Writing, and Rounding Whole Numbers: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

BUSINESS

An organization that makes or sells goods or services to earn money.

REVENUE

The total money a business brings in from sales, before any costs are paid.

COST

The amount paid to make or buy an item before it is sold.

KEY TERMS

Whole numbers
The numbers 0, 1, 2, 3, and so on, with no fractions or decimals.

Place value
The value a digit has because of its position in the number.

Period
A group of three places (units, thousands, millions, ...) separated by commas.

Rounding
Replacing a number with a nearby, simpler number to a chosen place.

FORMULAS & RULES

Rounding rule

```
Digit to the right ≥ 5 → round up; ≤ 4 → keep
```

All digits to the right of the rounding place become 0.

IN PLAIN WORDS

1. Find the place you are rounding to.
2. Look at the digit just to its right.
3. 5 or more: bump up by 1. 4 or less: leave it.
4. Turn every digit after it into 0.

Try it with real numbers: `48,627 to the nearest thousand → look at 6 → 8 becomes 9 → 49,000`

Worked examples

Example 1 Writing a number in words

Write 3,206,450 in words.

SOLUTION

1. Split into periods: 3 | 206 | 450.
2. Millions period: three million.
3. Thousands period: two hundred six thousand.
4. Units period: four hundred fifty.

> **Answer:** Three million, two hundred six thousand, four hundred fifty

Example 2 Rounding attendance

Round 48,627 to the nearest thousand and to the nearest hundred.

SOLUTION

1. Nearest thousand: the thousands digit is 8. The digit to its right is 6, which is 5 or more, so 8 becomes 9: 49,000.
2. Nearest hundred: the hundreds digit is 6. The digit to its right is 2, which is 4 or less, so 6 stays: 48,600.

> **Answer:** 49,000 (thousand); 48,600 (hundred)

Practice exercises 1.1

1. Write 7,045 in words.
2. Round 6,349 to the nearest hundred.
3. Round 82,501 to the nearest thousand.
4. What is the place value of the 5 in 352,108?
5. Write "two hundred thousand, nine" as a numeral.

RECAP QUIZ 1.1

1. Round 1,950 to the nearest hundred.

- ○ a. 1,900
- ○ b. 2,000
- ○ c. 1,950
- ○ d. 2,100

2. Which digit is in the thousands place of 64,812?

- ○ a. 6
- ○ b. 4
- ○ c. 8
- ○ d. 1

3. "Ninety thousand, ninety" as a numeral is:

- ○ a. 9,090
- ○ b. 90,900
- ○ c. 90,090
- ○ d. 900,090

SECTION 1.1 RECAP

- A digit's value depends on its place — each move left is 10 times greater.

- Rounding looks at the digit to the right: 5 or more rounds up, 4 or less stays.

Answer key with explanations: Section 1.1

PRACTICE EXERCISES

1. Seven thousand, forty-five
2. 6,300
3. 83,000
4. Ten thousands (the 5 is worth 50,000)
5. 200,009

RECAP QUIZ

1. **2,000**: The digit right of the hundreds place is 5, so 9 hundreds rounds up to 10 hundreds: 2,000.
2. **4**: Reading right to left: units 2, tens 1, hundreds 8, thousands 4.
3. **90,090**: 90 in the thousands period and 090 in the units period: 90,090.

1.2

Adding and Subtracting Whole Numbers

LEARNING OBJECTIVES

After this section you will be able to:

1. Add whole numbers and check the sum.
2. Subtract whole numbers, including borrowing, and check the difference.
3. Estimate sums by rounding first.

REAL-LIFE SITUATION

Lunch at the corner market

Dev buys lunch for himself and two friends and pays with a $20 bill. Before he walks away he wants to know: what was the total, and is his change right?

BROAD ST. CORNER MARKET
10/14 · Thank you!

1 × Turkey hoagie @ $8.00	$8.00
2 × Soda @ $2.00	$4.00
3 × Chips @ $1.00	$3.00
TOTAL	**$15.00**
CASH	$20.00
CHANGE	$5.00

Figure 1.2 Corner market receipt (whole-dollar prices)

The concept

To add, line up the digits by place value and add right to left. When a column adds to 10 or more, write the ones digit and carry the tens digit to the next column.

To subtract, line up the larger number (minuend) on top. When a top digit is smaller than the one below it, borrow 1 from the next place to the left, which adds 10 to the current place.

Always check. Addition checks by adding the column in the opposite direction. Subtraction checks by adding the difference back to the subtrahend; you should get the minuend.

Adding and Subtracting Whole Numbers: solution map

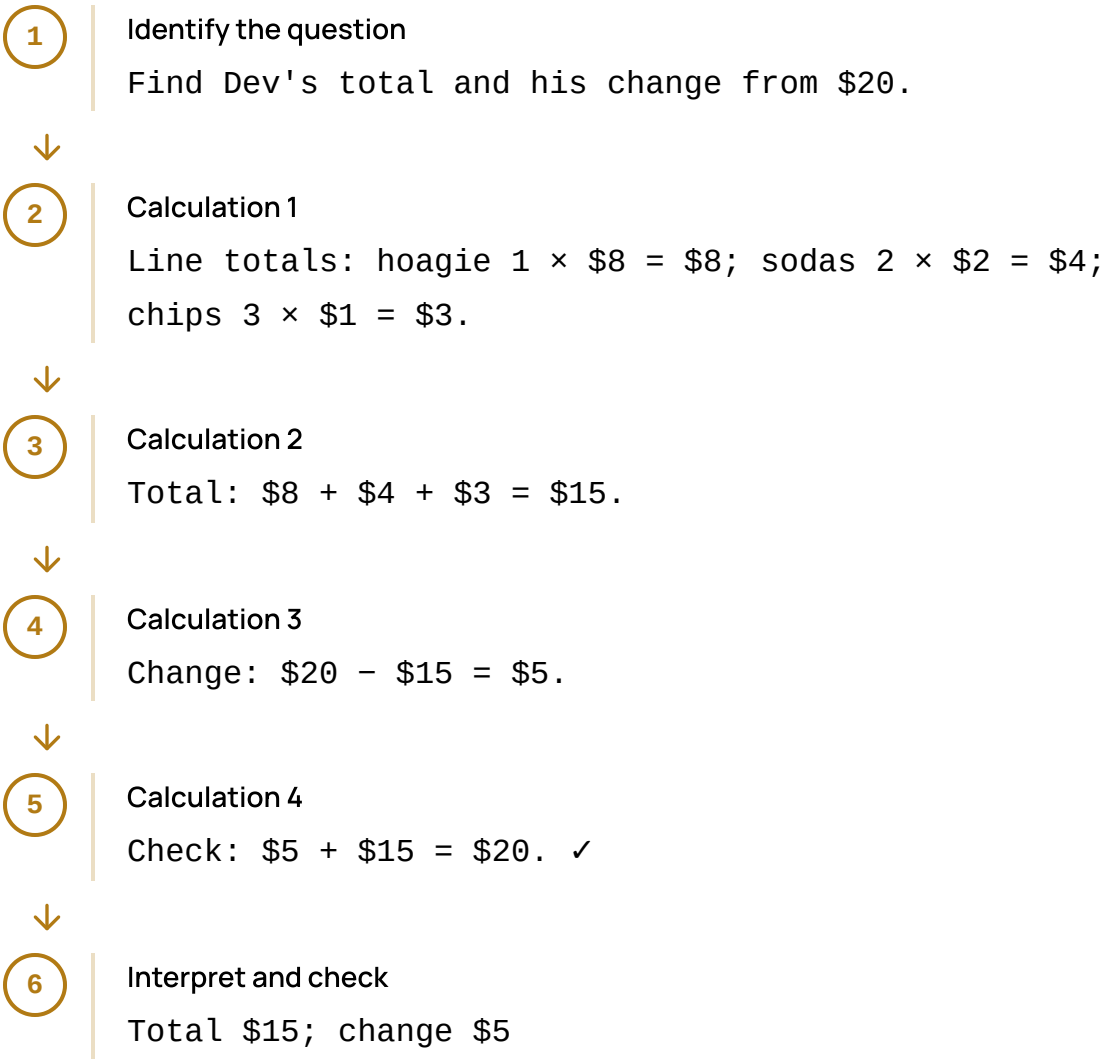


Figure 1.2.V Adding and Subtracting Whole Numbers: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

EXPENSE

Money a business spends to run, such as rent, supplies, or wages.

PROFIT

What is left when expenses are subtracted from revenue.

BUDGET

A plan that shows expected income and how it will be spent.

KEY TERMS

Addends
The numbers being added.

Sum
The result of addition.

Minuend
The number you subtract from (the starting amount).

Subtrahend
The number being subtracted.

Difference
The result of subtraction.

FORMULAS & RULES

Addition

```
Addend + Addend = Sum
```

THE LETTERS ARE JUST STAND-INS FOR NUMBERS

`Addend` = each amount you are putting together

`Sum` = the total of everything

IN PLAIN WORDS

1. Line up the amounts by place value.
2. Add them together.
3. The answer is the total (sum).

Try it with real numbers: `$8 hoagie + $4 sodas + $3 chips = $15 total`

Subtraction

```
Minuend − Subtrahend = Difference
```

THE LETTERS ARE JUST STAND-INS FOR NUMBERS

`Minuend` = what you started with

`Subtrahend` = what you spent or took away

`Difference` = what is left

IN PLAIN WORDS

1. Start with what you had.
2. Take away what you spent.
3. What remains is the difference.

Try it with real numbers: `$20 paid − $15 lunch = $5 change`

Subtraction check

```
Difference + Subtrahend = Minuend
```

THE LETTERS ARE JUST STAND-INS FOR NUMBERS

`Difference` = what was left

`Subtrahend` = what you spent

`Minuend` = what you started with

IN PLAIN WORDS

1. Add what is left to what you spent.
2. You should land back on what you started with.
3. If not, redo the subtraction.

Try it with real numbers: `$5 change + $15 lunch = $20 ✓`

Worked examples

Example 1 Checking the receipt

Find Dev's total and his change from $20.

SOLUTION

1. Line totals: hoagie 1 × $8 = $8; sodas 2 × $2 = $4; chips 3 × $1 = $3.
2. Total: $8 + $4 + $3 = $15.
3. Change: $20 − $15 = $5.
4. Check: $5 + $15 = $20. ✓

Answer: Total $15; change $5

Example 2 Subtracting with borrowing

Jada saved $1,245 over the summer and spent $389 on sneakers. How much is left?

SOLUTION

1. Ones: 5 − 9 cannot be done; borrow from the tens. 15 − 9 = 6.
2. Tens: 3 (after lending) − 8 cannot be done; borrow from the hundreds. 13 − 8 = 5.
3. Hundreds: 1 (after lending) − 3 cannot be done; borrow from the thousands. 11 − 3 = 8.
4. Thousands: 0. Result 856.
5. Check: 856 + 389 = 1,245. ✓

Answer: $856

Practice exercises 1.2

1. 4,508 + 2,976
2. 10,000 − 3,587
3. A basketball team scored 87, 94, and 79 points in three games. What is the total?
4. An account had $2,350. You withdrew $875. What is the new balance?
5. Estimate 612 + 389 by rounding each to the nearest hundred, then find the exact sum.

RECAP QUIZ 1.2

1. 503 − 278 =

- ○ a. 225
- ○ b. 235
- ○ c. 335
- ○ d. 275

2. How do you check a subtraction answer?

- ○ a. Subtract again
- ○ b. Add the difference to the subtrahend
- ○ c. Multiply the numbers
- ○ d. Round both numbers

3. In 1,999 + 1 = 2,000, the number 2,000 is the:

- ○ a. Addend
- ○ b. Minuend
- ○ c. Sum
- ○ d. Difference

SECTION 1.2 RECAP

- Line up digits by place value before adding or subtracting.

- Check subtraction by adding the difference back to the smaller number.

Answer key with explanations: Section 1.2

PRACTICE EXERCISES

1. 7,484
2. 6,413
3. 260 points
4. $1,475
5. Estimate 600 + 400 = 1,000; exact 1,001

RECAP QUIZ

1. **225**: Check: 225 + 278 = 503.
2. **Add the difference to the subtrahend**: Difference + subtrahend must equal the minuend.
3. **Sum**: The result of addition is the sum.

1.3

Multiplying, Dividing, and the Word-Problem Blueprint

LEARNING OBJECTIVES

After this section you will be able to:

1. Multiply and divide whole numbers, including remainders.
2. Decide what a remainder means in a real situation.

3. Use the four-part blueprint to dissect a word problem.

REAL-LIFE SITUATION

The candy-bar fundraiser

The youth league orders 24 cases of candy bars. Each case holds 36 bars, and each bar sells for $2. The coach needs to know how many bars there are, how much money a sellout brings in, and how many bags of 25 bars she can pack for sellers.

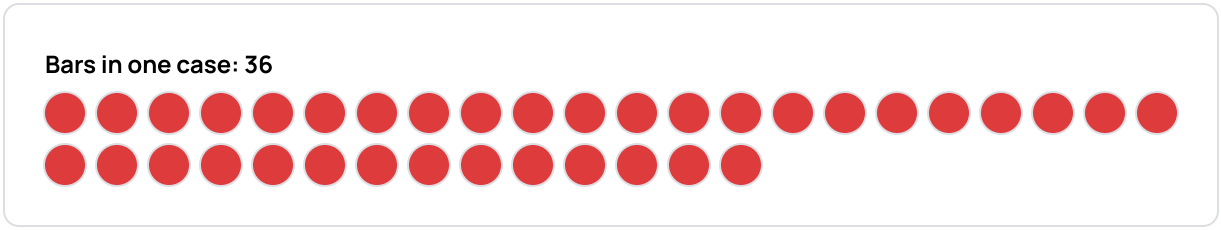


Figure 1.3 One case: 36 bars (6 rows of 6)

The concept

Multiplication is repeated addition. 24 cases of 36 bars means 36 added 24 times, which is 24 × 36.

Division splits a total into equal groups. When the total does not split evenly, the leftover is the remainder. In real life, decide whether the remainder is dropped (only full bags count), rounded up (you need another bus), or reported (14 bars left over).

The Blueprint. Before you calculate, write four things: (1) Facts given, (2) What you are solving for, (3) Steps to take, (4) Key points to watch. This keeps you from using the wrong numbers or answering the wrong question.

Multiplying, Dividing, and the Word-Problem Blueprint: solution map

1. Identify the question

   `How many bars are in 24 cases, and how much does a sellout raise?`

2. Calculation 1

   `Facts: 24 cases, 36 bars per case, $2 per bar.`

3. Calculation 2

   `Solving for: total bars, total dollars.`

4. Calculation 3

   `24 × 36 = 864 bars.`

5. Calculation 4

   `864 × $2 = $1,728.`

6. Interpret and check

   `864 bars; $1,728`

Figure 1.3.V Multiplying, Dividing, and the Word-Problem Blueprint: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

GROSS PROFIT

Revenue minus the direct cost of the items sold, before other expenses.

NET PROFIT

The final profit after every cost is paid — sometimes called the bottom line.

BALANCE

The amount of money in an account at a given moment.

KEY TERMS

Factors
Numbers being multiplied.

Product
The result of multiplication.

Dividend
The number being divided.

Divisor
The number you divide by.

Quotient
The result of division.

Remainder
What is left when a dividend does not divide evenly.

FORMULAS & RULES

Multiplication

```
Factor × Factor = Product
```

THE LETTERS ARE JUST STAND-INS FOR NUMBERS

`Factor` = the numbers being multiplied, like how many groups and how many in each

`Product` = the total

IN PLAIN WORDS

1. Count how many groups you have.
2. Count how many are in each group.
3. Multiply them to get the total.

Try it with real numbers: `24 cases × 36 bars per case = 864 bars`

Division check

```
Divisor × Quotient + Remainder = Dividend
```

THE LETTERS ARE JUST STAND-INS FOR NUMBERS

`Divisor` = the size of each group or bag

`Quotient` = how many full groups you made

`Remainder` = the loose leftovers

`Dividend` = the total you started with

IN PLAIN WORDS

1. Multiply full groups by the group size.
2. Add the leftovers.
3. You should get the starting total back.

Try it with real numbers: `25 bars per bag × 34 bags + 14 left over = 864 bars ✓`

Worked examples

Example 1 Total bars and sales

How many bars are in 24 cases, and how much does a sellout raise?

SOLUTION

1 Facts: 24 cases, 36 bars per case, $2 per bar.

2 Solving for: total bars, total dollars.

3 24 × 36 = 864 bars.

4 864 × $2 = $1,728.

Answer: 864 bars; $1,728

Example 2 Packing bags

Bars are packed in bags of 25 for each seller. How many full bags, and how many bars are left?

SOLUTION

1 864 ÷ 25: 25 × 34 = 850, and 864 − 850 = 14.

2 Quotient 34, remainder 14.

3 Check: 25 × 34 + 14 = 864. ✓

4 Only full bags go to sellers, so the remainder is reported as leftover.

Answer: 34 full bags, 14 bars left over

Example 3 Blueprint: paycheck after rent

Marcus earns $15 per hour and works 32 hours per week. His share of rent is $1,100 per month. Using 4 weeks per month, how much is left after rent?

SOLUTION

1 Facts: $15/hour, 32 hours/week, 4 weeks, rent $1,100.

2 Solving for: money left after rent for the month.

3 Steps: weekly pay = 15 × 32 = $480. Monthly = 480 × 4 = $1,920. Left = 1,920 − 1,100 = $820.

4 Key point: this ignores taxes, so real take-home will be lower.

> **Answer:** $820 (before taxes)

Practice exercises 1.3

1. 125 × 48
2. 7,236 ÷ 6
3. 400 students ride buses that each hold 52. How many buses are needed?
4. Three concert tickets cost $45 each, plus one $20 parking pass. What is the total?
5. Write the blueprint (facts, solving for, steps, key points) and solve: $1,000 shared equally by 8 people.

RECAP QUIZ 1.3

1. 56 × 10 =

- ○ a. 56
- ○ b. 506
- ○ c. 560
- ○ d. 5,600

2. 95 ÷ 4 =

- ○ a. 23 R3
- ○ b. 24 R1
- ○ c. 23 R1
- ○ d. 22 R7

3. The first step of the blueprint is:

- ○ a. Calculate
- ○ b. Identify the facts given

○ c. Round the answer

○ d. Guess and check

SECTION 1.3 RECAP

- Estimate first, then compute, then compare — the estimate catches wild errors.
- The word-problem blueprint: find what is known, what is asked, and the operation that connects them.

Answer key with explanations: Section 1.3

PRACTICE EXERCISES

1. 6,000
2. 1,206
3. 400 ÷ 52 = 7 R36, so 8 buses (the remainder means one more bus)
4. 3 × 45 + 20 = $155
5. Facts: $1,000, 8 people. Solving for: each share. Steps: 1,000 ÷ 8. Key point: equal shares. Answer $125 each

RECAP QUIZ

1. **560**: Multiplying by 10 attaches one zero.
2. **23 R3**: 4 × 23 = 92, and 95 − 92 = 3.
3. **Identify the facts given**: Know your facts before choosing operations.

CHAPTER 2

Fractions

Fractions describe parts of something. You use them when you split a bill, follow a recipe, or save part of every paycheck. This chapter covers what fractions mean, how to rewrite them, and how to add, subtract, multiply, and divide them.

2.1

Types of Fractions and Conversions

LEARNING OBJECTIVES

After this section you will be able to:

1. Name the numerator and denominator.
2. Classify fractions as proper, improper, or mixed numbers.
3. Convert between improper fractions and mixed numbers.

REAL-LIFE SITUATION

What's in the candy bag?

A bag of 12 hard candies has 5 cherry, 4 lemon, and 3 lime. What fraction of the bag is each flavor?



Figure 2.1 A 12-piece candy bag by flavor

The concept

A fraction has a numerator (top) that counts the parts you are talking about and a denominator (bottom) that counts the equal parts in one whole. Cherry candies are 5 of the 12 pieces, so cherry is 5/12 of the bag.

A proper fraction is less than 1 (numerator smaller than denominator). An improper fraction is 1 or more (numerator equal to or larger than denominator). A mixed number combines a whole number and a proper fraction, like 3 2/5.

To turn an improper fraction into a mixed number, divide: the quotient is the whole number and the remainder goes over the original denominator. To go back, multiply the whole number by the denominator and add the numerator.

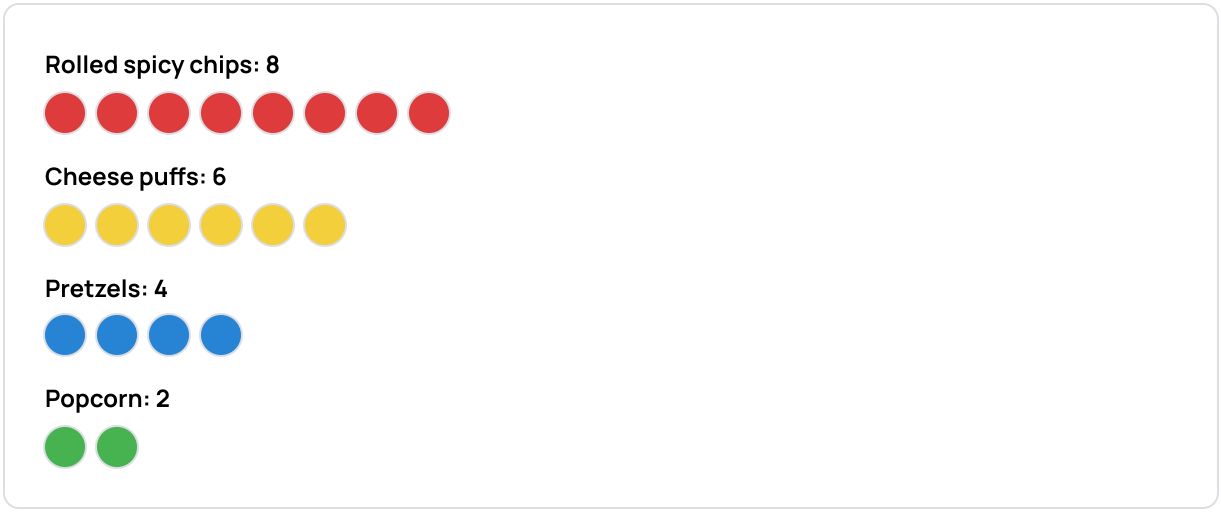


Figure 2.1b A 20-bag snack variety pack (generic flavors, hypothetical)

Fraction → decimal → percent



`3/8 = 0.375 = 37.5%`

1. Count 3 shaded parts out of 8 equal parts: 3/8.
2. Divide numerator by denominator: 3 ÷ 8 = 0.375.
3. Multiply the decimal by 100: 37.5%. The representation uses equal-size parts.

Original WealthWiseGuys diagram · Hypothetical teaching quantities.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

INVENTORY

The stock of goods a business holds ready to sell.

DISCOUNT

An amount taken off a regular price.

MARKUP

The amount added to a cost to set a selling price; the markup rate uses cost as its base.

KEY TERMS

Numerator
The top number; how many parts are counted.

Denominator
The bottom number; how many equal parts make one whole.

Proper fraction
Numerator less than denominator; value less than 1.

Improper fraction
Numerator equal to or greater than denominator; value 1 or more.

Mixed number
A whole number plus a proper fraction.

FORMULAS & RULES

Improper → mixed

```
a/b = q + r/b
```

THE LETTERS ARE JUST STAND-INS FOR NUMBERS

- **a** = the top number (how many pieces you have)
- **b** = the bottom number (how many pieces make one whole)
- **q** = how many full wholes you can make
- **r** = the pieces left over

IN PLAIN WORDS

1. Divide the top by the bottom.

2. The answer is your whole number.
3. The leftover goes on top; the bottom stays the same.

Try it with real numbers: `7/3 → 7 ÷ 3 = 2 with 1 left over → 2 1/3`

Mixed → improper

```
W n/d = (W × d + n)/d
```

THE LETTERS ARE JUST STAND-INS FOR NUMBERS

W = the whole number in front

n = the top number of the fraction

d = the bottom number of the fraction

IN PLAIN WORDS

1. Multiply the whole number by the bottom.
2. Add the top number.
3. Put that answer over the same bottom number.

Try it with real numbers: `2 1/3 → 2 × 3 = 6, then 6 + 1 = 7 → 7/3`

Worked examples

Example 1 Flavor fractions

What fraction of the bag is cherry? Lemon?

SOLUTION

1. Cherry: 5 of 12 → 5/12.
2. Lemon: 4 of 12 → 4/12, which simplifies to 1/3 (divide both by 4).

Answer: Cherry 5/12; lemon 4/12 = 1/3

Example 2 The snack variety pack

In Figure 2.1b, what fraction of the 20 bags are rolled spicy chips? What fraction are not chips?

SOLUTION

1. Spicy chips: 8 of 20 → 8/20.
2. GCD of 8 and 20 is 4: 8/20 = 2/5.
3. Not chips: 20 − 8 = 12 bags → 12/20 = 3/5.
4. Check: 2/5 + 3/5 = 5/5 = 1 whole pack. ✓

> **Answer:** 2/5 spicy chips; 3/5 other snacks

Example 3 Improper to mixed

Convert 17/5 to a mixed number.

SOLUTION

1. 17 ÷ 5 = 3 remainder 2.
2. Whole number 3, fraction 2/5.

> **Answer:** 3 2/5

Example 4 Mixed to improper

Convert 4 3/8 to an improper fraction.

SOLUTION

1. $4 \times 8 = 32$.
2. $32 + 3 = 35$.
3. Keep the denominator 8.

Answer: 35/8

Practice exercises 2.1

1. Convert 22/7 to a mixed number.
2. Convert 2 5/6 to an improper fraction.
3. What fraction of the candy bag is lime, in lowest terms?
4. Classify 9/9.
5. Convert 13/4 to a mixed number.
6. In the 20-bag snack pack, what fraction are pretzels or popcorn, in lowest terms?

RECAP QUIZ 2.1

1. Which is a proper fraction?

- ○ a. 9/8
- ○ b. 7/8
- ○ c. 8/8
- ○ d. 1 1/8

2. 3 1/2 as an improper fraction:

- ○ a. 4/2
- ○ b. 6/2
- ○ c. 7/2
- ○ d. 5/2

3. In 5/12, the numerator is:

- ○ a. 5
- ○ b. 12
- ○ c. 17
- ○ d. 7

SECTION 2.1 RECAP

- A fraction is part of a whole: the numerator counts parts, the denominator names their size.
- Mixed numbers and improper fractions convert both ways using multiplication and addition.

Answer key with explanations: Section 2.1

PRACTICE EXERCISES

1. 3 1/7
2. 17/6
3. 3/12 = 1/4
4. Improper (it equals 1)
5. 3 1/4
6. 6/20 = 3/10

RECAP QUIZ

1. **7/8**: 7 is less than 8.
2. **7/2**: 3 × 2 + 1 = 7, over 2.
3. **5**: The numerator is on top.

2.2

Equivalent Fractions, Lowest Terms, and Adding and Subtracting

LEARNING OBJECTIVES

After this section you will be able to:

1. Reduce fractions to lowest terms using the greatest common divisor.
2. Find the least common denominator (LCD).
3. Add and subtract fractions and mixed numbers.

REAL-LIFE SITUATION

Same pizza, different cuts

Three equally sized pizzas are cut into equal slices: one into 2, one into 4, and one into 8. Eating 1 of 2, 2 of 4, or 4 of 8 slices means you ate the same amount.

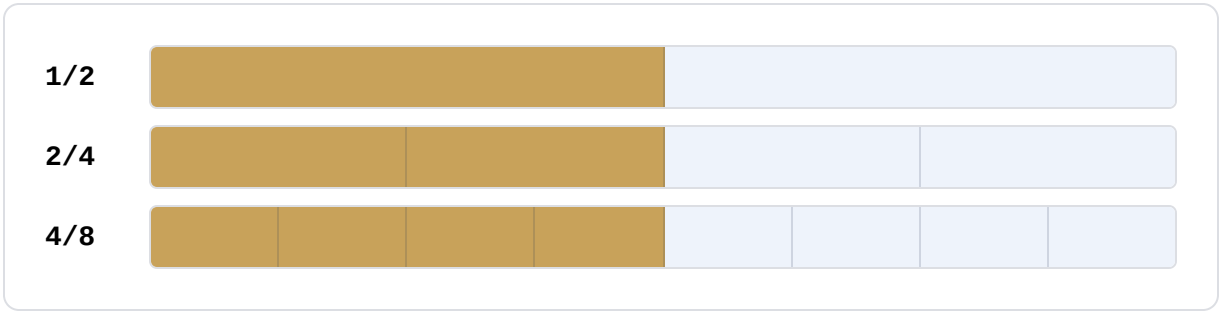


Figure 2.2 Equivalent fractions: 1/2 = 2/4 = 4/8

The concept

Equivalent fractions name the same amount. Multiply or divide the numerator and denominator by the same nonzero number and the value does not change.

A fraction is in lowest terms when the only number dividing both the top and bottom is 1. Divide both by their greatest common divisor (GCD) to get there in one step.

You can only add or subtract fractions that share a denominator. Find the LCD (the smallest number both denominators divide into), raise each fraction to it, then add or subtract the numerators. With mixed numbers, borrow 1 whole from the whole number when the top fraction is too small.

Equivalent Fractions, Lowest Terms, and Adding and Subtracting: solution map

1. Identify the question
   `Reduce 24/36 to lowest terms.`
2. Calculation 1
   `Divisors of 24 and 36 include 2, 3, 4, 6, 12. The GCD is 12.`
3. Calculation 2
   `24 ÷ 12 = 2; 36 ÷ 12 = 3.`
4. Interpret and check
   `2/3`

Figure 2.2.V Equivalent Fractions, Lowest Terms, and Adding and Subtracting: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

COST

The amount paid to make or buy an item before it is sold.

SELLING PRICE

The price a customer pays for an item or service.

REVENUE

The total money a business brings in from sales, before any costs are paid.

KEY TERMS

Equivalent fractions
Different fractions with the same value.

Greatest common divisor (GCD)
The largest number that divides two numbers evenly.

Lowest terms
A fraction whose numerator and denominator share no divisor except 1.

Least common denominator (LCD)
The smallest number every denominator divides into evenly.

FORMULAS & RULES

Reduce

$$a/b = (a \div \text{GCD})/(b \div \text{GCD})$$

Raise

$$a/b = (a \times k)/(b \times k)$$

Like denominators

$$a/d \pm c/d = (a \pm c)/d$$

Worked examples

Example 1 Reduce

Reduce 24/36 to lowest terms.

SOLUTION

1. Divisors of 24 and 36 include 2, 3, 4, 6, 12. The GCD is 12.
2. 24 ÷ 12 = 2; 36 ÷ 12 = 3.

> **Answer:** 2/3

Example 2 Add with an LCD

1/4 + 1/6

SOLUTION

1. Multiples of 6: 6, 12. 4 divides 12, so LCD = 12.
2. 1/4 = 3/12; 1/6 = 2/12.
3. 3/12 + 2/12 = 5/12.

> **Answer:** 5/12

Example 3 Subtract mixed numbers with borrowing

3 1/2 − 1 3/4

SOLUTION

1. LCD 4: 3 1/2 = 3 2/4.
2. 2/4 is smaller than 3/4, so borrow 1 whole: 3 2/4 = 2 6/4.

3 $2\,6/4 - 1\,3/4 = 1\,3/4$.

Answer: $1\,3/4$

Practice exercises 2.2

1. Reduce $18/42$.
2. Raise $3/5$ to a fraction with denominator 40.
3. Find the LCD of $1/3$ and $1/8$.
4. $2/3 + 1/4$
5. $5\,1/6 - 2\,2/3$

RECAP QUIZ 2.2

1. GCD of 16 and 40:

- a. 4
- b. 8
- c. 2
- d. 16

2. $1/2 + 1/3 =$

- a. $2/5$
- b. $5/6$
- c. $1/6$
- d. $2/6$

3. Which is equivalent to 3/4?

- ○ a. 6/12
- ○ b. 9/12
- ○ c. 4/3
- ○ d. 3/8

SECTION 2.2 RECAP

- Equivalent fractions multiply (or divide) numerator and denominator by the same number.
- Add and subtract only when denominators match — find a common denominator first.

Answer key with explanations: Section 2.2

PRACTICE EXERCISES

1. 3/7 (GCD 6)
2. 24/40
3. 24
4. 8/12 + 3/12 = 11/12
5. 31/6 − 16/6 = 15/6 = 2 1/2

RECAP QUIZ

1. **8**: 8 divides both; 16 does not divide 40.
2. **5/6**: 3/6 + 2/6 = 5/6.
3. **9/12**: Multiply top and bottom by 3.

2.3

Multiplying and Dividing Fractions

LEARNING OBJECTIVES

After this section you will be able to:

1. Multiply fractions and mixed numbers, canceling when possible.
2. Divide fractions using the reciprocal.
3. Find a fraction of a dollar amount.

REAL-LIFE SITUATION

Scaling a cookie recipe for a bake sale

Aaliyah's cookie recipe uses 3/4 cup of sugar per batch. She is making 2 1/2 batches for a bake sale and has 6 yards of ribbon to cut into 3/4-yard pieces for gift bags.



Figure 2.3 Recipe card (one batch)

The concept

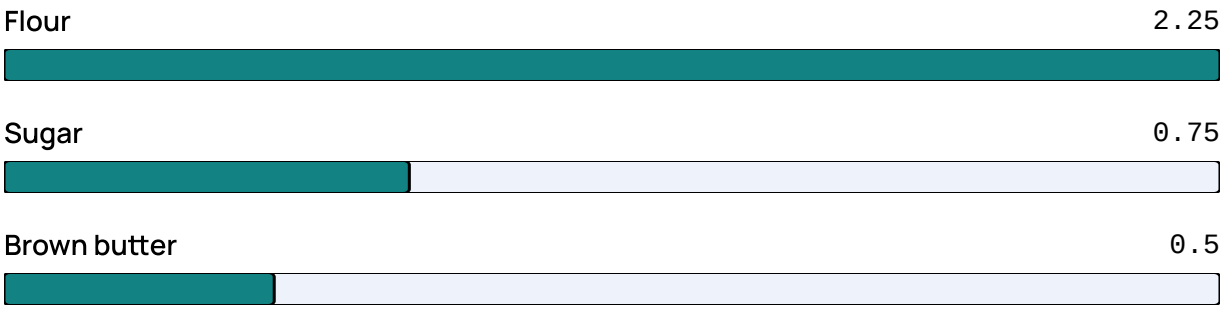
To multiply fractions, multiply the numerators and multiply the denominators. Change any mixed numbers to improper fractions first. You

can cancel a factor that appears in any numerator and any denominator before multiplying.

To divide, keep the first fraction, change division to multiplication, and flip the second fraction (use its reciprocal).

The word "of" in a money problem usually means multiply: 2/5 of $450 is 2/5 × 450.

Recipe quantities in cups



Bars share a zero baseline.

Figure 2.3.V Recipe quantities in cups. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

PROFIT
What is left when expenses are subtracted from revenue.

SALES TAX
A percentage added by the government to the price of many purchases.

BREAK-EVEN POINT
The sales level where revenue exactly equals total cost, so there is no profit and no loss.

KEY TERMS

Reciprocal
A fraction flipped upside down; a number times its reciprocal equals 1.

Canceling
Dividing a numerator and a denominator by a common factor before multiplying.

FORMULAS & RULES

Multiply

```
a/b × c/d = (a × c)/(b × d)
```

Divide

```
a/b ÷ c/d = a/b × d/c
```

Worked examples

Example 1 Scaling sugar

How much sugar for 2 1/2 batches?

SOLUTION

1. 2 1/2 = 5/2.
2. 3/4 × 5/2 = 15/8.
3. 15/8 = 1 7/8.

> **Answer:** 1 7/8 cups

Example 2 Cutting ribbon

How many 3/4-yard pieces come from 6 yards?

SOLUTION

1. 6 ÷ 3/4 = 6/1 × 4/3.
2. Cancel 3: 2/1 × 4/1 = 8.

> **Answer:** 8 pieces

Example 3 A fraction of a paycheck

Jordan saves 2/5 of a $450 paycheck. How much is saved?

SOLUTION

1. 2/5 × 450 = 900/5.
2. 900 ÷ 5 = 180.

> **Answer:** $180

Practice exercises 2.3

1. 2/3 × 9/10
2. 4 ÷ 2/3
3. 1 1/2 × 1 1/3
4. 7/8 ÷ 7/16
5. 3/8 of 64 students are in band. How many?

RECAP QUIZ 2.3

1. The reciprocal of 5/9 is:

- ○ a. 5/9
- ○ b. 9/5
- ○ c. −5/9
- ○ d. 1/9

2. 1/2 ÷ 1/4 =

- ○ a. 1/8
- ○ b. 2
- ○ c. 1/2
- ○ d. 4

3. 3/4 of $20 is:

- ○ a. $5
- ○ b. $12
- ○ c. $15
- ○ d. $16

SECTION 2.3 RECAP

- To multiply fractions, multiply across; to divide, multiply by the reciprocal.
- Simplify before multiplying to keep the numbers small.

Answer key with explanations: Section 2.3

PRACTICE EXERCISES

1. 18/30 = 3/5
2. 4 × 3/2 = 6
3. 3/2 × 4/3 = 2
4. 7/8 × 16/7 = 2

5. 24 students

RECAP QUIZ

1. **9/5**: Flip numerator and denominator.
2. **2**: 1/2 × 4/1 = 2.
3. **$15**: 20 ÷ 4 × 3 = 15.

CHAPTER 3

Decimals

Money is written in decimals. This chapter shows how to read and round decimals, convert them to and from fractions, compute with them on a real receipt, and compare prices with unit pricing.

3.1

Reading, Rounding, and Converting Decimals

LEARNING OBJECTIVES

After this section you will be able to:

1. Name decimal places: tenths, hundredths, thousandths.
2. Round decimals, including to the nearest cent.
3. Convert fractions to decimals and decimals to fractions.

REAL-LIFE SITUATION

Why gas prices have three decimals

A gas station posts regular at $3.479 per gallon. Your card is charged in cents, so the station rounds the final total to two decimal places.



Figure 3.1 Posted fuel prices (dollars per gallon)

The concept

Places to the right of the decimal point are tenths, hundredths, thousandths, and ten-thousandths. In money, the hundredths place is cents.

Rounding decimals follows the same rule as whole numbers: look one place to the right of the rounding place. Digits after the rounding place are dropped (not replaced with zeros).

A fraction is a division problem: numerator ÷ denominator. To change a decimal to a fraction, write the digits over 10, 100, or 1,000 (one zero per decimal place) and reduce.

Reading, Rounding, and Converting Decimals: solution map

1. Identify the question

   `Round $3.479 to the nearest cent.`

2. Calculation 1

   `Cents are the hundredths place: 7.`

3. Calculation 2

   `The digit to the right is 9, so round up: 3.48.`

4. Interpret and check

   `$3.48`

Figure 3.1.V Reading, Rounding, and Converting Decimals: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

SALES TAX
A percentage added by the government to the price of many purchases.

DISCOUNT
An amount taken off a regular price.

COST
The amount paid to make or buy an item before it is sold.

KEY TERMS

Decimal point
The dot separating whole units from parts of a unit.

Tenths, hundredths, thousandths
The first, second, and third places right of the decimal point.

Repeating decimal
A decimal whose digits repeat forever, such as 0.666…

FORMULAS & RULES

Fraction → decimal

```
a/b = a ÷ b
```

Decimal → fraction

```
0.65 = 65/100 = 13/20
```

Worked examples

Example 1 Nearest cent

Round $3.479 to the nearest cent.

SOLUTION

1. Cents are the hundredths place: 7.
2. The digit to the right is 9, so round up: 3.48.

Answer: $3.48

Example 2 Fraction to decimal

Write 3/8 as a decimal.

SOLUTION

1. 3 ÷ 8: 8 into 30 is 3 (24, remainder 6); into 60 is 7 (56, remainder 4); into 40 is 5.
2. Result 0.375.

> **Answer:** 0.375

Example 3 Decimal to fraction

Write 0.65 as a fraction in lowest terms.

SOLUTION

1. Two decimal places → over 100: 65/100.
2. GCD of 65 and 100 is 5: 13/20.

> **Answer:** 13/20

Practice exercises 3.1

1. Round 12.3462 to the nearest hundredth.
2. Write 7/20 as a decimal.
3. Write 0.125 as a fraction in lowest terms.
4. Write 2/3 as a decimal rounded to the nearest hundredth.
5. Write 0.07 in words.

RECAP QUIZ 3.1

1. Round 4.0951 to the nearest tenth.

- ○ a. 4.0
- ○ b. 4.1
- ○ c. 4.09
- ○ d. 4.2

2. 1/4 as a decimal:

- ○ a. 0.4
- ○ b. 0.14
- ○ c. 0.25
- ○ d. 0.75

3. Which is largest?

- ○ a. 0.45
- ○ b. 0.499
- ○ c. 0.5
- ○ d. 0.05

SECTION 3.1 RECAP

- Each decimal place is a power of ten: tenths, hundredths, thousandths.
- To compare decimals, line up the decimal points and compare place by place.

Answer key with explanations: Section 3.1

PRACTICE EXERCISES

1. 12.35
2. 0.35
3. 1/8
4. 0.67

5. Seven hundredths

RECAP QUIZ

1. **4.1**: Tenths digit 0, next digit 9 → round up to 4.1.
2. **0.25**: 1 ÷ 4 = 0.25.
3. **0.5**: Compare tenths first: 5 tenths beats 4 tenths.

3.2

Adding, Subtracting, Multiplying, and Dividing Decimals

LEARNING OBJECTIVES

After this section you will be able to:

1. Add and subtract decimals by lining up decimal points.
2. Multiply decimals by counting decimal places.
3. Divide by a decimal by moving the decimal point.

REAL-LIFE SITUATION

Back-to-school receipt

Kayla buys school supplies at a store that charges 6% sales tax and pays with $30. She wants to verify the subtotal, tax, total, and change.

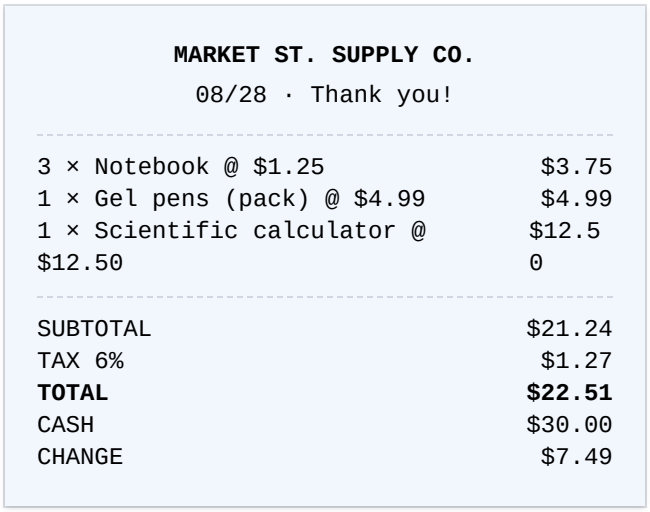


Figure 3.2 School-supply receipt with 6% sales tax

The concept

Adding and subtracting: line up the decimal points, fill empty places with zeros, and work like whole numbers. The decimal point in the answer goes straight down.

Multiplying: multiply as if there were no decimal points, then count the total decimal places in both factors. The product gets that many decimal places.

Dividing: if the divisor is a decimal, move its decimal point right until it is a whole number, and move the dividend's decimal point the same number of places. Then divide normally.

Adding, Subtracting, Multiplying, and Dividing Decimals: solution map

1. Identify the question
   `Find the subtotal, tax, total, and change from $30.`
2. Calculation 1
   `Notebooks: 3 × 1.25 = 3.75.`
3. Calculation 2
   `Subtotal: 3.75 + 4.99 + 12.50 = 21.24.`
4. Calculation 3
   `Tax: 21.24 × 0.06 = 1.2744 → $1.27.`
5. Calculation 4
   `Total: 21.24 + 1.27 = 22.51.`
6. Calculation 5
   `Change: 30.00 − 22.51 = 7.49.`
7. Interpret and check
   `Subtotal $21.24, tax $1.27, total $22.51, change $7.49`

Figure 3.2.V Adding, Subtracting, Multiplying, and Dividing Decimals: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

SELLING PRICE

The price a customer pays for an item or service.

REVENUE

The total money a business brings in from sales, before any costs are paid.

EXPENSE

Money a business spends to run, such as rent, supplies, or wages.

KEY TERMS

Subtotal
Total of items before tax.

Sales tax
A percent of the subtotal added by the state or city.

FORMULAS & RULES

Sales tax

```
Tax = Subtotal × rate (as a decimal)
```

Total

```
Total = Subtotal + Tax
```

Decimal places in a product

```
places(a × b) = places(a) + places(b)
```

Worked examples

Example 1 Verifying the receipt

Find the subtotal, tax, total, and change from $30.

SOLUTION

1. Notebooks: 3 × 1.25 = 3.75.
2. Subtotal: 3.75 + 4.99 + 12.50 = 21.24.
3. Tax: 21.24 × 0.06 = 1.2744 → $1.27.
4. Total: 21.24 + 1.27 = 22.51.
5. Change: 30.00 − 22.51 = 7.49.

> **Answer:** Subtotal $21.24, tax $1.27, total $22.51, change $7.49

Example 2 Multiplying

4.6×0.35

SOLUTION

1. 46 × 35 = 1,610.
2. Decimal places: 1 + 2 = 3.
3. 1.610 = 1.61.

> **Answer:** 1.61

Example 3 Dividing by a decimal

A $18.75 ride is shared by distance: each unit costs $2.50. How many units? $(18.75 \div 2.5)$

SOLUTION

1. Move both decimals one place: 187.5 ÷ 25.
2. 25 × 7 = 175; remainder 12.5; 12.5 ÷ 25 = 0.5.
3. 7.5.

Answer: 7.5

Practice exercises 3.2

1. 14.6 + 3.75 + 0.08
2. 50 − 17.86
3. 2.4 × 1.5
4. 9.6 ÷ 0.12
5. A 12-pack of water costs $6.60. What is the price per bottle?

RECAP QUIZ 3.2

1. 0.3 × 0.2 =

- ○ a. 0.6
- ○ b. 0.06
- ○ c. 0.006
- ○ d. 6

2. 5 − 0.25 =

- ○ a. 4.75
- ○ b. 4.25
- ○ c. 5.25
- ○ d. 0.25

3. 1.2 ÷ 0.4 =

- ○ a. 0.3
- ○ b. 3

○ c. 30

○ d. 0.03

SECTION 3.2 RECAP

- Line up the decimal points to add or subtract decimals.
- When multiplying, count the total decimal places; when dividing, move the point to make a whole-number divisor.

Answer key with explanations: Section 3.2

PRACTICE EXERCISES

1. 18.43
2. 32.14
3. 3.6
4. 80
5. $0.55

RECAP QUIZ

1. **0.06**: 3 × 2 = 6 with two decimal places.
2. **4.75**: 5.00 − 0.25 = 4.75.
3. **3**: 12 ÷ 4 = 3.

3.3

Powers of Ten and Unit Pricing

LEARNING OBJECTIVES

After this section you will be able to:

1. Multiply and divide decimals by 10, 100, and 1,000 mentally.
2. Compute unit prices.
3. Choose the better buy.

REAL-LIFE SITUATION

Which cereal is the better deal?

Two boxes of the same cereal sit side by side. The bigger box costs more, but is it cheaper per ounce?

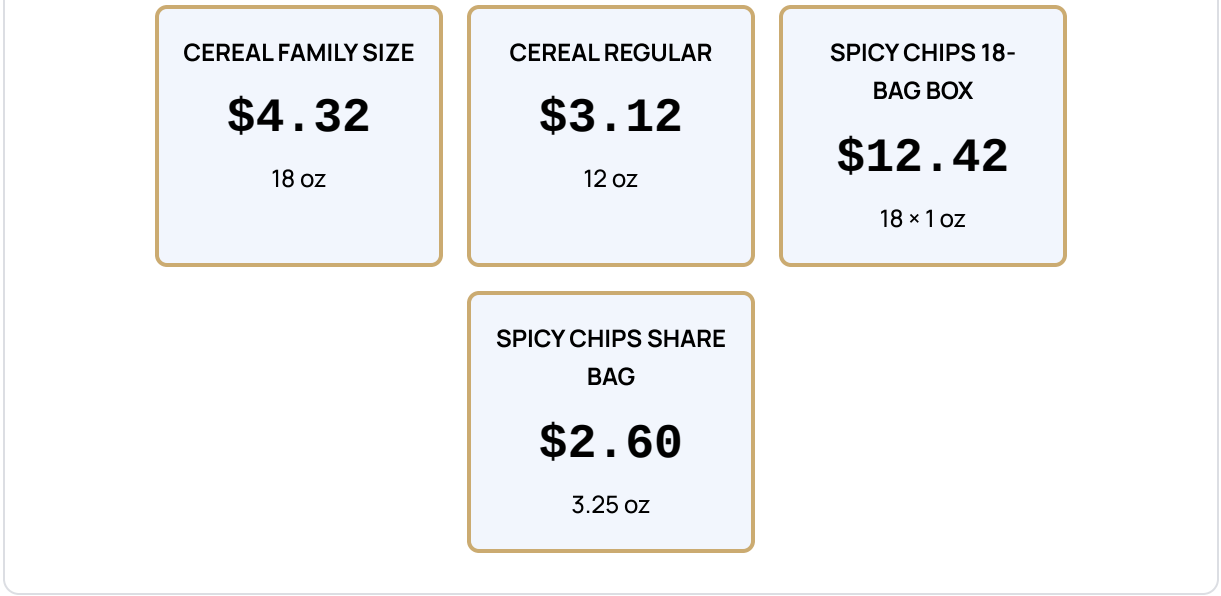


Figure 3.3 Shelf tags: cereal and snack packs (hypothetical prices)

The concept

Multiplying by 10, 100, or 1,000 moves the decimal point right 1, 2, or 3 places. Dividing moves it left. Add zeros as placeholders when needed.

Unit price is the cost of one unit (one ounce, one pound, one item). It lets you compare different package sizes fairly. Stores often print unit prices on shelf tags; check them.

Powers of Ten and Unit Pricing: solution map

1. Identify the question
   `3.45 × 100 and 72.5 ÷ 1,000`
2. Calculation 1
   `× 100: move right two places → 345.`
3. Calculation 2
   `÷ 1,000: move left three places → 0.0725.`
4. Interpret and check
   `345; 0.0725`

Figure 3.3.V Powers of Ten and Unit Pricing: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

BALANCE

The amount of money in an account at a given moment.

INVOICE

A document listing goods or services sold and the amount owed.

PAYMENT TERMS

The rules in an agreement about when a bill must be paid and any discounts for paying early.

KEY TERMS

Unit price
Price for one unit of measure.

Better buy
The option with the lower unit price, assuming you will use it all.

FORMULAS & RULES

Unit price

```
Unit price = Total price ÷ Number of units
```

× 10^n

```
Move the decimal n places right
```

÷ 10^n

```
Move the decimal n places left
```

Worked examples

Example 1 — Powers of ten

3.45×100 and $72.5 \div 1{,}000$

SOLUTION

1. × 100: move right two places → 345.
2. ÷ 1,000: move left three places → 0.0725.

> **Answer:** 345; 0.0725

Example 2 — Snack-pack unit pricing

A box of 18 one-ounce bags of rolled spicy chips costs $12.42. A single 3.25 oz share bag costs $2.60 (hypothetical prices). Which costs less per ounce?

SOLUTION

1. Box: 18 bags × 1 oz = 18 oz. 12.42 ÷ 18 = 0.69 → $0.69/oz.
2. Share bag: 2.60 ÷ 3.25 = 0.80 → $0.80/oz.
3. 0.69 < 0.80, so the box is cheaper per ounce — if you will actually eat 18 bags before they go stale.

> **Answer:** The 18-bag box, $0.69/oz vs $0.80/oz

Example 3 — Cereal comparison

Which box has the lower price per ounce?

SOLUTION

1. Family: $4.32 \div 18 = 0.24 \rightarrow \$0.24/\text{oz}$.
2. Regular: $3.12 \div 12 = 0.26 \rightarrow \$0.26/\text{oz}$.
3. $0.24 < 0.26$.

Answer: The 18 oz family size is the better buy

Practice exercises 3.3

1. $0.062 \times 1{,}000$
2. $485 \div 100$
3. An 8 lb bag of rice costs $11.92. What is the price per pound?
4. Which is the better buy: 6 juice boxes for $4.50 or 10 for $7.20?
5. Printing costs $0.035 per flyer. What do 1,000 flyers cost?

RECAP QUIZ 3.3

1. 5.6 ÷ 10 =

- ○ a. 56
- ○ b. 0.56
- ○ c. 0.056
- ○ d. 5.06

2. A 24 oz bottle costs $6.00. Unit price:

- ○ a. $0.25/oz
- ○ b. $4.00/oz
- ○ c. $0.30/oz
- ○ d. $0.20/oz

3. $0.9 \times 100 =$

- ○ a. 9
- ○ b. 0.009
- ○ c. 90
- ○ d. 900

SECTION 3.3 RECAP

- Multiplying by a power of ten moves the decimal point one place per zero.
- Unit price = total price ÷ quantity — the fair way to compare package sizes.

Answer key with explanations: Section 3.3

PRACTICE EXERCISES

1. 62
2. 4.85
3. $1.49
4. 6 for $0.75 each vs 10 for $0.72 each → the 10-pack
5. $35

RECAP QUIZ

1. **0.56**: Move the decimal one place left.
2. **$0.25/oz**: 6 ÷ 24 = 0.25.
3. **90**: Move the decimal two places right.

3.4

Decimals and Percents in Online Shopping

LEARNING OBJECTIVES

After this section you will be able to:

1. Convert between percents and decimals.
2. Find a sale price from a percent-off tag.
3. Compute an online cart total with a promo code and sales tax.

REAL-LIFE SITUATION

Checkout time

Maya fills an online cart with earbuds, two phone cases, and a charging cable. A promo code takes 20% off the cart, and 6% sales tax is added at checkout. Before she taps "Place order" she wants to know what she will really pay. The store and prices are hypothetical.

Your cart (4 items)

Wireless earbuds × 1	$79.99
Phone case × 2	$29.00
USB-C charging cable × 1	$9.99
Subtotal	$118.98
Promo 20% off	−$23.80
Est. tax 6%	$5.71
Order total	**$100.89**

Hypothetical store and prices.

Figure 3.4 Online shopping cart with promo code (hypothetical)

The concept

Percent means "per hundred." To change a percent to a decimal, drop the % sign and move the decimal point two places left: 20% = 0.20, 6% = 0.06, 7.5% = 0.075.

A percent-off sale removes that percent of the original price. You can find the discount and subtract, or multiply by what is left: 25% off means you pay 75%, or 0.75 of the price.

At checkout, stores usually apply the promo code first and then charge sales tax on the discounted amount. Round money to the nearest cent at each step shown on the receipt.



Figure 3.5 Sale tag on a hoodie (hypothetical)

Decimals and Percents in Online Shopping: solution map

1. Identify the question
   `A $60.00 hoodie is 25% off. What is the sale price?`
2. Calculation 1
   `25% = 0.25.`
3. Calculation 2
   `Discount: 60.00 × 0.25 = 15.00.`
4. Calculation 3
   `Sale price: 60.00 − 15.00 = 45.00.`
5. Calculation 4
   `Shortcut check: 60.00 × 0.75 = 45.00. ✓`
6. Interpret and check
   `$45.00`

Figure 3.4.V Decimals and Percents in Online Shopping: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

CHECKING ACCOUNT

A bank account for everyday spending, usually with a debit card and checks.

DEPOSIT

Money placed into a bank account.

WITHDRAWAL

Money taken out of a bank account.

KEY TERMS

Percent
A ratio out of 100.

Discount
The dollar amount taken off a price.

Sale price
Original price minus discount.

Promo code
A code that applies a discount to an online order.

FORMULAS & RULES

Percent → decimal

```
p% = p ÷ 100
```

Discount

```
Discount = Price × Rate
```

Sale price

```
Sale price = Price × (1 − Rate)
```

Cart total

```
Total = (Subtotal − Discount) × (1 + Tax rate)
```

Worked examples

Example 1 The sale tag

A $60.00 hoodie is 25% off. What is the sale price?

SOLUTION

1. 25% = 0.25.
2. Discount: 60.00 × 0.25 = 15.00.
3. Sale price: 60.00 − 15.00 = 45.00.
4. Shortcut check: 60.00 × 0.75 = 45.00. ✓

> **Answer:** $45.00

Example 2 Maya's cart

Find the order total in Figure 3.4.

SOLUTION

1. Subtotal: 79.99 + 2 × 14.50 + 9.99 = 79.99 + 29.00 + 9.99 = 118.98.
2. Promo: 118.98 × 0.20 = 23.796 → $23.80.
3. After discount: 118.98 − 23.80 = 95.18.

4 Tax: 95.18 × 0.06 = 5.7108 → $5.71.

5 Total: 95.18 + 5.71 = 100.89.

Answer: $100.89

Practice exercises 3.4

1. Write 8.5% as a decimal.
2. Sneakers priced $120.00 are 15% off. Find the sale price.
3. A $49.99 game is 40% off. Find the discount, rounded to the nearest cent.
4. Cart subtotal $64.00, promo 10% off, then 6% tax. Find the total.
5. Which is cheaper: $35 off a $140 jacket, or 30% off the same jacket?

RECAP QUIZ 3.4

1. 6% as a decimal is:

- ○ a. 0.6
- ○ b. 0.06
- ○ c. 6.0
- ○ d. 0.006

2. 25% off $80 leaves a price of:

- ○ a. $20
- ○ b. $55
- ○ c. $60
- ○ d. $75

3. At checkout, sales tax is usually charged on:

- ○ a. The original subtotal
- ○ b. The discounted amount
- ○ c. Only the discount
- ○ d. Shipping only

SECTION 3.4 RECAP

- A percent discount comes off the price; sales tax is added on top of the discounted amount.
- Work in steps: subtotal → discount → taxed amount → total.

Answer key with explanations: Section 3.4

PRACTICE EXERCISES

1. 0.085
2. 120 × 0.85 = $102.00
3. 49.99 × 0.40 = 19.996 → $20.00
4. 64 − 6.40 = 57.60; tax 3.456 → 3.46; total $61.06
5. $35 off → $105; 30% off → $98. The 30% deal is cheaper

RECAP QUIZ

1. **0.06**: Move the decimal two places left.
2. **$60**: 80 × 0.75 = 60.
3. **The discounted amount**: The promo comes off first; tax applies to what you actually pay for the items.

CHAPTER 4

Banking

A checking account is where most people's money lives day to day. This chapter shows how to write checks, keep a register, reconcile your records with the bank statement, and use online and mobile banking safely. All names, banks, and account numbers are fictional.

4.1

The Checking Account: Deposits, Checks, and the Register

LEARNING OBJECTIVES

After this section you will be able to:

1. Identify the parts of a check and write the amount in words.
2. Complete a deposit slip.
3. Keep a running balance in a check register.

REAL-LIFE SITUATION

Tyrell's first apartment

Tyrell opened a checking account when he moved into a shared apartment. He gets paid by direct deposit, pays his rent share and gym by check, and uses his debit card for transit.

Tyrell M. Brooks No. 1043
Philadelphia, PA Date: October 8, 2026

PAY TO THE ORDER OF

Philly Fitness Gym **$45.00**

Forty-five and 00/100 ———— DOLLARS

MEMO October membership *Tyrell M. Brooks*

⑆000000000⑆ 0000000000⑈ 1043

Figure 4.1 A completed personal check (fictional)

The concept

A check is a written order telling your bank to pay someone from your account. You are the drawer, the person or business you pay is the payee, and the bank is the drawee.

Write the amount twice: in numbers in the box, and in words on the line, with cents written as a fraction over 100. If they disagree, the bank generally uses the words. Draw a line after the words so nobody can add to them.

A check register is your own record. Every deposit is added and every check, debit card purchase, withdrawal, or fee is subtracted, keeping a running balance.

To deposit a check, the payee endorses it on the back. A blank endorsement (just a signature) lets anyone cash it. A restrictive endorsement, "For deposit only," is safer.

Date	No.	Description	Payment (−)	Deposit (+)	Balance
Balance forward					$820.00
10/1	DD	Paycheck (direct deposit)		$612.50	**$1,432.50**
10/3	1041	Rent share	$650.00		**$782.50**
10/5	DEB	SEPTA Key reload	$25.50		**$757.00**
10/8	1043	Philly Fitness Gym	$45.00		**$712.00**

Figure 4.2 Tyrell's check register, October

The Checking Account: Deposits, Checks, and the Register: solution map

1. Identify the question
   `Find each running balance in Tyrell's register (Figure 4.2).`
2. Calculation 1
   `Start $820.00.`
3. Calculation 2
   `10/1 deposit 612.50: 820.00 + 612.50 = 1,432.50.`
4. Calculation 3
   `10/3 check #1041, 650.00: 1,432.50 − 650.00 = 782.50.`
5. Calculation 4
   `10/5 debit SEPTA Key, 25.50: 782.50 − 25.50 = 757.00.`
6. Calculation 5
   `10/8 check #1043, 45.00: 757.00 − 45.00 = 712.00.`
7. Interpret and check
   `Ending balance $712.00`

Figure 4.1.V The Checking Account: Deposits, Checks, and the Register: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

CHECKING ACCOUNT

A bank account for everyday spending, usually with a debit card and checks.

SAVINGS ACCOUNT

A bank account meant to hold money for later, often earning a small amount of interest.

DEPOSIT

Money placed into a bank account.

KEY TERMS

Checking account
A bank account for everyday deposits and payments.

Drawer / payee / drawee
The account owner writing the check / who is paid / the bank paying it.

Deposit slip
A form listing cash and checks you are depositing.

Endorsement
The payee's signature on the back of a check, sometimes with instructions.

Check register
Your running log of deposits, payments, and balance.

FORMULAS & RULES

Running balance

```
New balance = Previous balance + Deposits − Payments
```

Net deposit

```
Cash + Coins + Checks − Cash received
```

Worked examples

Example 1 Keeping the register

Find each running balance in Tyrell's register (Figure 4.2).

SOLUTION

1. Start $820.00.
2. 10/1 deposit 612.50: 820.00 + 612.50 = 1,432.50.
3. 10/3 check #1041, 650.00: 1,432.50 − 650.00 = 782.50.
4. 10/5 debit SEPTA Key, 25.50: 782.50 − 25.50 = 757.00.
5. 10/8 check #1043, 45.00: 757.00 − 45.00 = 712.00.

> **Answer:** Ending balance $712.00

Example 2 Writing an amount in words

Write $1,208.07 as it appears on a check.

SOLUTION

1. Dollars in words: one thousand two hundred eight.
2. Cents over 100: 07/100.

> **Answer:** One thousand two hundred eight and 07/100 dollars

Practice exercises 4.1

1. Start with $300.00, deposit $125.40, then write a check for $89.99. What is the balance?
2. Write $63.50 in words as on a check.
3. On check #1043, who is the payee and who is the drawer?
4. Deposit slip: currency $140.00, coins $3.25, checks $210.00 and $56.30, less cash received $50.00. What is the net deposit?
5. Why is "For deposit only" safer than a plain signature?

RECAP QUIZ 4.1

1. In a register, a deposit:

- ○ a. Decreases the balance
- ○ b. Increases the balance
- ○ c. Does not change it
- ○ d. Is recorded only by the bank

2. "For deposit only" is a:

- ○ a. Blank endorsement
- ○ b. Restrictive endorsement
- ○ c. Memo
- ○ d. Stop-payment

3. The payee is:

- ○ a. The bank
- ○ b. The account owner
- ○ c. The person or business being paid
- ○ d. The cashier

SECTION 4.1 RECAP

- The register is a running record: deposits add, payments subtract.
- Record checks and card payments as soon as they happen.

Answer key with explanations: Section 4.1

PRACTICE EXERCISES

1. $335.41
2. Sixty-three and 50/100 dollars
3. Payee: Philly Fitness Gym. Drawer: Tyrell M. Brooks
4. 140 + 3.25 + 210 + 56.30 − 50 = $359.55
5. It is a restrictive endorsement, so the check can only be deposited into the payee's account, not cashed by someone who finds it

RECAP QUIZ

1. **Increases the balance**: Deposits are added.
2. **Restrictive endorsement**: It restricts what can be done with the check.
3. **The person or business being paid**: The payee receives the money.

4.2

Bank Statements and Reconciliation

LEARNING OBJECTIVES

After this section you will be able to:

1. Read a monthly bank statement.
2. Identify deposits in transit, outstanding checks, fees, and credits.

3. Reconcile the checkbook balance with the bank statement balance.

REAL-LIFE SITUATION

The numbers don't match

Tyrell's November statement says $1,186.40. His register says $1,240.15. Neither is wrong yet: some items have reached one record but not the other.

BANK SIDE

Statement balance	$1,186.40
+ Deposit in transit	$250.00
− Outstanding #1044	$75.00
− Outstanding #1046	$132.15
Adjusted bank	**$1,229.25**

CHECKBOOK SIDE

Checkbook balance	$1,240.15
+ Interest earned	$1.10
− Monthly service fee	$12.00
Adjusted checkbook	**$1,229.25**

Balanced ✓

Figure 4.3 Bank reconciliation worksheet

The concept

A bank statement lists everything the bank processed during the month. Your register lists everything you recorded. The two balances differ because of timing and because the bank may have items you did not know about.

On the bank side: add deposits in transit (you made them, the bank had not posted them by statement date) and subtract outstanding checks (you wrote them, they had not cleared).

On the checkbook side: add credits you did not record, such as interest, and subtract fees and NSF (returned) checks you did not record.

If the two adjusted balances are equal, the account is reconciled. If they are off by an amount divisible by 9, look for transposed digits, such as writing 54 instead of 45.

Bank Statements and Reconciliation: solution map

1. Identify the question

   `Use Figure 4.3.`

2. Calculation 1

   `Bank: 1,186.40 + 250.00 = 1,436.40.`

3. Calculation 2

   `Outstanding: 75.00 + 132.15 = 207.15. 1,436.40 − 207.15 = 1,229.25.`

4. Calculation 3

   `Checkbook: 1,240.15 − 12.00 + 1.10 = 1,229.25.`

5. Calculation 4

   `Both adjusted balances are $1,229.25. Reconciled. Tyrell records the fee and interest in his register.`

6. Interpret and check

   `Both adjusted balances: $1,229.25`

Figure 4.2.V Bank Statements and Reconciliation: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

WITHDRAWAL

Money taken out of a bank account.

BALANCE

The amount of money in an account at a given moment.

ACCOUNT STATEMENT

A bank's record of deposits, withdrawals, fees, and the balance for a period of time.

KEY TERMS

Bank statement
The bank's monthly record of your account activity.

Deposit in transit
A deposit you recorded that the bank had not yet posted.

Outstanding check
A check you wrote that has not yet cleared the bank.

Service charge
A fee the bank deducts for account services.

NSF check
Non-sufficient funds: a deposited check that bounced and was taken back out.

Reconciliation
Adjusting both balances until they agree.

FORMULAS & RULES

Adjusted bank balance

```
Statement balance + Deposits in transit − Outstanding checks
```

Adjusted checkbook balance

```
Checkbook balance + Credits − Fees − NSF items
```

Worked examples

Example 1 Reconciling Tyrell's account

Use Figure 4.3.

SOLUTION

1. Bank: 1,186.40 + 250.00 = 1,436.40.
2. Outstanding: 75.00 + 132.15 = 207.15. 1,436.40 − 207.15 = 1,229.25.
3. Checkbook: 1,240.15 − 12.00 + 1.10 = 1,229.25.
4. Both adjusted balances are $1,229.25. Reconciled. Tyrell records the fee and interest in his register.

> **Answer:** Both adjusted balances: $1,229.25

Example 2 A bounced check

Statement $540.00; outstanding check $60.00; checkbook $525.00. A $35.00 check Tyrell deposited bounced (NSF) and the bank charged a $10.00 returned-item fee.

SOLUTION

1. Bank: 540.00 − 60.00 = 480.00.
2. Checkbook: 525.00 − 35.00 − 10.00 = 480.00.

> **Answer:** Reconciled at $480.00

Practice exercises 4.2

1. Statement $932.10, deposit in transit $100.00, outstanding checks $45.60 and $18.90. Adjusted bank balance?
2. Checkbook $702.35, service fee $8.50, interest $0.42. Adjusted checkbook balance?
3. Is an outstanding check added or subtracted, and on which side?
4. You recorded a check as $72.00 but it cleared for $27.00. How should you fix the register?
5. Your balances differ by $18. What error might you look for?

RECAP QUIZ 4.2

1. A deposit in transit is:

- ○ a. Subtracted from the checkbook
- ○ b. Added to the bank balance
- ○ c. Ignored
- ○ d. Added to the checkbook

2. A monthly service fee is:

- ○ a. Added to the bank balance
- ○ b. Subtracted from the checkbook
- ○ c. An outstanding check
- ○ d. A credit

3. You are reconciled when:

- ○ a. The statement shows no fees
- ○ b. Both adjusted balances are equal
- ○ c. All checks have cleared
- ○ d. The balance is positive

SECTION 4.2 RECAP

- Reconciliation makes the bank's balance and your checkbook agree by accounting for what has not cleared yet.
- Deposits in transit add to the bank balance; outstanding checks subtract.

Answer key with explanations: Section 4.2

PRACTICE EXERCISES

1. 932.10 + 100 − 64.50 = $967.60
2. $694.27
3. Subtracted from the bank statement balance
4. Add $45.00 back to the register balance
5. 18 is divisible by 9, so look for transposed digits (for example 31 written as 13)

RECAP QUIZ

1. **Added to the bank balance**: The bank has not counted it yet.
2. **Subtracted from the checkbook**: You did not know about it, so your register must catch up.
3. **Both adjusted balances are equal**: Agreement of the adjusted balances is the goal.

4.3

Online and Mobile Banking

LEARNING OBJECTIVES

After this section you will be able to:

1. Tell the difference between current and available balance.

2. Use mobile check deposit and transfers correctly.
3. Avoid overdrafts and protect your account from scams.

REAL-LIFE SITUATION

Checking the app before buying sneakers

Nia opens her bank app. It shows a current balance and a smaller available balance, plus two pending purchases from the weekend.

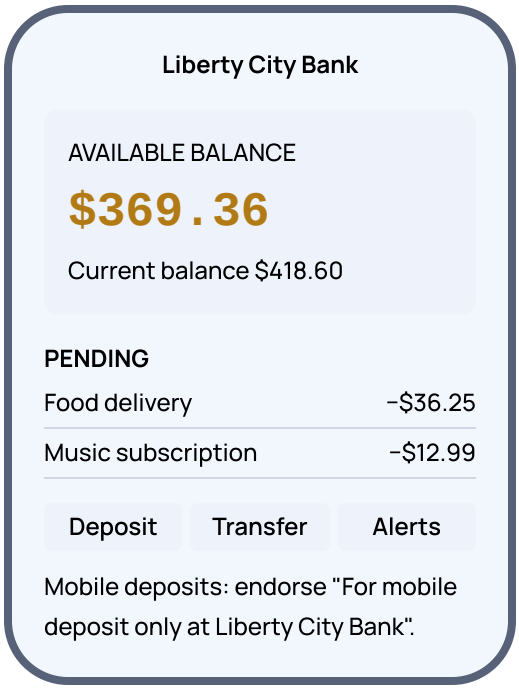


Figure 4.4 Mobile banking screen (fictional bank)

The concept

Current balance is what has fully posted. Available balance is what you can spend right now: current balance minus pending debits and holds. Spend based on available, and on what your register says is still coming.

Mobile check deposit: photograph the front and back after endorsing with your signature and the words your bank requires. Banks may make only

part of the deposit available right away. Keep the paper check until the deposit fully clears.

Direct deposit and transfers move money electronically. Person-to-person (P2P) payments are usually instant and hard to reverse, so only send money to people you know.

An overdraft happens when you spend more than is available. Banks may charge a fee or decline the purchase. Low-balance alerts help. Never share a one-time passcode: your real bank will not text or call asking for it.

Online and Mobile Banking: solution map

1. Identify the question
   `Use Figure 4.4.`
2. Calculation 1
   `Pending: 36.25 + 12.99 = 49.24.`
3. Calculation 2
   `Available: 418.60 − 49.24 = 369.36.`
4. Interpret and check
   `$369.36`

Figure 4.3.V Online and Mobile Banking: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

BANK RECONCILIATION

Comparing your own record of an account with the bank's statement and explaining any differences.

OUTSTANDING CHECK

A check you wrote that the recipient has not cashed yet.

OVERDRAFT

Spending more than the balance in an account, which may trigger fees.

KEY TERMS

Current balance
Money in the account from fully posted transactions.

Pending transaction
A purchase or deposit authorized but not yet posted.

Available balance
What you can spend now after pending items and holds.

Direct deposit
Pay sent electronically from an employer into your account.

Overdraft
Spending more than the available balance.

Two-factor authentication (2FA)
A second check, such as a texted code, to prove it is you.

FORMULAS & RULES

Available balance

```
Available = Current − Pending debits − Holds
```

Overdraft result

```
New balance = Available − Purchase − Fee
```

Worked examples

Example 1 Nia's available balance

Use Figure 4.4.

SOLUTION

1. Pending: 36.25 + 12.99 = 49.24.
2. Available: 418.60 − 49.24 = 369.36.

> **Answer:** $369.36

Example 2 The cost of an overdraft

Nia's available balance is $40.00. She buys a $55.00 hoodie and her bank charges a $35.00 overdraft fee.

SOLUTION

1. 40.00 − 55.00 = −15.00.
2. −15.00 − 35.00 = −50.00.
3. A $55 hoodie effectively cost $90.

> **Answer:** Balance −$50.00

Practice exercises 4.3

1. Current balance $1,050.00, pending debits $220.45. Available balance?
2. You mobile-deposit a $300 check. The bank makes $225 available today. How much is still on hold?

3. You get a text: "Your account is locked. Reply with the code we just sent." What should you do?
4. Direct deposit of $846.20 twice a month. Monthly total?
5. Balance $20.00, purchase $48.00, overdraft fee $35.00. New balance?

RECAP QUIZ 4.3

1. Your bank's code arrives by text. Who should you share it with?

- ○ a. Anyone who calls from the bank
- ○ b. Customer support on social media
- ○ c. No one
- ○ d. Friends you trust

2. Available balance is lower than current balance because of:

- ○ a. Interest
- ○ b. Pending transactions and holds
- ○ c. Direct deposits
- ○ d. Rounding

3. Before a mobile deposit, you should:

- ○ a. Shred the check
- ○ b. Endorse it as the bank requires
- ○ c. Mail it to the bank
- ○ d. Write VOID on it

SECTION 4.3 RECAP

- Available balance = current balance minus pending debits and holds.
- Review your account often — alerts and pending items prevent overdrafts.

Answer key with explanations: Section 4.3

PRACTICE EXERCISES

1. $829.55
2. $75
3. Do not reply or share the code. Contact the bank through its official app or the number on your card
4. $1,692.40
5. −$63.00

RECAP QUIZ

1. **No one**: One-time codes are for you alone.
2. **Pending transactions and holds**: Pending items reduce what you can spend.
3. **Endorse it as the bank requires**: An endorsement is required, and keep the check until it clears.

CHAPTER 5

Solving for the Unknown

Algebra lets you find a missing number: how many hours you need to work, how many weeks until you reach a savings goal, how much data fits your budget. This chapter treats an equation like a balance scale and builds up to real word problems.

5.1

Equations and the Balance Principle

LEARNING OBJECTIVES

After this section you will be able to:

1. Tell the difference between an expression and an equation.
2. Solve one-step equations with inverse operations.
3. Check a solution by substitution.

REAL-LIFE SITUATION

The mystery bag

On a balance scale, a closed bag of candy plus 7 loose pieces balances exactly 19 loose pieces. How many pieces are in the bag?

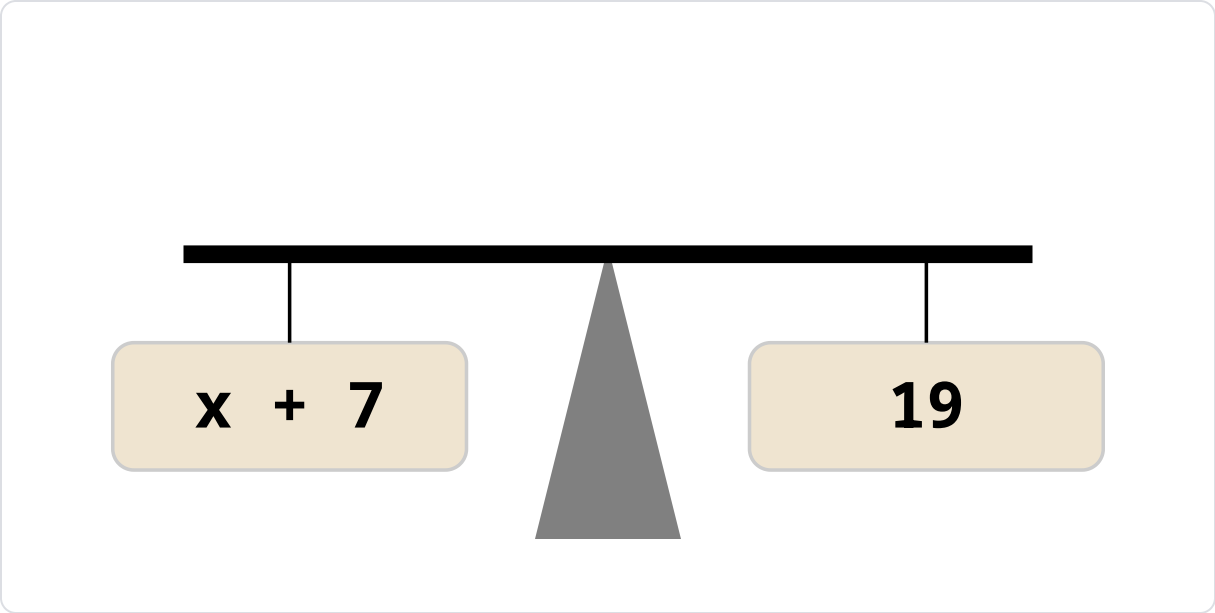


Figure 5.1 x + 7 balances 19

The concept

A variable, often x, stands for an unknown number. An expression, like x + 7, has no equals sign. An equation, like x + 7 = 19, says two expressions are equal.

Think of an equation as a balanced scale. Adding or subtracting the same number on both sides, or multiplying or dividing both sides by the same nonzero number, keeps it balanced. To get the variable alone, undo what is being done to it with the inverse operation: subtraction undoes addition, division undoes multiplication, and the reverse.

Always check by substituting your answer into the original equation.

Equations and the Balance Principle: solution map

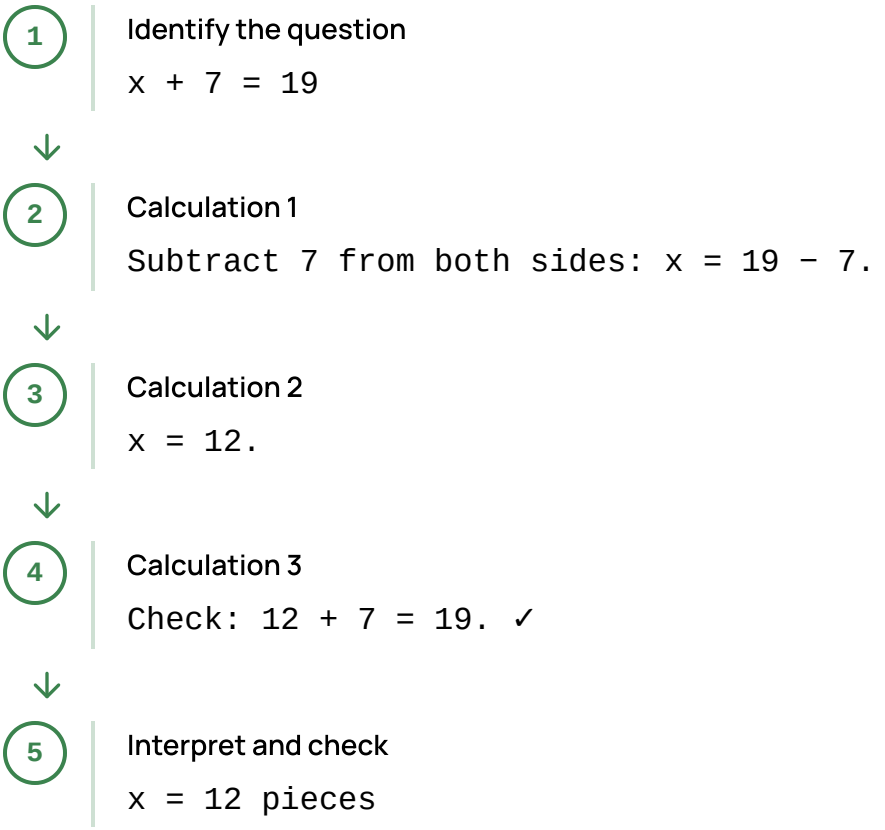


Figure 5.1.V Equations and the Balance Principle: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

VARIABLE COST

A cost that rises or falls with the number of items made or sold.

FIXED COST

A cost that stays the same no matter how many items are sold, like rent.

BREAK-EVEN POINT

The sales level where revenue exactly equals total cost, so there is no profit and no loss.

KEY TERMS

Variable
A letter standing for an unknown number.

Constant
A number that does not change.

Expression
Numbers, variables, and operations with no equals sign.

Equation
A statement that two expressions are equal.

Inverse operations
Operations that undo each other (+ and −, × and ÷).

FORMULAS & RULES

Addition equation

$$x + a = b \rightarrow x = b - a$$

Subtraction equation

$$x - a = b \rightarrow x = b + a$$

Multiplication equation

$$a \cdot x = b \rightarrow x = b \div a$$

Division equation

$$x \div a = b \rightarrow x = b \times a$$

Worked examples

Example 1 The mystery bag

$$x + 7 = 19$$

SOLUTION

1. Subtract 7 from both sides: x = 19 − 7.
2. x = 12.
3. Check: 12 + 7 = 19. ✓

> **Answer:** x = 12 pieces

Example 2 Multiplication

$$4n = 52$$

SOLUTION

1. Divide both sides by 4: n = 13.
2. Check: 4 × 13 = 52. ✓

> **Answer:** n = 13

Example 3 Division

$$n \div 5 = 9$$

SOLUTION

1. Multiply both sides by 5: n = 45.
2. Check: 45 ÷ 5 = 9. ✓

Answer: n = 45

Practice exercises 5.1

1. y − 14 = 30
2. 6m = 84
3. k ÷ 3 = 12
4. 25 + p = 61
5. 0.5x = 8

RECAP QUIZ 5.1

1. The inverse of multiplication is:

- ○ a. Addition
- ○ b. Subtraction
- ○ c. Division
- ○ d. Rounding

2. x − 9 = 4, so x =

- ○ a. 5
- ○ b. 13
- ○ c. −5
- ○ d. 36

3. 3x = 27, so x =

- ○ a. 9
- ○ b. 24
- ○ c. 30
- ○ d. 81

SECTION 5.1 RECAP

- An equation is a balance: whatever you do to one side, do to the other.
- Isolate the unknown by undoing operations in reverse order.

Answer key with explanations: Section 5.1

PRACTICE EXERCISES

1. y = 44
2. m = 14
3. k = 36
4. p = 36
5. x = 16

RECAP QUIZ

1. **Division**: Division undoes multiplication.
2. **13**: Add 9 to both sides.
3. **9**: Divide both sides by 3.

5.2

Multistep Equations

LEARNING OBJECTIVES

After this section you will be able to:

1. Combine like terms.
2. Use the distributive property.
3. Solve equations with variables on both sides.

REAL-LIFE SITUATION

Three bags and five extras

Three identical candy bags plus 5 loose pieces balance 26 loose pieces. Now there are two operations to undo.

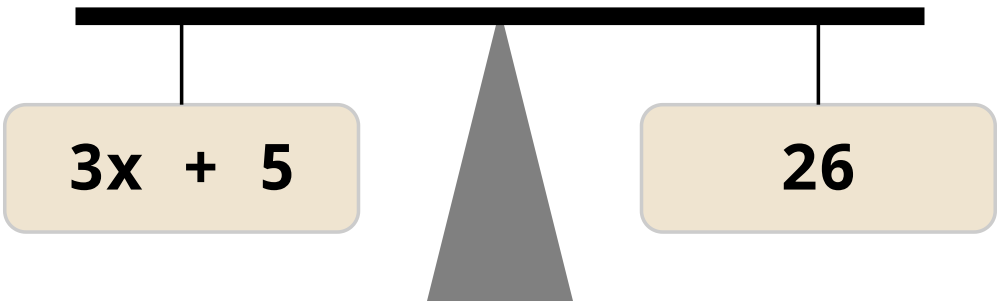


Figure 5.2 3x + 5 balances 26

The concept

Undo operations in reverse order. In 3x + 5, the x was multiplied by 3 and then 5 was added, so subtract 5 first, then divide by 3.

Like terms have the same variable part, such as 5x and 2x. Combine them by adding or subtracting their coefficients.

The distributive property says a(b + c) = ab + ac. You can also divide both sides by the outside number first when it divides evenly.

When the variable is on both sides, subtract one variable term from both sides so it appears on only one side.

Multistep Equations: solution map

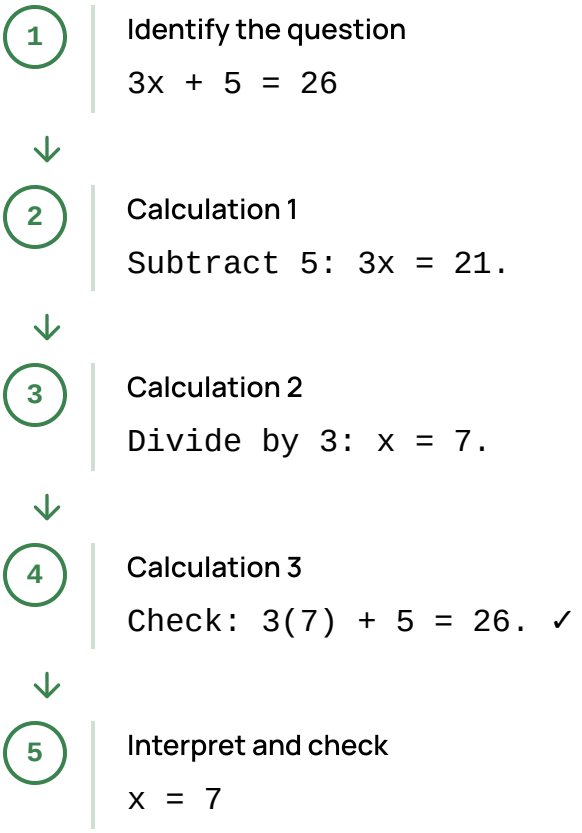


Figure 5.2.V Multistep Equations: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

EXPENSE

Money a business spends to run, such as rent, supplies, or wages.

REVENUE

The total money a business brings in from sales, before any costs are paid.

BUDGET

A plan that shows expected income and how it will be spent.

KEY TERMS

Coefficient
The number multiplying a variable, like 3 in 3x.

Like terms
Terms with identical variable parts.

Distributive property
a(b + c) = ab + ac.

FORMULAS & RULES

Two-step

```
ax + b = c → x = (c − b) ÷ a
```

Distribute

```
a(x + b) = ax + ab
```

Worked examples

Example 1 Two steps

$3x + 5 = 26$

SOLUTION

1. Subtract 5: 3x = 21.
2. Divide by 3: x = 7.
3. Check: 3(7) + 5 = 26. ✓

> **Answer:** x = 7

Example 2 Parentheses

$$2(x + 4) = 30$$

SOLUTION

1. Divide by 2: x + 4 = 15.
2. Subtract 4: x = 11.
3. Check: 2(15) = 30. ✓

> **Answer:** x = 11

Example 3 Like terms

$$5x - 2x + 6 = 24$$

SOLUTION

1. Combine: 3x + 6 = 24.
2. Subtract 6: 3x = 18.
3. x = 6.

Answer: x = 6

Example 4 Variables on both sides

$7x + 3 = 4x + 15$

SOLUTION

1. Subtract 4x: 3x + 3 = 15.
2. Subtract 3: 3x = 12.
3. x = 4.
4. Check: 31 = 31. ✓

Answer: x = 4

Practice exercises 5.2

1. 4a − 9 = 23
2. 3(b − 2) = 21
3. 8c + 2c = 150
4. 6d + 5 = 2d + 29
5. x ÷ 4 + 3 = 10

RECAP QUIZ 5.2

1. Which operation removes the +6 term from 2x + 6 = 20?

- ○ a. Divide by 2
- ○ b. Subtract 6

- ○ c. Add 6
- ○ d. Multiply by 2

2. $5(x - 1) = 20$, so $x =$

- ○ a. 3
- ○ b. 4
- ○ c. 5
- ○ d. 21

3. $9x - 4x = 35$, so $x =$

- ○ a. 5
- ○ b. 7
- ○ c. 35
- ○ d. 13

SECTION 5.2 RECAP

- Simplify each side first, then undo operations to isolate the unknown.
- Check the solution by substituting it back into the original equation.

Answer key with explanations: Section 5.2

PRACTICE EXERCISES

1. $a = 8$
2. $b = 9$
3. $c = 15$
4. $d = 6$
5. $x = 28$

RECAP QUIZ

1. **Subtract 6**: Undo the addition first.
2. **5**: $x - 1 = 4$, so $x = 5$.

3. **7**: $5x = 35$.

5.3

Word Problems with the Unknown

LEARNING OBJECTIVES

After this section you will be able to:

1. Translate words into algebraic expressions.
2. Use the blueprint to set up and solve an equation.
3. Check that the answer makes sense in the situation.

REAL-LIFE SITUATION

The phone bill

Imani's prepaid plan charges a flat $40 per month plus $8 for each gigabyte of data. Last month's bill was $72. How many gigabytes did she use?

Keystone Mobile — October bill

```
Base plan ............ $40.00
Data (? GB × $8.00) ... $32.00
Total due ............ $72.00
```

Figure 5.3 Monthly bill summary (fictional carrier)

The concept

Translate key words. "Sum", "more than", "increased by" → add. "Less than", "decreased by" → subtract ("5 less than n" is n − 5). "Times", "twice", "of" → multiply. "Per", "split equally" → divide. "Is" → equals.

Blueprint for algebra: (1) Let a variable represent the unknown and write what it means. (2) Write the other quantities using that variable. (3) Write and solve the equation. (4) Check the answer in the words of the problem, not just the equation.

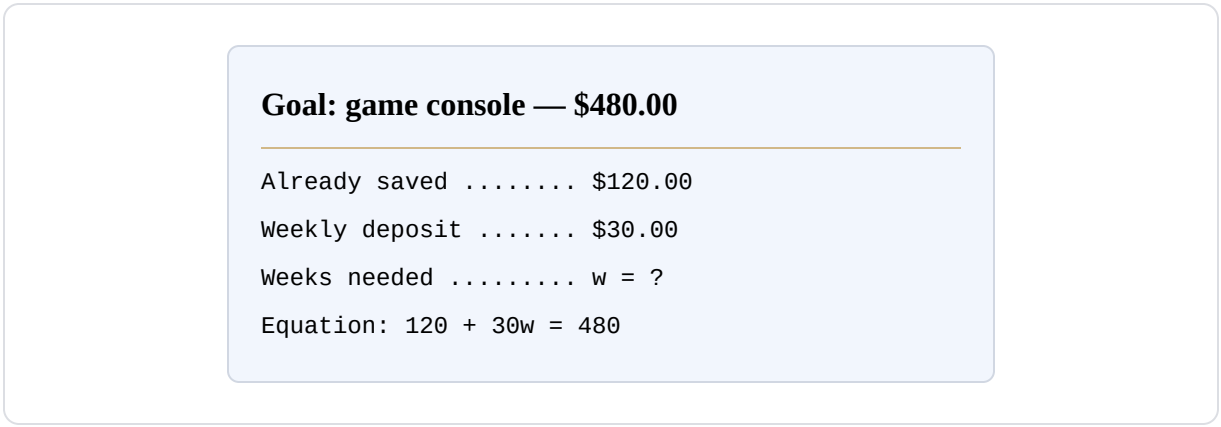


Figure 5.4 Savings goal tracker for a game console (hypothetical price)

Build the phone-plan equation

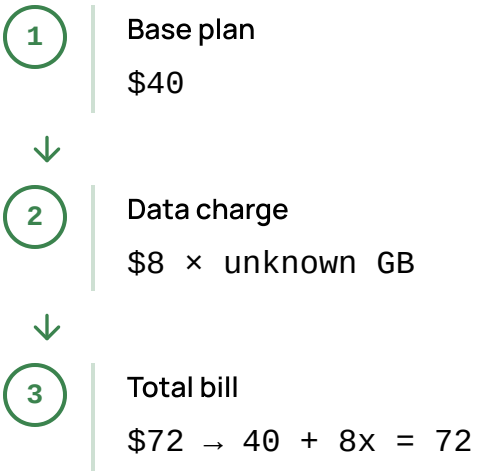


Figure 5.3.V Build the phone-plan equation. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

MONTHLY PAYMENT

The amount paid each month toward a loan or installment plan.

LOAN TERM

How long a loan lasts before it must be fully repaid.

TOTAL REPAYMENT COST

The full amount paid over a loan's life — principal plus all interest and fees.

KEY TERMS

Translate
Rewrite a sentence as an algebraic expression or equation.

Consecutive whole numbers
Whole numbers in a row, written n, n + 1, n + 2.

FORMULAS & RULES

Flat fee plus rate

$$\text{Total} = \text{Fixed fee} + \text{Rate} \times \text{Units}$$

Parts of a total

$$\text{Part}_1 + \text{Part}_2 = \text{Total}$$

Worked examples

Example 1 The phone bill

$40 + 8g = 72$

SOLUTION

1. Let g = gigabytes used.
2. Subtract 40: 8g = 32.
3. Divide by 8: g = 4.
4. Check: $40 + 4 × $8 = $72. ✓

> **Answer:** 4 GB

Example 2 Saving for a game console

Darius wants a $480 game console (hypothetical price). He has $120 saved and puts $30 a week from his part-time job into savings. How many weeks until he can buy it?

SOLUTION

1. Let w = number of weeks.
2. Saved so far + new savings = price: 120 + 30w = 480.
3. Subtract 120: 30w = 360.
4. Divide by 30: w = 12.
5. Check: 120 + 30 × 12 = 120 + 360 = 480. ✓

Answer: 12 weeks

Example 3 Splitting earnings

Tasha and Rob earned $540 together selling at a weekend market. Tasha earned twice as much as Rob. How much did each earn?

SOLUTION

1. Let r = Rob's earnings; Tasha = 2r.
2. r + 2r = 540 → 3r = 540 → r = 180.
3. Tasha = 2 × 180 = 360.
4. Check: 180 + 360 = 540. ✓

Answer: Rob $180, Tasha $360

Example 4 Gym classes

A gym charges $25 a month plus $10 per class. Ari's bill is $85. How many classes?

SOLUTION

1. 25 + 10c = 85.
2. 10c = 60.
3. c = 6.

Answer: 6 classes

Practice exercises 5.3

1. A number plus 18 is 47. Find the number.
2. You have $60 saved and add $15 a week. How many weeks until you have $240?
3. Three consecutive whole numbers add to 72. Find them.
4. Jordan has 3 times as many candies as Kim. Together they have 48. How many does each have?
5. A game bundle costs $360 (hypothetical). Kai has $90 and saves $45 a week. How many weeks?
6. A ride costs $3 to start plus $2 per mile. The fare was $19. How many miles?

RECAP QUIZ 5.3

1. "5 less than a number is 12" is:

- ○ a. $5 - n = 12$
- ○ b. $n - 5 = 12$
- ○ c. $n + 5 = 12$
- ○ d. $5n = 12$

2. "Twice a number" is:

- ○ a. $n + 2$
- ○ b. n^2
- ○ c. $2n$
- ○ d. $n \div 2$

3. The best way to check a word-problem answer is to:

- ○ a. Round it
- ○ b. Substitute it back into the situation
- ○ c. Solve again faster
- ○ d. Compare with a friend

SECTION 5.3 RECAP

- Name the unknown with a variable, then translate the words into an equation.
- A word-problem answer should be a sentence with units, checked against the question.

Answer key with explanations: Section 5.3

PRACTICE EXERCISES

1. $n + 18 = 47 \rightarrow 29$
2. $60 + 15w = 240 \rightarrow w = 12$ weeks
3. $n + (n + 1) + (n + 2) = 72 \rightarrow 3n + 3 = 72 \rightarrow n = 23$: 23, 24, 25
4. $k + 3k = 48 \rightarrow k = 12$. Kim 12, Jordan 36
5. $90 + 45w = 360 \rightarrow w = 6$ weeks
6. $3 + 2m = 19 \rightarrow m = 8$ miles

RECAP QUIZ

1. **$n - 5 = 12$**: "Less than" reverses the order: $n - 5$.
2. **$2n$**: Twice means multiply by 2.
3. **Substitute it back into the situation**: It must make the original statement true.

CHAPTER 6

Percentages and Their Applications

A percent describes a part relative to a whole, whether you are checking a survey or planning a purchase. Learn to choose the correct base before calculating discounts, taxes, tips, and changes. All scenarios and rates are hypothetical; round money to the nearest cent, with a half cent rounded up, and retain unrounded intermediate rates unless told otherwise.

6.1

Percent, Rate, and Base

LEARNING OBJECTIVES

After this section you will be able to:

1. Convert among fractions, decimals, and percents.
2. Identify the part, rate, and base in a situation.
3. Solve for a missing part, rate, or base.

REAL-LIFE SITUATION

A club surveys its members

Use this hypothetical Philadelphia-area scenario to connect the calculation to a decision. All people, businesses, amounts, and rates in this chapter are invented; tax and payroll rules are teaching assumptions, not current law.

A club surveys its members

```
Members surveyed: 80
Members choosing a weekend event: 28
Weekend share: 35%
```

Figure 6.1 A club surveys its members (hypothetical)

The concept

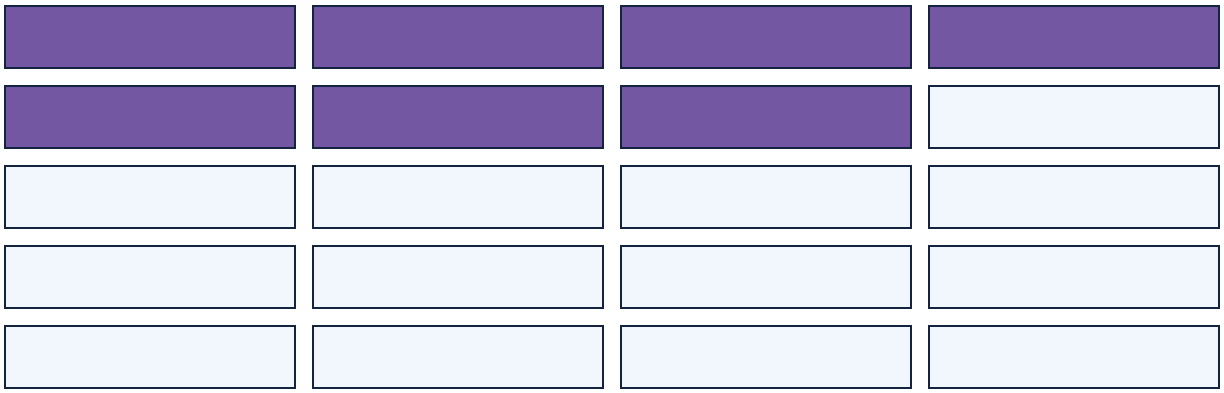
A percent is a ratio with a base of 100. The statement 35% means 35 out of every 100, not necessarily that exactly 100 people were counted. The same share is 0.35 or 35/100 = 7/20.

To change a percent to a decimal, divide its numerical value by 100. To change a decimal to a percent, multiply by 100 and attach the percent sign. For a fraction, divide numerator by denominator first. Values below 1% and above 100% are valid: context determines whether they make sense.

Every basic percent problem has a part, a base, and a rate. The base is the entire quantity described by the phrase "percent of." In the club survey, 80 is the base and 28 is the part. Labeling these quantities prevents accidentally dividing by the part.

Use part = base × rate with the rate in decimal form. If the rate or base is missing, undo multiplication by division. Check the scale: a rate below 100% should give a part smaller than a positive base. A zero base cannot be used to calculate a meaningful part-to-base rate.

Fraction → decimal → percent



7/20 = 0.35 = 35%

1. Count 7 shaded parts out of 20 equal parts: 7/20.
2. Divide numerator by denominator: 7 ÷ 20 = 0.35.
3. Multiply the decimal by 100: 35%. This matches 28 of 80 surveyed members.

Original WealthWiseGuys diagram · Hypothetical teaching quantities.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

DISCOUNT

An amount taken off a regular price.

SALES TAX

A percentage added by the government to the price of many purchases.

MARKUP

The amount added to a cost to set a selling price; the markup rate uses cost as its base.

KEY TERMS

Percent
A ratio expressed per hundred.

Base
The whole quantity used as the reference.

Part
The amount being compared with the whole.

Rate
The part-to-base ratio, often written as a percent.

FORMULAS & RULES

Find a part

```
Part = base × decimal rate
```

Find a rate

```
Decimal rate = part ÷ base
```

Find a base

```
Base = part ÷ decimal rate
```

Worked examples

Example 1 Read the survey

What percent of the 80 club members chose the weekend event if 28 selected it?

SOLUTION

1. Identify the part as 28 members and the base as 80 members.
2. Divide: 28 ÷ 80 = 0.35.

3. Convert: 0.35 × 100 = 35%.

> **Answer:** 35% of surveyed members chose the weekend event.

Example 2 Recover a fundraising goal

A club has raised $180, which is 30% of its goal. What is the full goal?

SOLUTION

1. Convert 30% to 0.30.
2. Divide the part by the rate: 180 ÷ 0.30 = 600.
3. Check: 600 × 0.30 = 180.

> **Answer:** The goal is $600.00.

Practice exercises 6.1

1. Write 3/8 as a decimal and percent.
2. Find 12% of $250.
3. A team wins 18 of 24 games. Find its winning percentage.
4. $45 is 15% of what amount?
5. A student writes 0.6% as 0.6. Correct the error and find 0.6% of $500.

RECAP QUIZ 6.1

1. What decimal equals 125%?

- ○ a. 0.125
- ○ b. 1.25

- ○ c. 12.5
- ○ d. 125

2. What is 20% of 60?

- ○ a. 3
- ○ b. 8
- ○ c. 12
- ○ d. 48

3. Nine is what percent of 30?

- ○ a. 30%
- ○ b. 21%
- ○ c. 3.33%
- ○ d. 270%

SECTION 6.1 RECAP

- Name the base before choosing an operation.
- Use decimal rates in calculations; multiply by 100 only to report a percent.

Answer key with explanations: Section 6.1

PRACTICE EXERCISES

1. 3 ÷ 8 = 0.375; 0.375 × 100 = 37.5%.
2. 12% = 0.12; 250 × 0.12 = $30.00.
3. 18 ÷ 24 = 0.75; multiply by 100 to get 75%.
4. 45 ÷ 0.15 = $300.00; 300 × 0.15 = 45 checks the result.
5. Divide by 100: 0.6% = 0.006, not 0.6. Then 500 × 0.006 = $3.00.

RECAP QUIZ

1. **1.25**: 125 ÷ 100 = 1.25; a percent can exceed 100%.
2. **12**: 60 × 0.20 = 12.

3. **30%**: 9 ÷ 30 = 0.30 = 30%.

6.2

Percent Change and Percentage Points

LEARNING OBJECTIVES

After this section you will be able to:

1. Compute increases and decreases using the original amount.
2. Distinguish percent change from percentage-point change.
3. Find an original amount from a changed amount.

REAL-LIFE SITUATION

A phone plan changes price

Use this hypothetical Philadelphia-area scenario to connect the calculation to a decision. All people, businesses, amounts, and rates in this chapter are invented; tax and payroll rules are teaching assumptions, not current law.



Figure 6.2 A phone plan changes price (hypothetical)

The concept

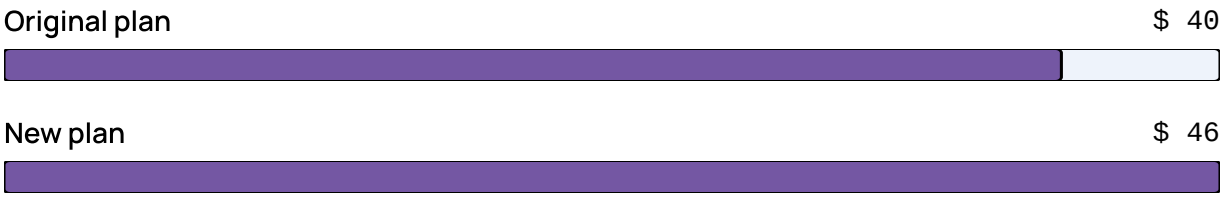
Percent change compares a difference with the original amount. Calculate new minus old first, then divide by old. A positive result describes an increase and a negative result describes a decrease. If the original amount is zero, ordinary percent change is undefined.

The denominator matters. A price rising from $40 to $46 increased by 15%, but a drop from $46 to $40 would use $46 as its base. Reversing the direction changes the percentage even when the dollar difference stays the same.

When comparing two rates, subtraction gives percentage points. A survey share moving from 20% to 25% rises by 5 percentage points. Its relative increase is 5 ÷ 20 = 25%. These statements answer different questions and should not be interchanged.

To work backward, use a multiplier. After a 15% increase, the new amount equals 1.15 times the old amount. After a 15% decrease it equals 0.85 times the old amount. Successive changes multiply; equal percentage increases and decreases do not generally cancel.

Compare with the original price



Bars share a zero baseline.

Figure 6.2.V Compare with the original price. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

PROFIT MARGIN

Profit shown as a percent of revenue; a 10% margin means $10 of profit per $100 of sales.

RETURN

The money or gain an investment produces, often shown as a percent.

ROI

Return on investment — the gain compared to what you put in, as a percent: (gain ÷ cost) × 100.

KEY TERMS

Percent change
Difference divided by the original amount, expressed as a percent.

Original amount
The starting value that supplies the comparison base.

Multiplier
The factor applied to the original value.

Percentage point
One unit of difference between values already expressed as percents.

FORMULAS & RULES

Percent change

```
Percent change = (new − old) ÷ old × 100%
```

Increase multiplier

```
New = old × (1 + decimal increase rate)
```

Decrease multiplier

```
New = old × (1 − decimal decrease rate)
```

Rate difference

```
Percentage-point change = new percent − old percent
```

Worked examples

Example 1 Compare two prices

A phone plan rises from $40 to $46 monthly. Find its dollar and percent increase.

SOLUTION

1. Subtract: 46 − 40 = $6.00.
2. Use the original base: 6 ÷ 40 = 0.15.
3. Convert: 0.15 × 100 = 15%.

> **Answer:** The monthly price increases $6.00, or 15%.

Example 2 Undo a reduction

A console accessory costs $72 after a 20% reduction. What was its original price?

SOLUTION

1. The remaining share is 100% − 20% = 80% = 0.80.
2. Solve 72 = original × 0.80: original = 72 ÷ 0.80 = $90.00.
3. Check the discount: 90 × 0.20 = $18.00; 90 − 18 = $72.00.

> **Answer:** The original price was $90.00.

Practice exercises 6.2

1. A ticket rises from $25 to $30. Find percent change.
2. A price drops from $80 to $60. Find percent change.
3. A response rate rises from 30% to 36%. Give both comparisons.
4. After a 10% increase a fee is $55. Find the old fee.
5. A $100 price rises 20%, then falls 20%. Does it return to $100?

RECAP QUIZ 6.2

1. A value falls from 50 to 40. Its percent change is:

- ○ a. −10%
- ○ b. −20%
- ○ c. −25%
- ○ d. 20%

2. A rate rises from 8% to 10%. The percentage-point increase is:

- ○ a. 20
- ○ b. 25
- ○ c. 2
- ○ d. 0.2

3. Which divisor recovers an original price after a 25% discount?

- ○ a. 0.75
- ○ b. 0.25
- ○ c. 1.25
- ○ d. 25

SECTION 6.2 RECAP

- Use the original amount as the denominator of percent change.

- Distinguish a difference in rates from a relative change in rates.

Answer key with explanations: Section 6.2

PRACTICE EXERCISES

1. Difference = 30 − 25 = 5; 5 ÷ 25 × 100% = 20% increase.
2. (60 − 80) ÷ 80 × 100% = −25%, a 25% decrease.
3. 36 − 30 = 6 percentage points; 6 ÷ 30 × 100% = 20% relative increase.
4. 55 ÷ 1.10 = $50.00; 50 × 1.10 = 55.
5. 100 × 1.20 = 120; 120 × 0.80 = $96.00. It is $4.00 lower because the second base is larger.

RECAP QUIZ

1. **−20%**: (40 − 50) ÷ 50 × 100% = −20%.
2. **2**: Subtract the rates as stated: 10 − 8 = 2 percentage points.
3. **0.75**: The sale price is 75% of the original, so divide by 0.75.

6.3

Discounts, Sales Tax, and Tips

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate a sale price and a tax-inclusive total.
2. Apply a stated tipping base correctly.
3. Compare successive discounts with a single discount.

REAL-LIFE SITUATION

A weekend meal budget

Use this hypothetical Philadelphia-area scenario to connect the calculation to a decision. All people, businesses, amounts, and rates in this chapter are invented; tax and payroll rules are teaching assumptions, not current law.

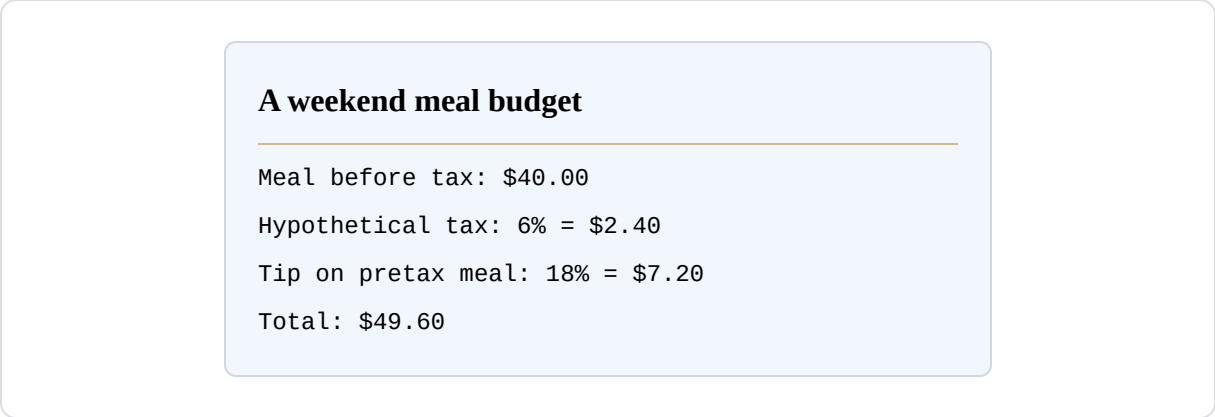


Figure 6.3 A weekend meal budget (hypothetical)

The concept

A discount reduces a stated price by a percent of that price. Find the discount amount first, round it to cents, and subtract it. The amount remaining is the sale price; a 25% discount leaves 75% of the original price.

In this chapter, taxable purchases are taxed after discounts. Find the tax using the discounted taxable subtotal, round that tax to cents, and add it to the subtotal. Actual taxability varies, so the fictional rate in a problem is an assumption rather than a statement of local law.

A tip also needs a clearly identified base. Here, a meal tip is calculated on the pretax meal amount, not on the tax. Add meal, tax, and tip as separate components. If people share the bill equally, divide the final total and account for any leftover cent.

Successive discounts apply to changing bases. A 20% discount followed by 10% off the reduced price leaves $0.80 \times 0.90 = 0.72$ of the original, an equivalent 28% discount, not 30%. For actual currency transactions, round each discount before moving to the next step; this can affect a final cent.

From meal price to amount paid

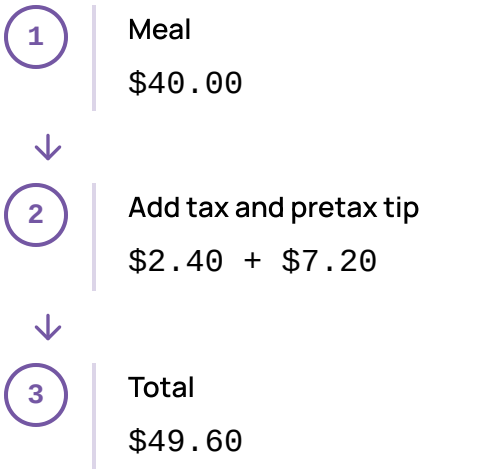


Figure 6.3.V From meal price to amount paid. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

YIELD

The income an investment pays, shown as a percent of its price.

INTEREST RATE

The percent used to calculate how much interest is earned or charged.

APR

Annual percentage rate — the yearly cost of borrowing, including interest and some fees, shown as a percent.

KEY TERMS

Discount
A reduction from a stated price.

Sale price
Price remaining after a discount.

Taxable subtotal
Amount to which the problem says tax applies.

Tip base
Amount used to calculate a gratuity.

Successive discounts
Discounts applied one after another to changing amounts.

FORMULAS & RULES

Discount amount

```
Discount = price × decimal discount rate
```

Sale price

```
Sale price = original price − discount
```

Tax and total

```
Total = taxable subtotal + rounded tax
```

Tip

```
Tip = stated tip base × decimal tip rate
```

Worked examples

Example 1 Discount before tax

A $60 backpack is 25% off. A hypothetical 6% tax applies to its sale price. Find the final total.

SOLUTION

1. Discount: 60 × 0.25 = $15.00.
2. Sale price: 60 − 15 = $45.00.
3. Tax: 45 × 0.06 = $2.70.
4. Total: 45 + 2.70 = $47.70.

> **Answer:** The final total is $47.70.

Example 2 A meal with a stated tip base

A $40 meal has hypothetical 6% tax and an 18% tip on its pretax price. What is paid?

SOLUTION

1. Tax: 40 × 0.06 = $2.40.
2. Tip: 40 × 0.18 = $7.20.
3. Add the components: 40 + 2.40 + 7.20 = $49.60.

> **Answer:** The payment is $49.60.

Practice exercises 6.3

1. Find the price of a $90 jacket after 30% off, before tax.
2. Add hypothetical 5% tax to a $24 taxable subtotal.
3. A $50 meal has hypothetical 8% tax and a 20% pretax tip. Find the total.
4. Apply 20% off, then another 10% off, to $100. Find the equivalent discount.

5. Split a $53.00 final bill equally among three friends to the nearest cent, keeping the sum correct.

RECAP QUIZ 6.3

1. A $50 item is 10% off before 6% hypothetical tax. The total is:

- ○ a. $47.70
- ○ b. $48.00
- ○ c. $45.00
- ○ d. $53.00

2. A 15% tip on a $20 pretax meal is:

- ○ a. $0.30
- ○ b. $1.50
- ○ c. $3.00
- ○ d. $23.00

3. Two successive 10% discounts on $100 leave:

- ○ a. $80.00
- ○ b. $81.00
- ○ c. $90.00
- ○ d. $99.00

SECTION 6.3 RECAP

- Calculate each percent on its stated base and round each monetary component.
- Successive percentage discounts multiply; they do not simply add.

Answer key with explanations: Section 6.3

PRACTICE EXERCISES

1. Discount = 90 × 0.30 = $27.00; sale price = 90 − 27 = $63.00.

2. Tax = 24 × 0.05 = $1.20; total = 24 + 1.20 = $25.20.
3. Tax = $4.00 and tip = $10.00; 50 + 4 + 10 = $64.00.
4. First price = $80.00; second discount = $8.00; final = $72.00. Savings = $28.00, or 28% of $100.
5. 53 ÷ 3 = 17.666...; three payments of $17.67 would total $53.01. Use $17.67, $17.67, and $17.66, totaling $53.00.

RECAP QUIZ

1. **$47.70**: Discount = $5; sale price = $45; tax = $2.70; total = $47.70.
2. **$3.00**: 20 × 0.15 = $3.00.
3. **$81.00**: 100 × 0.90 = 90; 90 × 0.90 = $81.00.

CHAPTER 7

Trade Discounts, Cash Discounts, and Purchasing

A low advertised price is only one part of a purchasing decision. Learn to read discount terms, account for returns and freight, and compare complete orders. All vendors, prices, dates, and terms are hypothetical; round each monetary discount and tax to cents, with a half cent rounded up.

7.1

Trade Discounts and Discount Series

LEARNING OBJECTIVES

After this section you will be able to:

1. Find a net price after a trade discount.
2. Apply a series of trade discounts in order.
3. Calculate a single equivalent discount.

REAL-LIFE SITUATION

A club buys event supplies

Use this hypothetical Philadelphia-area scenario to connect the calculation to a decision. All people, businesses, amounts, and rates in this chapter are invented; tax and payroll rules are teaching assumptions, not current law.

A club buys event supplies

```
Supplier list price: $800.00
Trade discounts: 25%, then 10%
Net merchandise price: $540.00
```

Figure 7.1 A club buys event supplies (hypothetical)

The concept

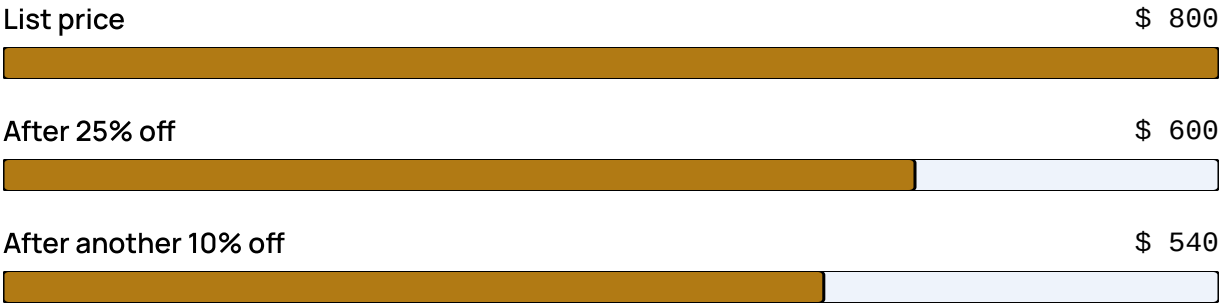
A trade discount reduces a supplier list price before the invoice amount is established. It may reflect a buyer category or order arrangement. A list price is a reference price, not necessarily the amount that any particular buyer pays.

For one discount, multiply list price by the discount rate and subtract that amount. The complementary rate, 1 minus the decimal discount rate, is the net price factor. Multiplying by this factor gives the same result before any rounding differences.

A discount series applies each discount to the price remaining after the previous discount. A 25% and 10% series does not mean 35% off. The second discount has a smaller base. On an $800 order, the discounts are $200 and $60, not $200 and $80.

The product of the net price factors gives the series net factor. Subtract it from 1 to find an equivalent single discount. This equivalent rate compares offers clearly, but actual invoices should follow the stated cent-rounding procedure for each discount. Freight and tax remain separate unless the contract says otherwise.

Successive discounts use a changing base



Bars share a zero baseline.

Figure 7.1.V Successive discounts use a changing base. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

INVOICE
A document listing goods or services sold and the amount owed.

PAYMENT TERMS
The rules in an agreement about when a bill must be paid and any discounts for paying early.

DISCOUNT
An amount taken off a regular price.

KEY TERMS

List price
Supplier reference price before trade discounts.

Trade discount
Price reduction used to establish the merchandise invoice price.

Net price
Merchandise amount remaining after trade discounts.

Net price factor
Fraction of a price left after discounts.

Equivalent discount
One discount rate matching a discount series before rounding.

FORMULAS & RULES

Single discount

```
Net price = list price × (1 − decimal discount rate)
```

Series factor

```
Net factor = (1 − d1) × (1 − d2)
```

Equivalent discount

```
Equivalent decimal discount = 1 − net factor
```

Worked examples

Example 1 One trade discount

A $600 supply order receives a 20% trade discount. Find its net merchandise price.

SOLUTION

1. Convert 20% to 0.20.
2. Discount: 600 × 0.20 = $120.00.
3. Net price: 600 − 120 = $480.00.

Answer: Net merchandise price: $480.00.

Example 2 A discount series

An $800 list-price order receives 25% and 10% trade discounts. Find its net price and equivalent discount.

SOLUTION

1. First discount: 800 × 0.25 = $200.00; remaining price = $600.00.
2. Second discount: 600 × 0.10 = $60.00; net price = $540.00.
3. Net factor: 0.75 × 0.90 = 0.675.
4. Equivalent discount: 1 − 0.675 = 0.325 = 32.5%.

Answer: Net price $540.00; equivalent discount 32.5%.

Practice exercises 7.1

1. Find the net price of a $250 order with a 12% trade discount.
2. Apply 20% and 5% trade discounts to $1,000.
3. Find the equivalent discount for 20% and 5%.
4. A $400 net price follows a 20% trade discount. Find list price.
5. Which costs less on a $500 list: 30% off or 20% then 15% off?

RECAP QUIZ 7.1

1. A 15% trade discount on $200 leaves:

- ○ a. $15
- ○ b. $30

- ○ c. $170
- ○ d. $185

2. The net factor for 10% then 20% off is:

- ○ a. 0.70
- ○ b. 0.72
- ○ c. 0.30
- ○ d. 0.02

3. Why are series discounts not added?

- ○ a. Each uses a new base
- ○ b. Rates are always rounded to whole numbers
- ○ c. Only the largest applies
- ○ d. List prices include freight

SECTION 7.1 RECAP

- Apply each trade discount to the remaining merchandise price.
- Use net factors to compare discount series.

Answer key with explanations: Section 7.1

PRACTICE EXERCISES

1. Discount = 250 × 0.12 = $30.00; net = $220.00.
2. First net = 1000 × 0.80 = $800.00; second discount = $40.00; final net = $760.00.
3. Net factor = 0.80 × 0.95 = 0.76; equivalent discount = 1 − 0.76 = 24%.
4. Net factor = 0.80; list price = 400 ÷ 0.80 = $500.00.
5. Single discount leaves 500 × 0.70 = $350.00. Series leaves 500 × 0.80 × 0.85 = $340.00, so the series saves $10.00 more.

RECAP QUIZ

1. **$170**: 200 − 200 × 0.15 = $170.

2. **0.72**: 0.90 × 0.80 = 0.72.
3. **Each uses a new base**: The second discount is based on the reduced price, not the original list price.

7.2

Cash Discounts and Invoice Terms

LEARNING OBJECTIVES

After this section you will be able to:

1. Interpret cash discount and net payment terms.
2. Determine eligibility using a stated counting convention.
3. Separate discount-eligible merchandise from excluded charges.

REAL-LIFE SITUATION

A supplier offers prompt-payment terms

Use this hypothetical Philadelphia-area scenario to connect the calculation to a decision. All people, businesses, amounts, and rates in this chapter are invented; tax and payroll rules are teaching assumptions, not current law.

A supplier offers prompt-payment terms

```
Invoice: April 3
Terms: 2/10, n/30
Day after invoice is day 1
Discount deadline: April 13; net deadline: May 3
```

Figure 7.2 A supplier offers prompt-payment terms (hypothetical)

The concept

A cash discount rewards payment within an early-payment window. Terms 2/10, n/30 mean a 2% discount if payment meets the 10-day deadline; otherwise the full net amount is due within 30 days. This is different from a trade discount, which determines the invoice merchandise price before payment timing is considered.

For every example here, count the day after the invoice date as day 1, use calendar days, and assume payment is received on the stated date. Holidays and weekends do not extend deadlines in these hypothetical contracts. Real contracts may define receipt, mailing, or electronic settlement differently.

The discount base is the eligible merchandise balance after applicable returns and allowances. In these examples, freight is excluded from the cash discount and tax is omitted unless a problem explicitly supplies it. Apply the discount only to the eligible amount, then add the excluded charges.

Paying early reduces the amount paid but also uses cash sooner. A buyer should check that funds are actually available and compare any borrowing cost with the savings; a discount does not automatically justify borrowing. If payment misses the discount deadline, use the undiscounted balance rather than a partial discount.

Invoice deadlines

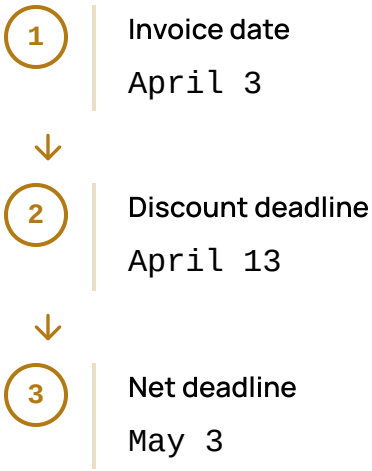


Figure 7.2.V Invoice deadlines. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

MARKDOWN

A reduction from the regular selling price, often to clear out inventory.

WHOLESALE

The price or market for buying goods in bulk, usually from a supplier.

RETAIL

The price or market for selling goods to the public, usually in small amounts.

KEY TERMS

Cash discount
Reduction for meeting an early-payment condition.

Discount period
Number of days in which discounted payment is permitted.

Net period
Time allowed for full payment without a cash discount.

Eligible balance
Amount to which a cash discount applies.

Invoice date
Starting reference date specified on the bill.

FORMULAS & RULES

Cash discount

```
Cash discount = eligible balance × decimal cash discount rate
```

Early payment

```
Payment = eligible balance − cash discount + excluded charges
```

Full payment

```
Full payment = eligible balance + excluded charges
```

Worked examples

Example 1 Read the calendar

An April 3 invoice for $1,000 has terms 2/10, n/30. Payment is received April 12. Find deadlines and payment.

SOLUTION

1. Count April 4 as day 1; day 10 is April 13.
2. Day 30 is May 3; April 12 is inside the discount period.

3. Discount: 1000 × 0.02 = $20.00.
4. Payment: 1000 − 20 = $980.00.

> **Answer:** Pay $980.00; the discount deadline is April 13 and the net deadline is May 3.

Example 2 Exclude freight

An invoice has $600 eligible merchandise and $40 freight, terms 3/10, n/30. Timely payment qualifies for the discount. Find the payment.

SOLUTION

1. Eligible base is $600.00, not $640.00.
2. Discount: 600 × 0.03 = $18.00.
3. Payment: 600 − 18 + 40 = $622.00.

> **Answer:** The timely payment is $622.00.

Practice exercises 7.2

1. Pay an $800 eligible invoice within terms 2/10, n/30. No other charges apply.
2. Pay that same $800 invoice on day 15.
3. A June 10 invoice has a 10-day discount window under this section's convention. What is the deadline?
4. Find early payment on $900 eligible merchandise plus $50 excluded freight with a 1% discount.
5. A timely payment of $490 settled merchandise only under a 2% cash discount. Find the eligible invoice balance.

RECAP QUIZ 7.2

1. Terms 1/10, n/30 offer which discount?

- ○ a. 10% within 1 day
- ○ b. 1% within 10 days
- ○ c. 30% within 10 days
- ○ d. 1% within 30 days

2. A $500 eligible balance receives 2% off. Payment is:

- ○ a. $490
- ○ b. $498
- ○ c. $480
- ○ d. $510

3. Under the stated convention, day 1 after a July 6 invoice is:

- ○ a. July 6
- ○ b. July 16
- ○ c. July 7
- ○ d. August 6

SECTION 7.2 RECAP

- Read both the discount deadline and the full-payment deadline.
- Do not discount excluded freight or other excluded charges.

Answer key with explanations: Section 7.2

PRACTICE EXERCISES

1. Discount = 800 × 0.02 = $16.00; payment = $784.00.
2. Day 15 is outside the 10-day discount period, so no discount applies: payment = $800.00.
3. June 11 is day 1; add 10 calendar days to June 10 to get June 20.

4. Discount = 900 × 0.01 = $9.00; payment = 900 − 9 + 50 = $941.00.
5. Payment is 98% of the balance; 490 ÷ 0.98 = $500.00.

RECAP QUIZ

1. **1% within 10 days**: The first number is the percent and the second is the discount window.
2. **$490**: Discount = 500 × 0.02 = $10; payment = $490.
3. **July 7**: Counting begins the calendar day after the invoice date.

7.3

Returns, Freight, and Total Purchasing Cost

LEARNING OBJECTIVES

After this section you will be able to:

1. Adjust an invoice for returns and allowances.
2. Calculate the complete delivered cost of an order.
3. Compare suppliers on cost per usable unit.

REAL-LIFE SITUATION

The club checks a delivery

Use this hypothetical Philadelphia-area scenario to connect the calculation to a decision. All people, businesses, amounts, and rates in this chapter are invented; tax and payroll rules are teaching assumptions, not current law.

The club checks a delivery

```
20 supply kits at $12.00: $240.00
Return 2 kits: credit $24.00
Remaining merchandise: $216.00
Freight: $18.00; total: $234.00
```

Figure 7.3 The club checks a delivery (hypothetical)

The concept

A purchase return removes merchandise from an order and creates a credit for its agreed invoice value. An allowance reduces the price of goods the buyer keeps, perhaps because of minor damage. Subtract these credits before calculating a cash discount when the contract makes only the adjusted balance eligible.

Freight is part of acquiring goods even when it is not eligible for a cash discount. For these problems, delivered cost means adjusted merchandise minus any qualifying cash discount plus freight and any stated tax. No tax is assumed when it is not supplied.

Purchasing comparisons should use the same quantity and quality. Cost per usable unit divides complete delivered cost by units actually retained and usable. Returned or unusable units do not belong in that denominator, even if the original order was larger.

The lowest unit cost is a mathematical comparison, not the entire purchasing decision. Delivery reliability, storage capacity, quality, and the cash needed for a large order also matter. State which costs and assumptions the comparison includes rather than presenting a low sticker price as the complete answer.

Purchasing cost bridge

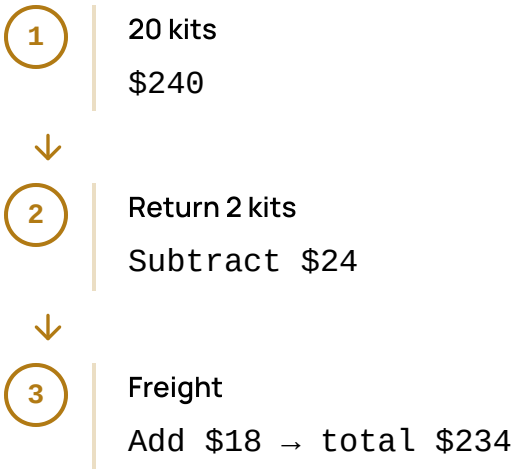


Figure 7.3.V Purchasing cost bridge. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

VENDOR
A company or person that supplies goods or services to a business.

COST
The amount paid to make or buy an item before it is sold.

INVENTORY
The stock of goods a business holds ready to sell.

KEY TERMS

Return
Goods sent back for a credit.

Allowance
Price reduction on goods retained.

Freight
Charge for transporting goods.

Delivered cost
Purchase cost including the specified delivery-related charges.

Usable unit cost
Total delivered cost divided by usable units retained.

FORMULAS & RULES

Adjusted merchandise

```
Adjusted merchandise = invoice merchandise − returns − allowances
```

Delivered cost

```
Delivered cost = adjusted merchandise − cash discount + freight + stated tax
```

Unit cost

```
Cost per usable unit = delivered cost ÷ usable units
```

Worked examples

Example 1 Return before paying

A club receives 20 kits at $12 each, returns 2 for full credit, and pays $18 freight. No discounts or tax apply. Find delivered cost per retained kit.

SOLUTION

1. Original merchandise: 20 × 12 = $240.00.
2. Return credit: 2 × 12 = $24.00; adjusted merchandise = $216.00.

3. Delivered cost: 216 + 18 = $234.00.
4. Usable kits: 20 − 2 = 18; 234 ÷ 18 = $13.00 per kit.

> **Answer:** Delivered cost is $234.00, or $13.00 per retained kit.

Example 2 Compare delivered offers

Vendor A offers 50 identical notebooks for $100 plus $15 freight. Vendor B offers 50 for $110 with free freight. No tax or discounts apply. Which costs less?

SOLUTION

1. A delivered cost: 100 + 15 = $115.00.
2. B delivered cost: 110 + 0 = $110.00.
3. Unit costs: 115 ÷ 50 = $2.30; 110 ÷ 50 = $2.20.
4. Difference: 115 − 110 = $5.00.

> **Answer:** Vendor B costs $5.00 less overall, or $0.10 less per notebook.

Practice exercises 7.3

1. Adjust $700 merchandise for an $80 return and a $20 allowance.
2. Apply a timely 2% cash discount to that $600 adjusted balance and add $30 excluded freight.
3. Thirty usable items cost $162 delivered. Find cost per unit.
4. Compare 40 identical items at $180 plus $20 freight with 40 at $196 delivered.
5. A $300 shipment contains 25 items, but 5 unusable items receive no credit. Find cost per usable item with no other charges.

RECAP QUIZ 7.3

1. A $400 invoice less a $50 return has an adjusted merchandise balance of:

- ○ a. $450
- ○ b. $350
- ○ c. $400
- ○ d. $50

2. Which belongs in delivered cost under the stated model?

- ○ a. Only list price
- ○ b. Only discounts
- ○ c. Merchandise and specified freight
- ○ d. Only the unit count

3. $144 delivered for 24 usable units costs how much per unit?

- ○ a. $6
- ○ b. $12
- ○ c. $120
- ○ d. $168

SECTION 7.3 RECAP

- Adjust merchandise before applying a qualifying cash discount.
- Compare full delivered costs using usable quantities.

Answer key with explanations: Section 7.3

PRACTICE EXERCISES

1. 700 − 80 − 20 = $600.00 adjusted merchandise.
2. Discount = 600 × 0.02 = $12.00; delivered payment = 600 − 12 + 30 = $618.00.
3. 162 ÷ 30 = $5.40 per usable item.

4. First offer totals $200.00, or $5.00 each; second is $196.00, or $4.90 each. The second saves $4.00.
5. Usable units = 25 − 5 = 20; 300 ÷ 20 = $15.00, not 300 ÷ 25 = $12.00.

RECAP QUIZ

1. **$350**: 400 − 50 = $350.
2. **Merchandise and specified freight**: Delivery charges must be included along with adjusted merchandise and any other stated costs.
3. **$6**: 144 ÷ 24 = $6 per unit.

CHAPTER 8

Markups, Markdowns, Pricing, and Break-Even Analysis

A busy food stand can collect plenty of cash and still lose money. This chapter connects cost, selling price, markdowns, and sales volume so that you can explain where a business result comes from. All businesses and amounts are hypothetical; round money to cents, with a half cent rounded up, and round required whole-unit sales upward.

8.1

Markup on Cost and Margin on Selling Price

LEARNING OBJECTIVES

After this section you will be able to:

1. Find a selling price from cost and markup.
2. Distinguish markup on cost from margin on selling price.
3. Recover price from a target margin.

REAL-LIFE SITUATION

A pop-up prices a tote

Use this hypothetical Philadelphia-area scenario to connect the calculation to a decision. All people, businesses, amounts, and rates in this chapter are

invented; tax and payroll rules are teaching assumptions, not current law.



Figure 8.1 A pop-up prices a tote (hypothetical)

The concept

Cost is the amount assigned to acquiring or producing an item under the model being used. Selling price is what the customer pays before any separately stated tax. Markup dollars are selling price minus cost; this difference must help cover operating expenses as well as profit.

A markup rate on cost uses cost as its denominator. If cost is $30 and price is $50, the $20 markup is 20 ÷ 30, or approximately 66.67% of cost. Label the base whenever reporting a markup percentage.

Margin on selling price uses selling price instead. For the same tote, 20 ÷ 50 = 40%. These two rates describe the same dollars but are not interchangeable. Applying a 40% markup to cost would produce a different selling price.

To achieve a target margin m, the cost must be the remaining 1 − m share of selling price. Divide cost by that remaining share. A positive cost cannot be covered at a margin of 100% in this simple model. A pricing calculation is a plan, not a guarantee that customers will buy.

Cost + markup = selling price



1. Cost: **$30.00** (60.0% of total)
2. Markup: **$20.00** (40.0% of total)

1. Start with $30 cost; add $20 markup to reach $50 selling price.
2. Markup uses cost as its base: 20 ÷ 30 = 66.67%.
3. Margin uses selling price as its base: 20 ÷ 50 = 40%. These are not interchangeable.

Original WealthWiseGuys diagram · Hypothetical teaching quantities.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

MARKUP

The amount added to a cost to set a selling price; the markup rate uses cost as its base.

PROFIT MARGIN

Profit shown as a percent of revenue; a 10% margin means $10 of profit per $100 of sales.

GROSS PROFIT

Revenue minus the direct cost of the items sold, before other expenses.

KEY TERMS

Cost
Amount assigned to acquiring or producing the item.

Selling price
Amount charged for the item before separately stated tax.

Markup dollars
Selling price minus cost.

Markup on cost
Markup dollars divided by cost.

Margin
Markup dollars divided by selling price in this model.

FORMULAS & RULES

Markup dollars

```
Markup = selling price − cost
```

Price from cost markup

```
Selling price = cost × (1 + decimal markup rate)
```

Margin

```
Decimal margin = markup ÷ selling price
```

Price from margin

```
Selling price = cost ÷ (1 − decimal margin)
```

Worked examples

Example 1 Apply markup on cost

An item costs $40. Set price using a 25% markup on cost.

SOLUTION

1 Markup: 40 × 0.25 = $10.00.

2. Price: 40 + 10 = $50.00.
3. Check markup rate: 10 ÷ 40 = 0.25 = 25%.

> **Answer:** Price: $50.00.

Example 2 Target a margin

A tote costs $30. Find the price for a 40% margin, then its markup rate on cost.

SOLUTION

1. Cost share: 1 − 0.40 = 0.60.
2. Price: 30 ÷ 0.60 = $50.00.
3. Markup: 50 − 30 = $20.00.
4. Markup on cost: 20 ÷ 30 × 100% ≈ 66.67%.

> **Answer:** Price $50.00; markup on cost approximately 66.67%.

Example 3 Real-world friction: resale through an online marketplace

A student buys sneakers for $60 and resells them for $100 on a hypothetical marketplace that keeps a 12% seller fee. The student also pays $8 to ship. Compare the simple margin with the real net profit and net margin.

SOLUTION

1. Simple (no-fee) profit: 100 − 60 = $40.00, a 40% margin on the $100 price.

2. Marketplace fee: 100 × 0.12 = $12.00.
3. Total selling costs: 12 + 8 = $20.00.
4. Net profit: 40 − 20 = $20.00.
5. Net margin: 20 ÷ 100 × 100% = 20%.
6. Judgment: fees and shipping cut the profit in half, so the price must clear all selling costs, not just the purchase cost.

> **Answer:** Net profit $20.00; net margin 20%, half of the 40% simple margin.

Practice exercises 8.1

1. Cost is $20 and price is $32. Find markup dollars.
2. For that item, find markup on cost and margin.
3. Price a $60 item with 30% markup on cost.
4. Price a $45 item for a 25% margin.
5. A seller says a 50% cost markup gives a 50% margin. Test with $10 cost.
6. Decision: A hoodie costs $25 and sells for $40 on a platform charging a 10% fee plus $7 shipping paid by the seller. Find net profit and decide whether the flip is worth it if the student wants at least $10 profit.
7. Decision: For that hoodie, what minimum price gives $10 net profit? (Price − 10% fee − $7 − $25 = $10.)

RECAP QUIZ 8.1

1. $12 cost and $18 price give markup dollars of:

- ○ a. $6
- ○ b. $12

○ c. $18

○ d. $30

2. A 20% margin on a $50 price represents:

○ a. $2.50

○ b. $10

○ c. $40

○ d. $60

3. For a target 30% margin, divide cost by:

○ a. 1.30

○ b. 0.30

○ c. 0.70

○ d. 30

SECTION 8.1 RECAP

- Markup and margin use different denominators.
- Markup dollars must cover expenses before they can become profit.

Answer key with explanations: Section 8.1

PRACTICE EXERCISES

1. 32 − 20 = $12.00.
2. 12 ÷ 20 = 60% markup on cost; 12 ÷ 32 = 37.5% margin.
3. Markup = 60 × 0.30 = $18.00; price = $78.00.
4. 45 ÷ (1 − 0.25) = 45 ÷ 0.75 = $60.00.
5. Price = 10 × 1.50 = $15.00; markup = $5.00; margin = 5 ÷ 15 ≈ 33.33%, not 50%.
6. Fee = 40 × 0.10 = $4.00; selling costs = 4 + 7 = $11.00; net profit = 40 − 25 − 11 = $4.00. Below the $10 goal, so No-Go at this price.
7. 0.90 × price = 10 + 7 + 25 = $42.00; price = 42 ÷ 0.90 ≈ $46.67.

RECAP QUIZ

1. **$6**: 18 − 12 = $6.
2. **$10**: 50 × 0.20 = $10 markup dollars.
3. **0.70**: Cost is the remaining 70% of price.

8.2

Markdowns and Pricing Decisions

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate a markdown and reduced price.
2. Measure markup remaining after a sale.
3. Find an original price from a markdown price.

REAL-LIFE SITUATION

A seasonal tote sale

Use this hypothetical Philadelphia-area scenario to connect the calculation to a decision. All people, businesses, amounts, and rates in this chapter are invented; tax and payroll rules are teaching assumptions, not current law.

A seasonal tote sale

```
Regular price: $80.00
Markdown: 25% = $20.00
Sale price: $60.00
Unit cost: $45.00; remaining markup: $15.00
```

Figure 8.2 A seasonal tote sale (hypothetical)

The concept

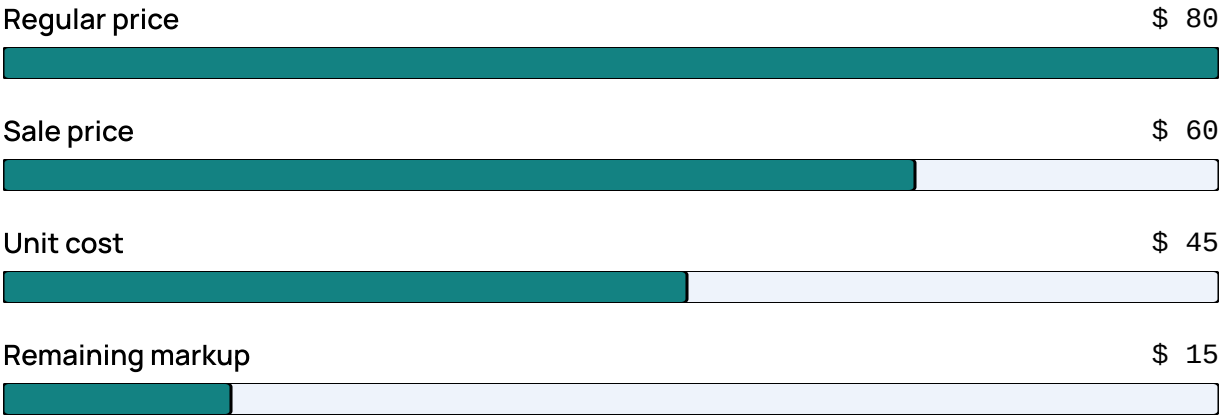
A markdown reduces a previously established selling price. Its rate is usually based on that regular price, not on the seller's cost. Calculate markdown dollars, round them to cents, and subtract them from the regular price.

A reduced selling price can still exceed cost. Compare the sale price with cost to find remaining markup dollars. That amount is not automatically profit because the seller may still need to cover rent, labor, payment fees, or other expenses.

A sale can also push price below cost. The difference is then a negative markup on that item under the stated cost model. A seller might accept this to clear storage, but the arithmetic should not hide the loss.

To recover a regular price from a sale price, divide by the remaining fraction after markdown. To evaluate several markdowns, apply them sequentially and round each monetary reduction. Compare actual final prices rather than simply adding advertised percentages.

Markdown and remaining markup



Bars share a zero baseline.

Figure 8.2.V Markdown and remaining markup. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

NET PROFIT

The final profit after every cost is paid — sometimes called the bottom line.

FIXED COST

A cost that stays the same no matter how many items are sold, like rent.

VARIABLE COST

A cost that rises or falls with the number of items made or sold.

KEY TERMS

Markdown
Reduction from a regular selling price.

Regular price
Selling price before the stated markdown.

Markdown rate
Markdown dollars divided by regular price.

Remaining markup
Sale price minus cost.

FORMULAS & RULES

Markdown

```
Markdown dollars = regular price × decimal markdown rate
```

Reduced price

```
Sale price = regular price − markdown dollars
```

Remaining markup

```
Remaining markup = sale price − cost
```

Recover regular price

```
Regular price = sale price ÷ (1 − decimal markdown rate)
```

Worked examples

Example 1 Check a sale

A tote regularly sells for $80 and costs $45. Find sale price and remaining markup after 25% off.

SOLUTION

1. Markdown: 80 × 0.25 = $20.00.
2. Sale price: 80 − 20 = $60.00.

3 Remaining markup: 60 − 45 = $15.00.

Answer: Sale price $60.00; remaining markup $15.00 before operating expenses.

Example 2 Recover a regular price

An item sells for $42 after a 30% markdown. Find its regular price.

SOLUTION

1 Remaining rate: 1 − 0.30 = 0.70.

2 Regular price: 42 ÷ 0.70 = $60.00.

3 Check: 60 × 0.30 = $18.00 markdown; 60 − 18 = $42.00.

Answer: Regular price: $60.00.

Practice exercises 8.2

1. Reduce a $120 price by 15%.
2. A $70 item is marked down to $56. Find the markdown rate.
3. An item costing $35 sells on clearance for $30. Find remaining markup.
4. A $72 sale price follows a 10% markdown. Find regular price.
5. Apply successive 25% and 20% markdowns to $100.

RECAP QUIZ 8.2

1. A 20% markdown on $90 is:

○ a. $18 ○ b. $20

○ c. $72

○ d. $108

2. Cost is $40 and sale price is $52. Remaining markup is:

○ a. $92

○ b. $12

○ c. $40

○ d. $52

3. A $45 price after 25% off came from:

○ a. $56.25

○ b. $33.75

○ c. $60

○ d. $70

SECTION 8.2 RECAP

- Markdowns use the regular selling price as their base.
- A positive remaining markup is not necessarily net profit.

Answer key with explanations: Section 8.2

PRACTICE EXERCISES

1. Markdown = 120 × 0.15 = $18.00; sale price = $102.00.
2. Markdown = 70 − 56 = $14.00; 14 ÷ 70 = 20%.
3. 30 − 35 = −$5.00; the sale is $5.00 below cost before other expenses.
4. 72 ÷ 0.90 = $80.00; the markdown was $8.00.
5. First reduction = $25.00, leaving $75.00; next reduction = $15.00, leaving $60.00. The combined markdown is 40%, not 45%.

RECAP QUIZ

1. **$18**: 90 × 0.20 = $18 markdown dollars.

2. **$12**: 52 − 40 = $12.

3. **$60**: 45 ÷ 0.75 = $60.

8.3

Contribution and Break-Even Analysis

LEARNING OBJECTIVES

After this section you will be able to:

1. Separate fixed and variable costs.
2. Find contribution per unit and break-even volume.
3. Calculate volume for a target operating profit.

REAL-LIFE SITUATION

A neighborhood food stand plans a day

Use this hypothetical Philadelphia-area scenario to connect the calculation to a decision. All people, businesses, amounts, and rates in this chapter are invented; tax and payroll rules are teaching assumptions, not current law.

A neighborhood food stand plans a day

```
Fixed daily cost: $240.00
Price per meal: $10.00
Variable cost per meal: $4.00
Contribution: $6.00; break-even: 40 meals
```

Figure 8.3 A neighborhood food stand plans a day (hypothetical)

The concept

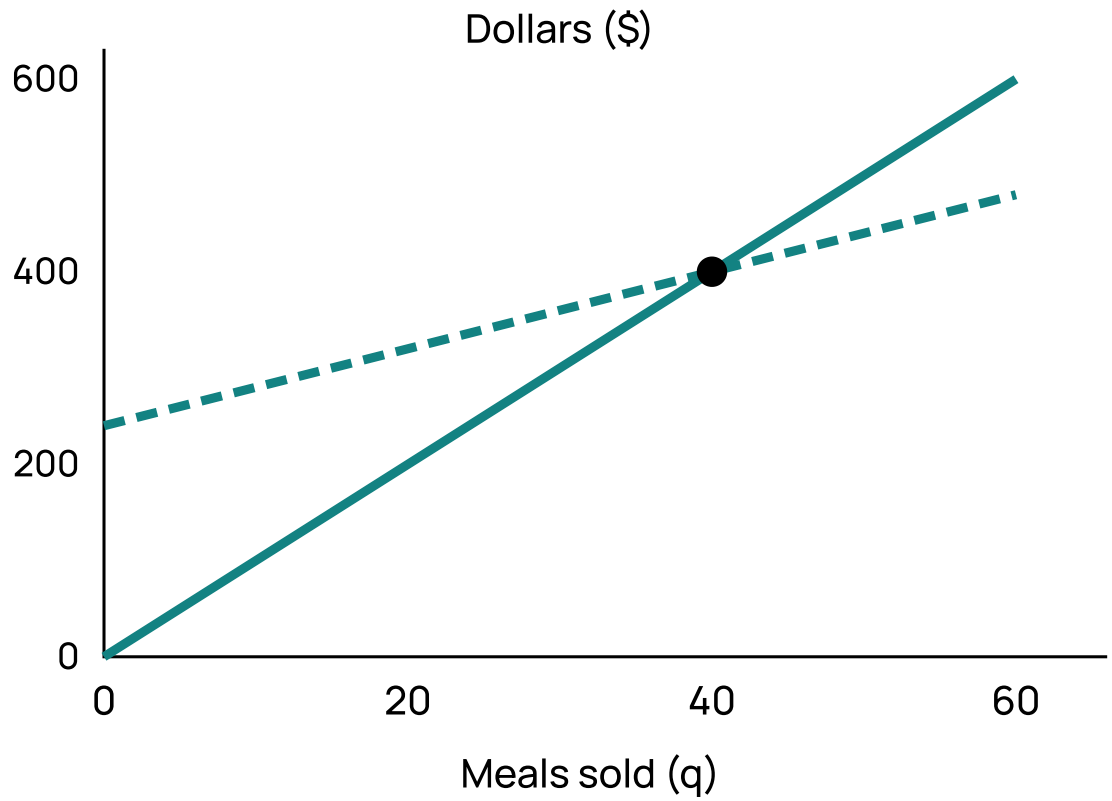
Fixed costs do not change with sales volume within the range being modeled. Variable costs change with each unit sold. A one-day permit may be fixed, while ingredients and a container may be variable. Classifications depend on the period and assumptions used.

Contribution per unit is selling price minus variable cost per unit. Each sale contributes this amount toward fixed costs. Only after all fixed costs are covered does additional contribution create operating profit in the model.

Break-even occurs when total revenue equals total fixed and variable costs. Divide fixed costs by positive contribution per unit. If the result is not a whole number, round up when deciding how many whole units must be sold to avoid a loss. If contribution is zero or negative and fixed costs are positive, no sales volume breaks even under these assumptions.

To plan for a target operating profit, add that target to fixed costs before dividing by contribution. This simple model assumes every unit made is sold, price and variable unit cost stay constant, and capacity is sufficient. Taxes, financing, and owner compensation are excluded unless explicitly included as costs.

Break-even: revenue meets total cost



Solid line: revenue = 10q. Dashed line: total cost = 240 + 4q.

1. Each meal contributes $10 − $4 = $6 toward fixed cost.
2. Break-even quantity = $240 ÷ $6 = 40 meals. At 40 meals, revenue and total cost both equal $400.
3. Below the crossing, cost exceeds revenue; above it, revenue exceeds cost. This linear model assumes constant prices and costs.

Original WealthWiseGuys diagram · Hypothetical teaching quantities.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

BREAK-EVEN POINT

The sales level where revenue exactly equals total cost, so there is no profit and no loss.

SELLING PRICE

The price a customer pays for an item or service.

COST

The amount paid to make or buy an item before it is sold.

KEY TERMS

Fixed cost
Cost constant over the modeled activity range.

Variable cost
Cost that changes with units sold.

Contribution per unit
Price minus variable cost per unit.

Break-even
Volume at which revenue equals modeled total cost.

Operating profit
Revenue less the modeled fixed and variable costs.

FORMULAS & RULES

Contribution

```
Contribution per unit = price − variable unit cost
```

Break-even

```
Break-even units = fixed cost ÷ contribution per unit
```

Profit

```
Operating profit = units × contribution per unit −
```

```
fixed cost
```

Target volume

```
Target units = (fixed cost + target profit) ÷
contribution per unit
```

Worked examples

Example 1 Cover the stand's costs

Fixed daily cost is $240. Meals sell for $10 with $4 variable cost each. Find break-even units and profit at 55 sales.

SOLUTION

1. Contribution: 10 − 4 = $6.00 per meal.
2. Break-even: 240 ÷ 6 = 40 meals.
3. At 55 meals, contribution = 55 × 6 = $330.00.
4. Operating profit: 330 − 240 = $90.00.

> **Answer:** Break-even is 40 meals; 55 meals produce $90.00 operating profit.

Example 2 Round required units upward

Fixed cost is $100, price is $9, and variable unit cost is $5. How many sales are needed for at least $50 operating profit?

SOLUTION

1. Contribution: 9 − 5 = $4.00.

2 Required contribution: 100 + 50 = $150.00.

3 Required volume: 150 ÷ 4 = 37.5; round up to 38 units.

4 Check: 38 × 4 − 100 = $52.00 profit; 37 × 4 − 100 = $48.00.

Answer: Sell at least 38 units.

Practice exercises 8.3

1. Find contribution when price is $15 and variable cost is $9.
2. Fixed cost is $180 with $6 contribution. Find break-even.
3. With those costs, find profit at 45 units.
4. Fixed cost is $250 and contribution is $8. Find minimum whole-unit break-even volume.
5. Price and variable cost are both $5 with $100 fixed cost. Can higher sales break even?

RECAP QUIZ 8.3

1. Price $12 and variable cost $7 give contribution of:

- ○ a. $19
- ○ b. $5
- ○ c. $7
- ○ d. $12

2. $90 fixed cost and $4 contribution require at least:

- ○ a. 22 units
- ○ b. 22.5 whole units
- ○ c. 23 units
- ○ d. 24 units

3. At break-even, operating profit is:

- ○ a. $0
- ○ b. Equal to revenue

○ c. Equal to variable cost

○ d. Always positive

SECTION 8.3 RECAP

- Positive contribution first covers fixed costs, then produces operating profit.
- Round required whole-unit volume upward and state the model assumptions.

Answer key with explanations: Section 8.3

PRACTICE EXERCISES

1. 15 − 9 = $6.00 per unit.
2. 180 ÷ 6 = 30 units.
3. 45 × 6 − 180 = 270 − 180 = $90.00.
4. 250 ÷ 8 = 31.25; round up to 32 units. Profit at 32 is 256 − 250 = $6.00.
5. Contribution = 5 − 5 = $0.00; profit = units × 0 − 100 = −$100.00 at every volume, so no.

RECAP QUIZ

1. **$5**: 12 − 7 = $5.
2. **23 units**: 90 ÷ 4 = 22.5; 23 is the smallest whole-unit volume covering costs.
3. **$0**: Revenue exactly covers the modeled fixed and variable costs.

CHAPTER 9

Wages, Salaries, Commissions, and Payroll

Your hourly rate is not the same thing as the deposit that reaches your account. Learn to calculate different kinds of earnings and trace gross pay through a hypothetical payroll statement. All workers, rates, overtime rules, and deductions are invented teaching examples, not current tax or employment law; round each monetary payroll component to cents, with a half cent rounded up.

9.1

Hourly Earnings and Overtime

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate straight-time wages.
2. Apply a stated overtime rule.
3. Convert hours and minutes to decimal hours.

REAL-LIFE SITUATION

A recreation-center time sheet

Use this hypothetical Philadelphia-area scenario to connect the calculation to a decision. All people, businesses, amounts, and rates in this chapter are invented; tax and payroll rules are teaching assumptions, not current law.

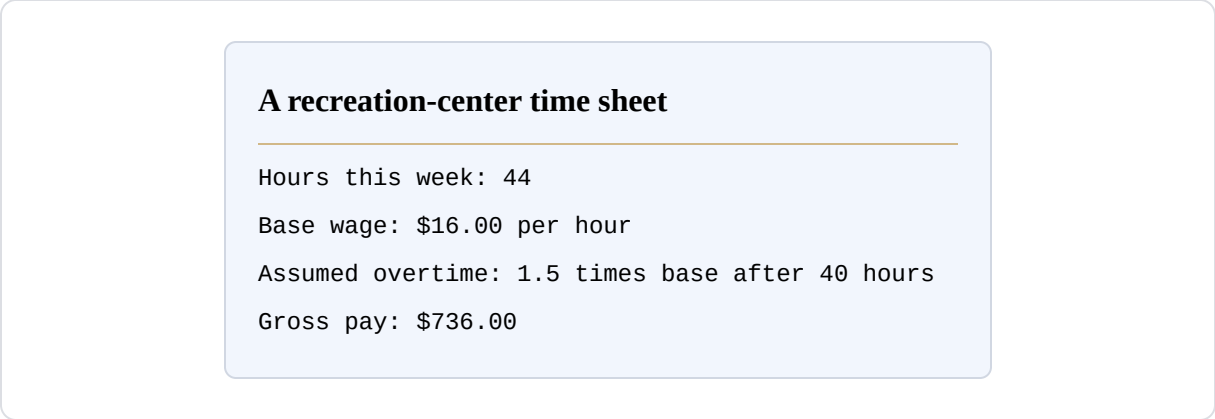


Figure 9.1 A recreation-center time sheet (hypothetical)

The concept

Hourly gross pay is earnings before deductions. Multiply paid hours by the hourly wage. Time records may list hours and minutes rather than decimal hours: 30 minutes is 30 ÷ 60 = 0.50 hour, not 0.30 hour.

Overtime calculations depend on the specific rules given. For this section only, assume 1.5 times base pay for hours above 40 in one week. These are fictional employment terms, not legal guidance. Count the first 40 hours at the regular rate and only the excess at the overtime rate.

Separate regular hours, overtime hours, and their rates before multiplying. The overtime rate of 1.5 times base already includes the regular wage for those hours; do not add another full base wage for the same overtime hours.

When combining shifts, subtract only breaks the problem explicitly labels unpaid. Keep the time calculation exact until you have decimal paid hours, then calculate each pay component and round to cents. The gross amount is not the same as take-home pay because deductions may follow.

Separate regular and overtime hours

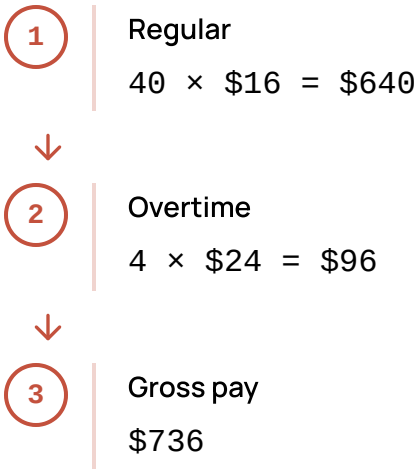


Figure 9.1.V Separate regular and overtime hours. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

PAYROLL
The system and records a business uses to pay its workers.

GROSS PAY
Total earnings before any deductions are taken out.

NET PAY
Take-home pay after taxes and other deductions.

KEY TERMS

Hourly wage
Pay per paid hour of work.

Gross pay
Earnings before employee deductions.

Regular hours
Hours compensated at the base rate under the stated rule.

Overtime premium
Additional pay rate above the base rate.

Decimal hours
Time expressed in hours plus a fraction of an hour.

FORMULAS & RULES

Decimal time

```
Decimal hours = whole hours + minutes ÷ 60
```

Regular wages

```
Regular wages = regular hours × base hourly rate
```

Overtime rate

```
Overtime rate = base rate × stated multiplier
```

Gross wages

```
Gross wages = regular wages + overtime wages
```

Worked examples

Example 1 Read paid time

A student works 12 hours 30 minutes at $15 per hour, with no unpaid breaks or overtime. Find gross pay.

SOLUTION

1. Convert minutes: 30 ÷ 60 = 0.50 hour.
2. Paid time: 12 + 0.50 = 12.50 hours.

3 Gross pay: 12.50 × 15 = $187.50.

Answer: Gross pay: $187.50.

Example 2 Apply the assumed overtime rule

A worker earns $16 per hour for 44 hours. Assume 1.5 times base for hours over 40. Find gross pay.

SOLUTION

1 Regular hours = 40; overtime hours = 44 − 40 = 4.

2 Regular pay: 40 × 16 = $640.00.

3 Overtime rate: 16 × 1.5 = $24.00 per hour.

4 Overtime pay: 4 × 24 = $96.00.

5 Gross pay: 640 + 96 = $736.00.

Answer: Gross pay: $736.00.

Practice exercises 9.1

1. Earn $14 per hour for 18 hours. Find gross pay.
2. Convert 7 hours 45 minutes to decimal hours.
3. Find pay for 7.75 hours at $20 per hour.
4. Use the section overtime rule for 42 hours at $18 per hour.
5. A shift lasts 8 hours including a 30-minute unpaid break. Pay is $16 per hour with no overtime. Find gross.

RECAP QUIZ 9.1

1. 15 minutes is how many hours?

- ○ a. 0.15
- ○ b. 0.25
- ○ c. 0.50
- ○ d. 1.5

2. Time-and-a-half on $20 per hour is:

- ○ a. $10
- ○ b. $20
- ○ c. $30
- ○ d. $40

3. Under the section rule, a 43-hour week has:

- ○ a. 3 overtime hours
- ○ b. 40 overtime hours
- ○ c. 43 overtime hours
- ○ d. No overtime hours

SECTION 9.1 RECAP

- Convert minutes using 60, not 100.
- Apply the stated overtime rule only to eligible hours.

Answer key with explanations: Section 9.1

PRACTICE EXERCISES

1. 18 × 14 = $252.00.
2. 45 ÷ 60 = 0.75; total = 7.75 hours.
3. 7.75 × 20 = $155.00.

4. Regular = 40 × 18 = $720.00; overtime rate = $27.00; overtime = 2 × 27 = $54.00; gross = $774.00.
5. Paid hours = 8 − 30 ÷ 60 = 7.5; gross = 7.5 × 16 = $120.00.

RECAP QUIZ

1. **0.25**: 15 ÷ 60 = 0.25 hour.
2. **$30**: 20 × 1.5 = $30 per overtime hour.
3. **3 overtime hours**: 43 − 40 = 3 hours above the assumed threshold.

9.2

Salaries and Commissions

LEARNING OBJECTIVES

After this section you will be able to:

1. Convert annual salary to periodic gross pay.
2. Calculate straight commission and base-plus-commission pay.
3. Apply tiered commission rates to the correct sales slices.

REAL-LIFE SITUATION

A sales assistant compares pay plans

Use this hypothetical Philadelphia-area scenario to connect the calculation to a decision. All people, businesses, amounts, and rates in this chapter are invented; tax and payroll rules are teaching assumptions, not current law.

A sales assistant compares pay plans

```
Annual salary example: $31,200.00
26 biweekly checks: $1,200.00 each
Separate sales plan: $300.00 base + 4% of $5,000 sales
Sales-plan gross: $500.00
```

Figure 9.2 A sales assistant compares pay plans (hypothetical)

The concept

A salary is a stated amount for a pay period or year rather than a direct product of hours and rate. For these examples, assume 52 weekly periods, 26 biweekly periods, 24 semimonthly periods, or 12 monthly periods per year. Actual payroll calendars and partial periods can differ.

Biweekly means every two weeks; semimonthly means twice a month. They do not produce the same number of payments under these assumptions. Divide annual salary by the stated count, retaining exact values until the payment is rounded. A real payroll may adjust a final cent to reconcile the annual amount.

Commission is pay based on eligible sales or another stated measure. Straight commission has no base pay in the problem; base-plus-commission adds the stated base earnings. Use eligible sales after any specified returns rather than assuming all sales qualify.

A tiered plan can pay different rates on different portions of sales. In the plans here, only the amount above a threshold receives the higher rate. Do not apply the higher rate to all sales unless the contract explicitly says to. A sales target does not guarantee a paycheck amount because the sales may not occur.

Two different pay arrangements

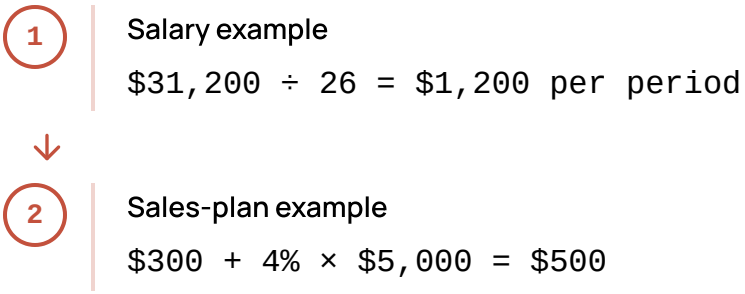


Figure 9.2.V Two different pay arrangements. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

OVERTIME
Extra pay, often 1.5 times the normal rate, for hours worked beyond a set limit.

COMMISSION
Pay based on a percentage of the sales a worker makes.

SALARY
A fixed amount of pay, usually stated per year, that does not depend on hours worked.

KEY TERMS

Salary
Stated compensation for a period rather than pay directly per hour.

Biweekly
Every two weeks; assumed 26 periods annually here.

Semimonthly
Twice monthly; 24 periods annually.

Commission
Pay calculated from eligible sales using a stated rate.

Tiered commission
Different rates applied to specified portions of eligible sales.

FORMULAS & RULES

Periodic salary

```
Gross pay per period = annual salary ÷ number of periods
```

Straight commission

```
Commission = eligible sales × decimal commission rate
```

Base plus commission

```
Gross pay = base pay + commission
```

Tiered commission

```
Commission = sum of (sales in each tier × tier rate)
```

Worked examples

Example 1 Compare pay schedules

An annual salary is $31,200. Find gross pay for 26 biweekly checks and 24 semimonthly checks.

SOLUTION

1. Biweekly count is 26 under the stated assumption.
2. Biweekly pay: 31200 ÷ 26 = $1,200.00.

3. Semimonthly pay: 31200 ÷ 24 = $1,300.00.
4. Check: 26 × 1200 = 24 × 1300 = $31,200.00.

Answer: Biweekly gross $1,200.00; semimonthly gross $1,300.00.

Example 2 Apply tiers to slices

A plan pays $200 base, 3% of the first $4,000 sales, and 5% of sales above $4,000. Find gross on $6,000 eligible sales.

SOLUTION

1. First-tier commission: 4000 × 0.03 = $120.00.
2. Excess sales: 6000 − 4000 = $2,000.00.
3. Second-tier commission: 2000 × 0.05 = $100.00.
4. Gross: 200 + 120 + 100 = $420.00.

Answer: Gross pay: $420.00.

Practice exercises 9.2

1. Convert $39,000 annual salary to 26 biweekly checks.
2. Find straight commission at 6% on $2,500 eligible sales.
3. Add a $300 base to 4% commission on $5,000 sales.
4. Gross sales are $4,000 with $500 returns. Commission is 8% of net eligible sales. Find commission.
5. A plan pays 2% on the first $1,000 and 4% on the excess. Find commission on $1,500.

RECAP QUIZ 9.2

1. How many semimonthly checks occur in a full year?

- ○ a. 12
- ○ b. 24
- ○ c. 26
- ○ d. 52

2. Five percent commission on $800 is:

- ○ a. $4
- ○ b. $16
- ○ c. $40
- ○ d. $160

3. In a marginal tier plan, the upper rate applies to:

- ○ a. Only sales in that tier
- ○ b. Every dollar of sales
- ○ c. Only base salary
- ○ d. No sales

SECTION 9.2 RECAP

- Distinguish biweekly from semimonthly pay.
- Apply commission rates to the eligible sales and tiers specified.

Answer key with explanations: Section 9.2

PRACTICE EXERCISES

1. 39000 ÷ 26 = $1,500.00 gross per check.
2. 2500 × 0.06 = $150.00.
3. Commission = $200.00; gross = 300 + 200 = $500.00.
4. Eligible sales = 4000 − 500 = $3,500.00; commission = 3500 × 0.08 = $280.00.

5. First tier = 1000 × 0.02 = $20.00; excess tier = 500 × 0.04 = $20.00; total = $40.00.

RECAP QUIZ

1. **24**: Two checks per month × 12 months = 24.
2. **$40**: 800 × 0.05 = $40.
3. **Only sales in that tier**: Each slice of sales receives the rate assigned to that slice.

9.3

Payroll Deductions and Net Pay

LEARNING OBJECTIVES

After this section you will be able to:

1. Read gross earnings and deductions on a pay stub.
2. Compute net pay from stated deductions.
3. Distinguish employee deductions from employer-only costs.

REAL-LIFE SITUATION

A hypothetical pay stub

Use this hypothetical Philadelphia-area scenario to connect the calculation to a decision. All people, businesses, amounts, and rates in this chapter are invented; tax and payroll rules are teaching assumptions, not current law.

A hypothetical pay stub

```
Gross earnings: $600.00
Assumed withholding: 12% of gross = $72.00
Other employee deduction: $25.00
Net pay: $503.00
```

Figure 9.3 A hypothetical pay stub (hypothetical)

The concept

A pay stub reconciles earnings with the amount paid to the employee. Gross pay includes the earnings components named in the problem. Employee deductions are amounts subtracted before determining net pay. Always check the pay period and distinguish current-period figures from year-to-date totals.

Withholding is money retained from a paycheck under a stated payroll procedure. The flat percentages in these exercises are fictional arithmetic assumptions, not current tax tables or a calculation of final tax liability. Actual tax withholding and tax owed are different concepts.

For this section only, percentage deductions apply to gross pay unless another base is explicitly provided. Fixed deductions are then subtracted as stated, with no assumed effect on taxable earnings. Round each deduction to cents before adding deductions and subtracting their total from gross.

Employer-only payroll charges increase an employer's compensation cost but do not reduce employee net pay in these problems. Keep the employer calculation separate. When reviewing a stub, recompute gross minus employee deductions and investigate a mismatch rather than assuming the printed deposit is correct.

Gross-to-net payroll

Gross pay
`$600.00`

↓

Deductions
`$72.00 + $25.00`

↓

Net pay
`$503.00`

1. Gross pay is earnings before deductions.
2. Subtract $72 withholding and $25 other deductions: $97 total.
3. Take-home pay = $600 − $97 = $503.00. Rates here are teaching assumptions, not current tax law.

Original WealthWiseGuys diagram · Hypothetical teaching quantities.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

DEDUCTION
Money removed from gross pay for taxes, benefits, or other reasons.

BUDGET
A plan that shows expected income and how it will be spent.

BALANCE
The amount of money in an account at a given moment.

KEY TERMS

Net pay
Gross earnings less employee deductions.

Withholding
Amount retained from earnings under a stated payroll procedure.

Fixed deduction
Specified dollar amount withheld per period.

Year-to-date
Cumulative total from the start of the year through the current period.

Employer-only cost
Employer expense not subtracted from employee pay in the model.

FORMULAS & RULES

Percentage deduction

```
Deduction = stated base × decimal deduction rate
```

Total deductions

```
Total deductions = sum of rounded employee deductions
```

Net pay

```
Net pay = gross pay − total employee deductions
```

Employer cost

```
Employer cost = gross pay + employer-only costs
```

Worked examples

Example 1 Reconcile a pay stub

Gross pay is $600. Assume 12% withholding on gross plus a $25 fixed deduction. Find net pay.

SOLUTION

1. Withholding: 600 × 0.12 = $72.00.
2. Total employee deductions: 72 + 25 = $97.00.
3. Net pay: 600 − 97 = $503.00.

> **Answer:** Net pay: $503.00 under these hypothetical rules.

Example 2 Keep employer costs separate

Gross pay is $800. Employee deductions are 10% of gross plus $30. Employer-only charges are 7% of gross. Find net pay and employer cost.

SOLUTION

1. Employee percentage deduction: 800 × 0.10 = $80.00.
2. Net pay: 800 − 80 − 30 = $690.00.
3. Employer-only charge: 800 × 0.07 = $56.00.
4. Employer cost: 800 + 56 = $856.00.

> **Answer:** Employee net pay is $690.00; employer cost is $856.00.

Practice exercises 9.3

1. Gross is $450 and total employee deductions are $65. Find net pay.
2. Gross is $500 with hypothetical 8% withholding and $20 fixed deduction. Find net.
3. Gross is $700 with separate hypothetical deductions of 10% and 3% of gross. Find net.
4. A stub lists gross $620, employee deductions $95, and deposit $535. Check it.
5. Gross is $900, employee deductions are $140, and employer-only costs are $72. Find net and employer cost.

RECAP QUIZ 9.3

1. $1,000 gross less $180 employee deductions gives net pay of:

- ○ a. $820
- ○ b. $1,180
- ○ c. $180
- ○ d. $1,000

2. Employer-only costs in these examples:

- ○ a. Reduce net pay
- ○ b. Increase employer cost
- ○ c. Are employee tips
- ○ d. Replace gross pay

3. Hypothetical 5% withholding on $640 is:

- ○ a. $3.20
- ○ b. $64
- ○ c. $32
- ○ d. $128

SECTION 9.3 RECAP

- Reconcile gross minus rounded employee deductions to net pay.

- Payroll assumptions here are invented, and employer-only costs stay separate.

Answer key with explanations: Section 9.3

PRACTICE EXERCISES

1. 450 − 65 = $385.00.
2. Withholding = $40.00; deductions = $60.00; net = $440.00.
3. Deductions = $70.00 and $21.00; total = $91.00; net = $609.00.
4. Expected net = 620 − 95 = $525.00. The listed deposit is $10.00 higher; investigate the discrepancy.
5. Net = 900 − 140 = $760.00; employer cost = 900 + 72 = $972.00. Do not subtract the $72 from net.

RECAP QUIZ

1. **$820**: 1000 − 180 = $820.
2. **Increase employer cost**: They are added to employer cost, not subtracted from the employee check.
3. **$32**: 640 × 0.05 = $32.

CHAPTER 10

Simple Interest and Borrowing

Borrowing can move a purchase into today, but repayment uses tomorrow's money. Learn to calculate simple interest, solve for missing loan terms, and compare offers on the same basis. All people, prices, rates, and loan terms in this chapter are hypothetical; round final money amounts to cents and retain unrounded intermediate values unless stated otherwise.

10.1

Principal, Rate, and Time

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate simple interest and maturity value.
2. Convert months and days to fractions of a year.
3. Distinguish a stated interest rate from a dollar interest charge.

REAL-LIFE SITUATION

A bike for a Philadelphia commute

A student considers a short-term loan for a used bicycle. The disclosure separates the amount borrowed from the cost of using it.

A bike for a Philadelphia commute

Principal: $600.00

Simple annual rate: 8%

Term: 9 months

Interest: $36.00; repayment: $636.00

Figure 10.1 A bike for a Philadelphia commute (hypothetical)

The concept

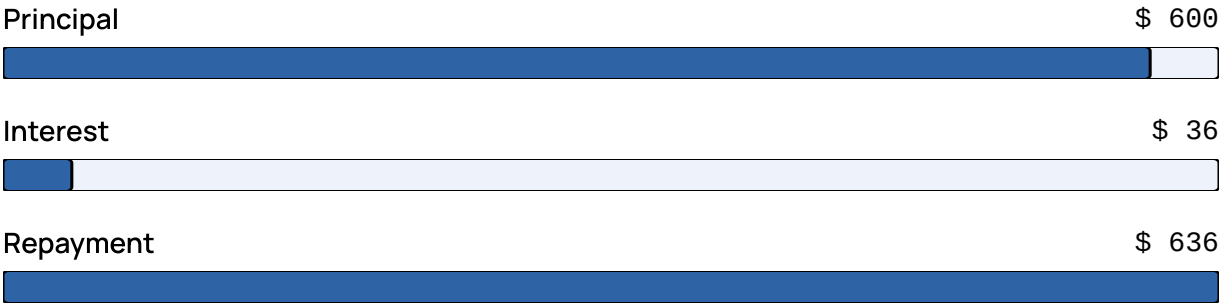
Simple interest is a charge for using an original amount of money, called principal. Unlike compound interest, it does not add earlier interest to the base for the next interest calculation. A lender quoting 8% annually is quoting a rate, not an $8 charge on every loan.

The formula I = P × r × t requires matching units. Write an annual percent as a decimal and express time in years. Nine months is 9 ÷ 12 year, so an annual rate applies for only three-fourths of a year.

For day-based loans, the agreement must specify a day-count convention. An actual/365 calculation divides actual elapsed days by 365; an actual/360 calculation divides those same days by 360. Neither denominator may be assumed just because a year usually has 365 days.

Maturity value is principal plus interest when the entire loan is repaid at the end. This model assumes no interim payments or fees. Read the payment terms before applying it to a real borrowing arrangement.

Principal and repayment



Bars share a zero baseline.

Figure 10.1.V Principal and repayment. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

PRINCIPAL

The original amount of money borrowed or invested, before interest.

SIMPLE INTEREST

Interest calculated only on the original principal, not on interest already earned.

INTEREST RATE

The percent used to calculate how much interest is earned or charged.

KEY TERMS

Principal
The original sum borrowed or invested.

Simple interest
Interest calculated on original principal only.

Annual rate

Interest per year expressed as a fraction of principal.

Term
The length of the loan.

Maturity value
Principal plus interest payable at the end of the term.

FORMULAS & RULES

Simple interest

$$I = P \times r \times t$$

Maturity value

$$M = P + I$$

Month conversion

$$t = \text{months} \div 12$$

Actual/365 time

$$t = \text{actual days} \div 365$$

Worked examples

Example 1 The bicycle loan

Find interest and repayment on $600 at 8% simple annual interest for 9 months.

SOLUTION

1 Convert 8% to r = 0.08.

2. Convert time: t = 9 ÷ 12 = 0.75 year.
3. I = 600 × 0.08 × 0.75 = $36.00.
4. M = 600 + 36 = $636.00.

> **Answer:** Interest is $36.00 and maturity value is $636.00.

Example 2 A day-count convention

A $1,200 loan charges 6% simple annual interest for 45 actual days using actual/365. Find the amount due.

SOLUTION

1. Use r = 0.06 and t = 45 ÷ 365 year.
2. I = 1,200 × 0.06 × 45 ÷ 365 = $8.876712… .
3. Round the interest to $8.88.
4. M = 1,200 + 8.88 = $1,208.88.

> **Answer:** The borrower owes $1,208.88.

Practice exercises 10.1

1. Find interest on $800 at 5% for 6 months.
2. Find maturity value on $1,500 at 4% for 2 years.
3. A $900 loan runs 60 actual days at 9%, actual/360. Find interest.
4. Compare actual/360 and actual/365 interest on $730 at 10% for 36 actual days.
5. A friend calculates 500 × 0.06 × 4 for a four-month loan. Correct the interest.

RECAP QUIZ 10.1

1. Interest on $200 at 6% for one year is:

- a. $6.00
- b. $12.00
- c. $206.00
- d. $212.00

2. Three months expressed in years is:

- a. 3
- b. 0.03
- c. 0.25
- d. 4

3. A loan has principal $400 and interest $18. What is its maturity value?

- a. $18
- b. $382
- c. $400
- d. $418

SECTION 10.1 RECAP

- Match annual rates with time measured in years.
- Simple interest uses original principal; maturity value adds principal and interest.

Answer key with explanations: Section 10.1

PRACTICE EXERCISES

1. $t = 6 \div 12 = 0.5$; $I = 800 \times 0.05 \times 0.5 = \$20.00$.
2. $I = 1{,}500 \times 0.04 \times 2 = \$120.00$; $M = 1{,}500 + 120 = \$1{,}620.00$.
3. $t = 60 \div 360 = 1/6$; $I = 900 \times 0.09 \times 1/6 = \$13.50$.

4. Actual/360: 730 × 0.10 × 36 ÷ 360 = $7.30. Actual/365: 730 × 0.10 × 36 ÷ 365 = $7.20. The first costs $0.10 more.
5. Four months is 4 ÷ 12 year, not 4 years. I = 500 × 0.06 × 4 ÷ 12 = $10.00, not $120.00.

RECAP QUIZ

1. **$12.00**: 200 × 0.06 × 1 = $12.00; repayment would be $212.00.
2. **0.25**: 3 ÷ 12 = 0.25 year.
3. **$418**: M = P + I = 400 + 18 = $418.

10.2

Solving for Missing Loan Terms

LEARNING OBJECTIVES

After this section you will be able to:

1. Rearrange the simple-interest formula.
2. Find principal, annual rate, or time from a stated interest charge.
3. Check answers by substituting into the original equation.

REAL-LIFE SITUATION

Working backward from a charge

A student sees a $45 interest charge on a $1,000 loan lasting nine months. Working backward reveals the annual rate.

Working backward from a charge

```
P = $1,000.00; I = $45.00
t = 9 ÷ 12 = 0.75 year
r = 45 ÷ (1,000 × 0.75) = 0.06 = 6%
```

Figure 10.2 Working backward from a charge (hypothetical)

The concept

The simple-interest equation can answer more than the interest question. If three quantities are known, isolate the fourth using inverse operations. Dividing both sides by the same nonzero product preserves equality.

To find principal, divide interest by rate times time. To find the annual rate, divide interest by principal times time. The resulting rate is a decimal; multiply by 100 to express it as a percent.

To find time, divide interest by principal times annual rate. That answer is in years. Multiply years by 12 for months, or by the agreement's day-count denominator for days. A fractional month is a mathematical result, not permission to change a contractual due date.

Check a rearranged formula by substitution. Also check scale: charging $45 on $1,000 over less than a year should imply an annual rate above 4.5%, not below it.

Recover the annual interest rate

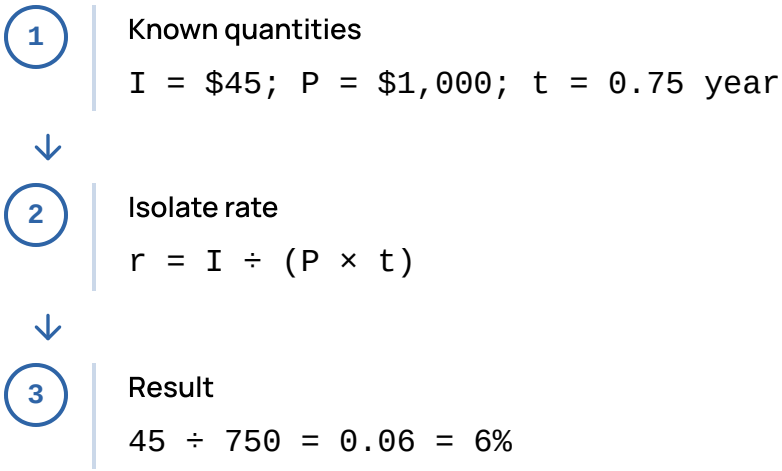


Figure 10.2.V Recover the annual interest rate. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

LOAN TERM

How long a loan lasts before it must be fully repaid.

TOTAL REPAYMENT COST

The full amount paid over a loan's life — principal plus all interest and fees.

APR

Annual percentage rate — the yearly cost of borrowing, including interest and some fees, shown as a percent.

KEY TERMS

Inverse operation
An operation that undoes another, such as division undoing multiplication.

Unknown
A quantity to be determined.

Decimal rate
A percent divided by 100.

Substitution
Replacing symbols with values to check an equation.

FORMULAS & RULES

Find principal

```
P = I ÷ (r × t)
```

Find rate

```
r = I ÷ (P × t)
```

Find time

```
t = I ÷ (P × r)
```

Worked examples

Example 1 Recover the annual rate

A $1,000 loan earns $45 simple interest in 9 months. Find the annual rate.

SOLUTION

1. t = 9 ÷ 12 = 0.75 year.
2. r = 45 ÷ (1,000 × 0.75) = 0.06.
3. Convert 0.06 to 6%.
4. Check: 1,000 × 0.06 × 0.75 = $45.00.

Answer: The annual simple-interest rate is 6%.

Example 2 How long was the money borrowed?

A $750 principal incurs $30 interest at 8% simple annual interest. Find the term in months.

SOLUTION

1. P × r = 750 × 0.08 = 60 dollars per year.
2. t = 30 ÷ 60 = 0.5 year.
3. Months = 0.5 × 12 = 6.
4. Check: 750 × 0.08 × 6 ÷ 12 = $30.00.

Answer: The term is 6 months.

Practice exercises 10.2

1. Find principal if interest is $24 at 6% for 8 months.
2. A $500 loan costs $15 interest over 6 months. Find the annual rate.
3. How many months produce $48 interest on $1,200 at 8%?
4. A $2,000 loan has maturity value $2,090 after 9 months. Find its annual simple rate.
5. At 9% actual/360, how many days produce $18 interest on $800?

RECAP QUIZ 10.2

1. Which formula finds principal?

- ○ a. $I \times r \times t$
- ○ b. $I \div (r \times t)$
- ○ c. $I \div P$
- ○ d. $P + I$

2. $20 interest on $400 over half a year implies which annual rate?

- ○ a. 5%
- ○ b. 2.5%
- ○ c. 10%
- ○ d. 20%

3. A calculated term of 1.5 years equals:

- ○ a. 18 months
- ○ b. 15 months
- ○ c. 6 months
- ○ d. 1.5 months

SECTION 10.2 RECAP

- Rearrange before substituting values.
- Rate answers are decimals until converted to percentages; time answers follow the rate unit.

Answer key with explanations: Section 10.2

PRACTICE EXERCISES

1. $t = 8 \div 12$; $P = 24 \div (0.06 \times 8 \div 12) = \$600.00$.
2. $r = 15 \div (500 \times 0.5) = 0.06 = 6\%$ annually.
3. $t = 48 \div (1{,}200 \times 0.08) = 0.5$ year; $0.5 \times 12 = 6$ months.
4. $I = 2{,}090 - 2{,}000 = \$90$; $r = 90 \div (2{,}000 \times 0.75) = 0.06 = 6\%$.
5. $t = 18 \div (800 \times 0.09) = 0.25$ year; days $= 0.25 \times 360 = 90$.

RECAP QUIZ

1. **$I \div (r \times t)$**: Dividing $I = P \times r \times t$ by $r \times t$ isolates P.

2. **10%**: r = 20 ÷ (400 × 0.5) = 0.10 = 10%.
3. **18 months**: 1.5 × 12 = 18 months.

10.3

Fees, Proceeds, and Fair Loan Comparisons

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate usable proceeds and total borrowing cost.
2. Compare offers for equal cash needs and equal terms.
3. Distinguish a simple annualized cost indicator from a disclosed APR.

REAL-LIFE SITUATION

Two ways to fund a repair

A neighborhood repair worker needs $1,000 for one year. Both offers provide the same usable cash, but one adds an origination fee.

Two ways to fund a repair

```
Offer A: $1,000 cash; 8% simple interest; no fee
Offer B: $1,000 cash; 6% simple interest; $30
fee paid at maturity
A repayment: $1,080.00; B repayment: $1,090.00
```

Figure 10.3 Two ways to fund a repair (hypothetical)

The concept

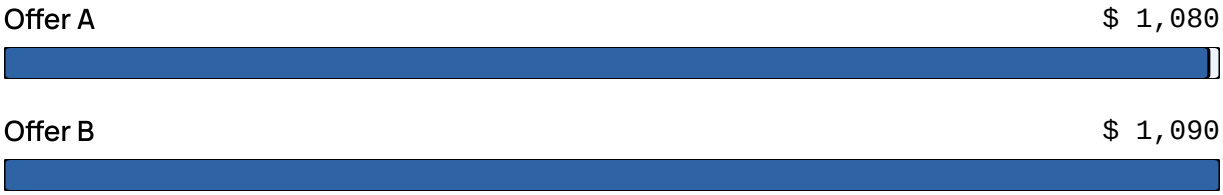
A low stated rate does not necessarily mean a low total cost. Origination fees, processing fees, and other required charges add to interest. Identify which charges are required, when they are paid, and whether they are financed.

Usable proceeds are the money actually available for the intended purchase. A $1,000 face amount with a $40 fee withheld delivers only $960. If the borrower needs $1,000 cash, that loan is not directly comparable with a loan that delivers the full amount.

For a single repayment, a classroom annualized cost indicator divides total borrowing cost by usable proceeds and by years. It is useful for transparent comparisons under the stated assumptions, but it is not a substitute for a legally defined annual percentage rate. APR calculations and disclosures depend on the cash-flow pattern and applicable rules.

Compare equal usable cash, equal terms, and all required payments. Affordability is a separate question: a mathematically cheaper loan can still be difficult to repay. All fee arrangements here are hypothetical examples, not statements of current lending law.

Compare total repayments, including fees



Bars share a zero baseline.

Figure 10.3.V Compare total repayments, including fees. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

LOAN
Money borrowed that must be repaid, usually with interest.

MONTHLY PAYMENT
The amount paid each month toward a loan or installment plan.

CREDIT
The ability to buy now and pay later, or money borrowed that must be repaid.

KEY TERMS

Origination fee
A charge for arranging a loan.

Usable proceeds
Cash the borrower can actually use after withheld charges.

Borrowing cost
Total required outflow minus usable cash received.

APR

A standardized annual cost measure whose calculation follows applicable disclosure rules.

Single-payment loan
A loan repaid in one payment at maturity.

FORMULAS & RULES

Net proceeds

```
Proceeds = face amount − withheld fees
```

Single-payment cost

```
Cost = maturity payment + separate required fees − proceeds
```

Classroom annualization

```
Annualized indicator = cost ÷ proceeds ÷ t
```

Worked examples

Example 1 Compare equal proceeds

Compare the two one-year $1,000-cash offers in the figure.

SOLUTION

1. A interest = 1,000 × 0.08 × 1 = $80.00.
2. A maturity payment = 1,000 + 80 = $1,080.00.
3. B interest = 1,000 × 0.06 × 1 = $60.00.
4. B maturity payment = 1,000 + 60 + 30 = $1,090.00.

5 Cost difference = 90 − 80 = $10.00.

> **Answer:** Offer A costs $10.00 less despite its higher stated rate.

Example 2 A withheld fee

A one-year loan has face amount $1,000, a $40 withheld fee, and $80 interest due at maturity. Find proceeds, cost, and the classroom annualized indicator.

SOLUTION

1 Proceeds = 1,000 − 40 = $960.00.

2 Maturity payment = 1,000 + 80 = $1,080.00.

3 Cost = 1,080 − 960 = $120.00.

4 Indicator = 120 ÷ 960 ÷ 1 = 0.125 = 12.5%.

> **Answer:** Usable cash is $960.00; cost is $120.00; the indicator is 12.5%, not a claimed legal APR.

Practice exercises 10.3

1. A $700 face loan withholds $25 and requires $735 at maturity. Find proceeds and cost.
2. A $500-cash six-month loan requires $525 at maturity with no other fees. Find its classroom annualized indicator.
3. Compare one-year $800-cash loans: A charges 7% with no fees; B charges 5% plus $20 due at maturity.
4. A lender deducts $30 from a $600 loan. Does it fully fund a $600 expense?
5. A one-year $900-cash loan charges 4% interest plus a $15 fee at maturity. Find repayment and total cost.

RECAP QUIZ 10.3

1. A $1,000 face loan withholds $50. Usable proceeds are:

- ○ a. $1,050
- ○ b. $1,000
- ○ c. $950
- ○ d. $50

2. Which is essential for a fair comparison?

- ○ a. Compare rates only
- ○ b. Equal usable cash and term
- ○ c. Ignore fees
- ○ d. Use the smallest payment only

3. $600 received and $648 repaid one year later means what cost?

- ○ a. $48
- ○ b. $648
- ○ c. $600
- ○ d. $1,248

SECTION 10.3 RECAP

- Compare total costs, not rates alone.
- A withheld fee reduces usable cash; a classroom annualized indicator is not automatically the disclosed APR.

Answer key with explanations: Section 10.3

PRACTICE EXERCISES

1. Proceeds = 700 − 25 = $675.00; cost = 735 − 675 = $60.00.
2. Cost = 525 − 500 = $25; t = 0.5; indicator = 25 ÷ 500 ÷ 0.5 = 0.10 = 10%.
3. A cost = 800 × 0.07 = $56.00. B cost = 800 × 0.05 + 20 = $60.00. A costs $4.00 less.

4. Proceeds = 600 − 30 = $570.00. The shortfall is 600 − 570 = $30.00, so no.
5. Interest = 900 × 0.04 = $36.00; repayment = 900 + 36 + 15 = $951.00; cost = 951 − 900 = $51.00.

RECAP QUIZ

1. **$950**: 1,000 − 50 = $950.
2. **Equal usable cash and term**: Equal cash and term keep the comparison focused on total cost.
3. **$48**: 648 − 600 = $48 of borrowing cost.

CHAPTER 11

Promissory Notes and Financial Obligations

A promissory note turns a borrowing arrangement into a written schedule of obligations. This chapter connects the words on a note to dates, interest, discount proceeds, and payment records. All notes, people, rates, fees, and legal arrangements are hypothetical teaching examples, not legal advice or current law; round money to cents unless stated otherwise.

11.1

Reading Notes and Finding Maturity Dates

LEARNING OBJECTIVES

After this section you will be able to:

1. Identify the parties and key terms on a note.
2. Find a due date for a day-based note.
3. Calculate maturity value under a specified convention.

REAL-LIFE SITUATION

A food-cart equipment note

A Philadelphia food-cart operator signs a 60-day note for equipment. The document states the annual rate, day-count basis, and single maturity

payment.

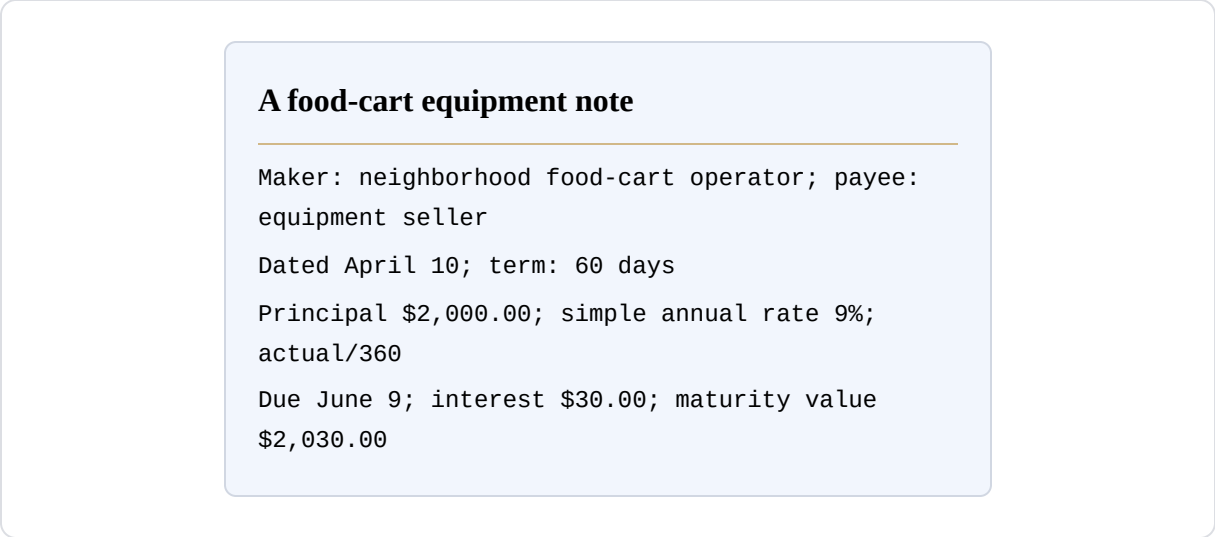


Figure 11.1 A food-cart equipment note (hypothetical)

The concept

A promissory note is a written promise to pay a specified amount under specified terms. The maker promises payment; the payee is the party named to receive it. Principal, date, rate, term, and payment instructions help translate that promise into a cash obligation.

In the day-count method used here, exclude the date the note is issued and include the maturity date. Count actual calendar days rather than pretending that each month has the same length. A 60-day note is not necessarily a two-calendar-month note.

The calendar determines how many days elapse; the interest convention determines the denominator used to convert those days to years. An actual/360 note still counts actual days even though interest uses a 360-day year. A note stated in calendar months instead follows its month-based due-date terms.

Before signing, identify any security, guarantor, late charges, or early-payment terms. These can change obligations without changing the printed principal. Weekend and holiday adjustments depend on the agreement and applicable rules; the classroom examples make no such adjustments.

Follow the note to maturity

1. Issue
   `April 10; principal $2,000`
2. Term
   `60 days; interest $30`
3. Maturity
   `June 9; amount due $2,030`

Figure 11.1.V Follow the note to maturity. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

PROMISSORY NOTE
A written promise to repay a specific amount by a certain date.

MATURITY DATE
The date a loan or note must be fully repaid.

BANK DISCOUNT
Interest deducted from a loan in advance, so the borrower receives less than the face amount.

KEY TERMS

Promissory note
A written promise to pay under stated terms.

Maker

The party promising to pay the note.

Payee
The party named to receive payment.

Maturity date
The date payment becomes due.

Collateral
Property pledged to secure an obligation.

Guarantor
A party who undertakes responsibility under a guarantee if its conditions are met.

FORMULAS & RULES

Day-based interest

```
I = P × r × (days ÷ year basis)
```

Maturity value

```
M = P + I
```

Days remaining

```
Remaining days = stated term − days already counted
```

Worked examples

Example 1 Date and value of a note

Find the maturity date and value of the figure's April 10, 60-day, $2,000 note at 9%, actual/360.

SOLUTION

1 April 11–30 contributes 30 − 10 = 20 days.

2. May contributes 31 days; 60 − 20 − 31 = 9 days remain in June.
3. The maturity date is June 9; exclude April 10 and include June 9.
4. I = 2,000 × 0.09 × 60 ÷ 360 = $30.00.
5. M = 2,000 + 30 = $2,030.00.

> **Answer:** Payment of $2,030.00 is due June 9.

Example 2 A year-end note

A $1,460 note dated November 20 runs 45 days at 5%, actual/365. Find due date and maturity value; assume no holiday adjustment.

SOLUTION

1. November 21–30 contributes 10 days.
2. December adds 31 days, leaving 45 − 10 − 31 = 4 days.
3. Maturity is January 4 of the following year.
4. I = 1,460 × 0.05 × 45 ÷ 365 = $9.00.
5. M = 1,460 + 9 = $1,469.00.

> **Answer:** The note matures January 4 for $1,469.00.

Practice exercises 11.1

1. Find the due date of a 30-day note dated September 12.
2. Find the due date of a 20-day note dated February 20 in a nonleap year.

3. Find maturity value for $900 at 8% for 90 days, actual/360.
4. A maker signs a note payable to an equipment seller. Who promises payment and who receives it?
5. A note dated May 5 is due June 4. Count days and find interest on $1,200 at 6%, actual/360.

RECAP QUIZ 11.1

1. Who makes the promise to pay?

- ○ a. Payee
- ○ b. Maker
- ○ c. Witness
- ○ d. Customer of the payee

2. A 10-day note dated April 25 matures:

- ○ a. May 4
- ○ b. May 5
- ○ c. May 10
- ○ d. April 30

3. $600 at 6% for 60 days, actual/360, has interest of:

- ○ a. $6
- ○ b. $36
- ○ c. $60
- ○ d. $606

SECTION 11.1 RECAP

- Read the parties, rate, term, and payment provisions together.
- Exclude the issue date and include the due date for these day-based notes.

Answer key with explanations: Section 11.1

PRACTICE EXERCISES

1. September contributes 30 − 12 = 18 days; 30 − 18 = 12 days remain. The due date is October 12.
2. February contributes 28 − 20 = 8 days; 20 − 8 = 12 days remain. Maturity is March 12.
3. I = 900 × 0.08 × 90 ÷ 360 = $18.00; M = 900 + 18 = $918.00.
4. The maker promises payment; the named equipment seller is the payee. These roles follow the promise, not the physical possession of the equipment.
5. May 6–31 gives 26 days; June 1–4 gives 4, totaling 30. I = 1,200 × 0.06 × 30 ÷ 360 = $6.00.

RECAP QUIZ

1. **Maker**: The maker is the party promising payment.
2. **May 5**: April 26–30 is 5 days; May 1–5 supplies the other 5.
3. **$6**: 600 × 0.06 × 60 ÷ 360 = $6.

11.2

Bank Discount and Cash Proceeds

LEARNING OBJECTIVES

After this section you will be able to:

1. Distinguish interest-bearing notes from bank-discount notes.
2. Calculate a discount and net proceeds.
3. Measure cost relative to the cash actually received.

REAL-LIFE SITUATION

Cash withheld before borrowing begins

A small vendor signs a non-interest-bearing discount note. Its face amount is the promised repayment; the lender subtracts the bank discount before handing over cash.

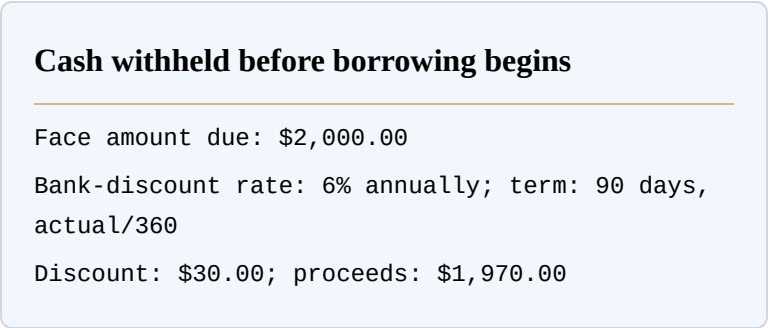


Figure 11.2 Cash withheld before borrowing begins (hypothetical)

The concept

An interest-bearing note usually advances principal and adds interest at maturity. A bank-discount note instead subtracts its financing charge in advance. In the non-interest-bearing notes used here, face amount is also the maturity amount; no additional interest is added at maturity.

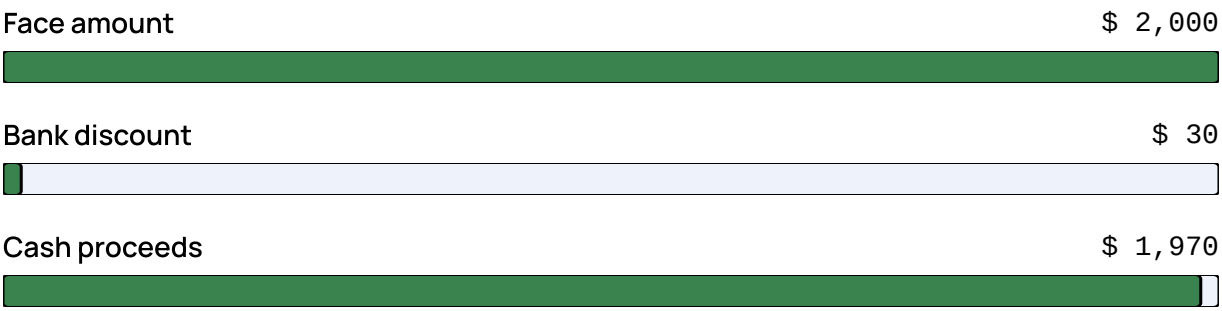
Bank discount is calculated on maturity value, not on usable cash. Multiply the maturity amount by the discount rate and time. Then subtract that discount to find proceeds. The borrower receives less than the amount promised for repayment.

A bank-discount rate understates the charge as a fraction of money actually received. A classroom simple annualized proceeds-based rate divides the discount by proceeds and by years. This indicator is not asserted to be a statutory APR.

A business can also sell an existing interest-bearing note before maturity. First determine that note's maturity value, then discount that maturity

value for the remaining time. The two intervals and possibly the two rates serve different purposes.

Face amount is not cash received



Bars share a zero baseline.

Figure 11.2.V Face amount is not cash received. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

PRINCIPAL
The original amount of money borrowed or invested, before interest.

INTEREST
The cost of borrowing money, or the earnings on money saved or invested.

COLLATERAL
Property a borrower promises to a lender if the loan is not repaid, such as a car.

KEY TERMS

Face amount
The amount written on a note; for a non-interest-bearing discount note, the maturity obligation.

Bank discount
A financing charge deducted in advance from maturity value.

Discount rate
The annual rate applied to maturity value to calculate bank discount.

Proceeds
Cash received after discount is withheld.

Remaining term
Time from the discount date to the maturity date.

FORMULAS & RULES

Bank discount

$$D = M \times d \times t$$

Proceeds

$$\text{Proceeds} = M - D$$

Proceeds-based indicator

$$\text{Annualized indicator} = D \div \text{proceeds} \div t$$

Worked examples

Example 1 A discounted note

Find discount, proceeds, and the annualized proceeds-based indicator for the figure's $2,000 note.

SOLUTION

1. $t = 90 \div 360 = 0.25$ year.
2. $D = 2{,}000 \times 0.06 \times 0.25 = \$30.00$.

3. Proceeds = 2,000 − 30 = $1,970.00.
4. Indicator = 30 ÷ 1,970 ÷ 0.25 = 0.0609137… .
5. Express as approximately 6.09% annually.

> **Answer:** The borrower receives $1,970.00 and owes $2,000.00; the indicator is about 6.09%.

Example 2 Selling an interest-bearing note

A $3,000 note earns 8% simple interest over 120 days, actual/360. It is sold with 30 days remaining at a 6% bank-discount rate, actual/360. Find sale proceeds.

SOLUTION

1. Original interest = 3,000 × 0.08 × 120 ÷ 360 = $80.00.
2. Maturity value = 3,000 + 80 = $3,080.00.
3. Remaining time = 30 ÷ 360 year.
4. Bank discount = 3,080 × 0.06 × 30 ÷ 360 = $15.40.
5. Sale proceeds = 3,080 − 15.40 = $3,064.60.

> **Answer:** The seller receives $3,064.60; the maker still owes $3,080.00 at maturity.

Practice exercises 11.2

1. A $1,000 non-interest-bearing note is discounted at 8% for 90 days, actual/360. Find discount and proceeds.

2. A maturity amount of $1,500 has $45 withheld. Find proceeds and the one-year proceeds-based indicator.
3. Find proceeds for a $2,400 discount note at 5% for 60 days, actual/360.
4. An existing note has maturity value $4,080 and 45 days remaining. Find bank discount at 4%, actual/360.
5. A student discounts $2,000 principal instead of its $2,040 maturity value for 30 days at 6%, actual/360. Correct the proceeds.

RECAP QUIZ 11.2

1. A discount note with face $500 and discount $10 provides:

- ○ a. $510
- ○ b. $500
- ○ c. $490
- ○ d. $10

2. Bank discount on an existing note is based on:

- ○ a. Original principal only
- ○ b. Maturity value
- ○ c. Proceeds only
- ○ d. Interest only

3. On a discount note, the proceeds-based annualized indicator is generally:

- ○ a. Below the discount rate
- ○ b. Equal to zero
- ○ c. Above the discount rate
- ○ d. Always the legal APR

SECTION 11.2 RECAP

- Bank discount is withheld up front and computed on maturity value.

- Use only the remaining time when discounting an existing note.

Answer key with explanations: Section 11.2

PRACTICE EXERCISES

1. D = 1,000 × 0.08 × 90 ÷ 360 = $20.00; proceeds = 1,000 − 20 = $980.00.
2. Proceeds = 1,500 − 45 = $1,455.00; indicator = 45 ÷ 1,455 ÷ 1 ≈ 3.09%.
3. D = 2,400 × 0.05 × 60 ÷ 360 = $20.00; proceeds = $2,380.00.
4. Use maturity value: D = 4,080 × 0.04 × 45 ÷ 360 = $20.40.
5. D = 2,040 × 0.06 × 30 ÷ 360 = $10.20; proceeds = 2,040 − 10.20 = $2,029.80. Discount applies to maturity value.

RECAP QUIZ

1. **$490**: Proceeds = 500 − 10 = $490.
2. **Maturity value**: D = M × d × t uses maturity value M.
3. **Above the discount rate**: The same charge is divided by proceeds smaller than maturity value, making this indicator larger.

11.3

Partial Payments and Obligation Records

LEARNING OBJECTIVES

After this section you will be able to:

1. Allocate partial payments under a stated interest-first rule.
2. Calculate later interest on reduced principal.

3. Separate contractual fees from principal and interest.

REAL-LIFE SITUATION

A payment before maturity

A hypothetical note permits partial payments without penalty. Its stated allocation rule applies a payment first to accrued interest and then to principal, with actual/360 simple interest.

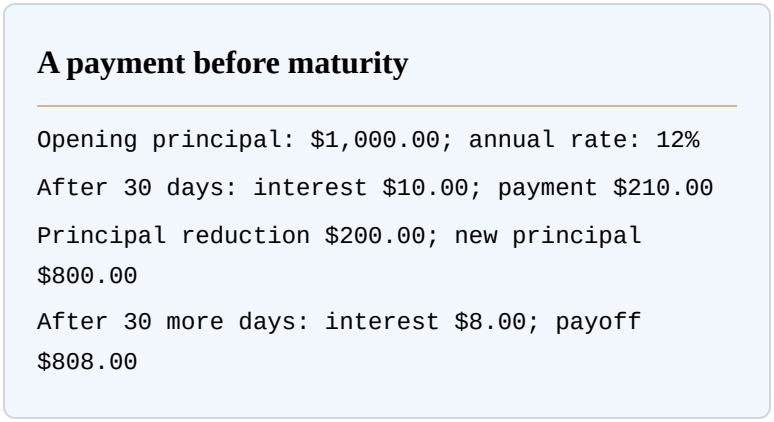


Figure 11.3 A payment before maturity (hypothetical)

The concept

A payment record should distinguish principal, accrued interest, and fees. A payment is not automatically a principal reduction. Under this section's hypothetical interest-first rule, first calculate interest since the last transaction, then apply the payment to interest and use any remainder to reduce principal.

After principal changes, later simple interest is computed on the reduced outstanding principal. Do not continue charging on the original principal after a qualifying reduction, and do not charge a full original term again after every payment.

The examples assume each partial payment exceeds accrued interest. If a payment does not cover interest, the agreement must specify how unpaid

interest is handled; do not silently capitalize it or declare the obligation satisfied. These examples do not establish a legal allocation rule.

Late fees and other obligations must follow the stated agreement and applicable rules. For classroom records, show a hypothetical fee separately and specify whether it earns interest. A clean record preserves dates, opening principal, elapsed time, interest, payment, and closing principal.

Record a partial payment

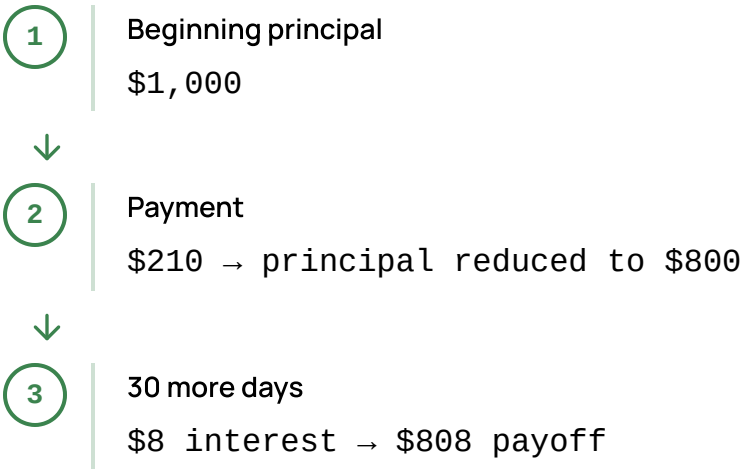


Figure 11.3.V Record a partial payment. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

LOAN TERM

How long a loan lasts before it must be fully repaid.

DEFAULT

Failing to repay a loan as agreed, which can harm credit and lead to collection.

PAYMENT TERMS

The rules in an agreement about when a bill must be paid and any discounts for paying early.

KEY TERMS

Accrued interest
Interest earned or owed since the last relevant date.

Partial payment
A payment smaller than the full outstanding obligation.

Interest-first allocation
A stated method applying payment to accrued interest before principal.

Principal reduction
The portion of a payment that decreases outstanding principal.

Payoff
The amount required to settle the obligation at a specified time.

FORMULAS & RULES

Accrued interest

```
I = outstanding principal × r × t
```

Principal reduction

```
Reduction = payment − accrued interest
```

New balance

```
New principal = old principal − reduction
```

Payoff

```
Payoff = outstanding principal + accrued interest +
unpaid separate fees
```

Worked examples

Example 1 Two periods, one partial payment

Use the figure's $1,000 note at 12%: pay $210 after 30 days and settle 30 days later, actual/360.

SOLUTION

1. First-period interest = 1,000 × 0.12 × 30 ÷ 360 = $10.00.
2. Payment toward principal = 210 − 10 = $200.00.
3. New principal = 1,000 − 200 = $800.00.
4. Second-period interest = 800 × 0.12 × 30 ÷ 360 = $8.00.
5. Payoff = 800 + 8 = $808.00; total paid = 210 + 808 = $1,018.00.

> **Answer:** The final payment is $808.00; total interest is $18.00.

Example 2 A separately stated fee

A $600 note at 6%, actual/360, is settled after 60 days. A hypothetical $15 fee is due and earns no interest. Find payoff.

SOLUTION

1. $t = 60 \div 360 = 1/6$ year.
2. Accrued interest $= 600 \times 0.06 \times 1/6 = \$6.00$.
3. Keep the non-interest-bearing fee separate: $15.00.
4. Payoff $= 600 + 6 + 15 = \$621.00$.

Answer: Payoff is $621.00, including $6.00 interest and a $15.00 fee.

Practice exercises 11.3

1. On $1,200 at 9%, actual/360, pay $318 after 60 days. Find the new principal.
2. The new principal is $900 at 9%. Find payoff 40 days later, actual/360.
3. A $150 payment meets $12 accrued interest. How much reduces principal?
4. A statement shows $450 principal, $7.50 interest, and a separate $10 non-interest-bearing fee. Find payoff.
5. A $5 payment arrives when accrued interest is $8. Can the standard reduction formula be treated as a principal increase automatically?

RECAP QUIZ 11.3

1. A $100 payment with $9 accrued interest reduces principal by:

- ○ a. $100
- ○ b. $109
- ○ c. $91
- ○ d. $9

2. After a valid principal reduction, later simple interest uses:

- ○ a. Original principal forever
- ○ b. Reduced outstanding principal
- ○
- ○

c. Total past payments

d. Only the fee

3. $300 principal, $4 interest, and $6 separate fee require payoff of:

- ○ a. $304
- ○ b. $306
- ○ c. $310
- ○ d. $290

SECTION 11.3 RECAP

- Under the stated interest-first rule, only payment beyond accrued interest reduces principal.
- Use a new time interval and the updated principal after each payment.

Answer key with explanations: Section 11.3

PRACTICE EXERCISES

1. I = 1,200 × 0.09 × 60 ÷ 360 = $18; principal reduction = 318 − 18 = $300; new principal = $900.00.
2. I = 900 × 0.09 × 40 ÷ 360 = $9.00; payoff = 900 + 9 = $909.00.
3. Interest gets $12; principal receives 150 − 12 = $138.00.
4. Payoff = 450 + 7.50 + 10 = $467.50; the fee is added once, not treated as principal.
5. No. Interest exceeds payment by 8 − 5 = $3.00. Consult the agreement for unpaid-interest treatment; do not automatically capitalize the $3 or reduce principal.

RECAP QUIZ

1. **$91**: 100 − 9 = $91 reaches principal.
2. **Reduced outstanding principal**: The new principal is the base for the next interval.
3. **$310**: 300 + 4 + 6 = $310.

CHAPTER 12

Compound Interest, Exponents, and Present Value

A cash amount means more when its date is clear. This chapter builds timeline models for future expenses and savings, using hypothetical people, rates, and amounts rather than promised returns. Money is rounded to cents at the end unless a minimum-goal deposit must be rounded upward.

12.1

Exponents and Compound Growth

LEARNING OBJECTIVES

After this section you will be able to:

1. Place the relevant cash flows on a timeline.
2. Calculate the modeled value using a matching periodic rate.
3. Interpret the result and identify the model's assumptions.

REAL-LIFE SITUATION

A student savings timeline

A Philadelphia student uses a hypothetical fixed-rate model to compare cash flows for future school or equipment costs.

A student savings timeline

```
Initial deposit: $1,000.00
Annual rate: 5%; annual compounding; 2 years
Year 1: $1,050.00; year 2: $1,102.50
```

Figure 12.1 A student savings timeline (hypothetical)

The concept

An exponent tells how many times a factor is multiplied by itself. In $(1 + i)^n$, i is the rate per compounding period and n counts periods, not necessarily years. Square means two factors; it does not mean multiply the factor by two.

Compound interest adds each period's interest to the balance, so later interest is earned on both original principal and earlier interest. At a positive fixed rate this produces more growth than simple interest over multiple periods.

For a single deposit with no withdrawals or fees, multiply principal by the accumulation factor. Keep all calculator digits until the final result. These models round money only at the end; a provider that rounds each period can produce a slightly different result.

All rates and cash flows are hypothetical classroom assumptions, not investment advice or promised returns. Retain full precision through intermediate calculations and round final money to cents; for a minimum savings target, round the deposit upward instead. Draw the timeline before calculating and explain what the result means.

Compound interest timeline

Year 0	Year 1	Year 2
`$1,000.00`	`$1,050.00`	`$1,102.50`
Principal	Interest: $50.00	Interest: $52.50

1. Year 0: deposit $1,000.
2. Year 1: multiply by 1.05; $50 interest gives $1,050.
3. Year 2: multiply the new balance by 1.05; $52.50 interest gives $1,102.50. Growth is not guaranteed in an investment.

Original WealthWiseGuys diagram · Hypothetical teaching quantities.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

COMPOUND INTEREST

Interest earned on both the original amount and the interest already earned.

PRESENT VALUE

What a future amount of money is worth today at a given interest rate.

FUTURE VALUE

What an amount of money today will be worth at a later date with interest.

KEY TERMS

Exponent
The number of repeated factors.

Periodic rate
The rate applying in one compounding period.

Compound interest
Interest on principal and previously credited interest.

Future value
The modeled balance at a later date.

FORMULAS & RULES

Exponents and Compound Growth

$$FV = P \times (1 + i)^n$$

Related relationship

$$Interest = FV - P$$

Worked examples

Example 1 The timeline in the figure

Find future value of $1,000.00 at 5% annually compounded for 2 years.

SOLUTION

1. Use P = $1,000.00, i = 0.05, and n = 2.
2. Growth factor = $1.05^2 = 1.1025$.
3. FV = 1000 × 1.1025 = $1,102.50.
4. Interest = $1,102.50 − $1,000.00 = $102.50.

> **Answer:** $1,102.50

Example 2 A different amount

Find future value of $2,000.00 at 5% annually compounded for 2 years.

SOLUTION

1. Use P = $2,000.00, i = 0.05, and n = 2.
2. Growth factor = $1.05^2 = 1.1025$.
3. FV = 2000 × 1.1025 = $2,205.00.
4. Interest = $2,205.00 − $2,000.00 = $205.00.

> **Answer:** $2,205.00

Practice exercises 12.1

1. Find future value of $1,000.00 at 5% annually compounded for 2 years.
2. Find future value of $2,000.00 at 5% annually compounded for 2 years.
3. Find future value of $500.00 at 5% annually compounded for 2 years.
4. Find future value of $3,000.00 at 5% annually compounded for 2 years.
5. Find future value of $4,000.00 at 5% annually compounded for 2 years.

RECAP QUIZ 12.1

1. Find future value of $1,000.00 at 5% annually compounded for 2 years.

- ○ a. $1,102.50
- ○ b. $1,000.00
- ○ c. $900.00
- ○ d. $1,500.00

2. Which practice is correct?

- ○ a. Use an annual rate in every month
- ○ b. Match the rate period to the timeline
- ○ c. Round every growth factor to two decimals
- ○ d. Treat an assumed return as guaranteed

3. How should intermediate growth factors be handled?

- ○ a. Round to whole numbers
- ○ b. Always replace them with 1
- ○ c. Retain full precision
- ○ d. Ignore them

SECTION 12.1 RECAP

- Draw the timeline and match the rate to each interval.
- Keep intermediate precision; distinguish modeled values from guaranteed outcomes.

Answer key with explanations: Section 12.1

PRACTICE EXERCISES

1. Use P = \$1,000.00, i = 0.05, and n = 2. Growth factor = 1.05^2 = 1.1025. FV = 1000 × 1.1025 = \$1,102.50. Interest = \$1,102.50 − \$1,000.00 = \$102.50.
2. Use P = \$2,000.00, i = 0.05, and n = 2. Growth factor = 1.05^2 = 1.1025. FV = 2000 × 1.1025 = \$2,205.00. Interest = \$2,205.00 − \$2,000.00 = \$205.00.
3. Use P = \$500.00, i = 0.05, and n = 2. Growth factor = 1.05^2 = 1.1025. FV = 500.0 × 1.1025 = \$551.25. Interest = \$551.25 − \$500.00 = \$51.25.
4. Use P = \$3,000.00, i = 0.05, and n = 2. Growth factor = 1.05^2 = 1.1025. FV = 3000 × 1.1025 = \$3,307.50. Interest = \$3,307.50 − \$3,000.00 = \$307.50.
5. Use P = \$4,000.00, i = 0.05, and n = 2. Growth factor = 1.05^2 = 1.1025. FV = 4000 × 1.1025 = \$4,410.00. Interest = \$4,410.00 − \$4,000.00 = \$410.00.

RECAP QUIZ

1. **$1,102.50**: Use P = $1,000.00, i = 0.05, and n = 2. Growth factor = $1.05^2 = 1.1025$. FV = 1000 × 1.1025 = $1,102.50. Interest = $1,102.50 − $1,000.00 = $102.50.
2. **Match the rate period to the timeline**: The periodic rate and period count must describe the same interval; assumptions are not guarantees.
3. **Retain full precision**: Retaining calculator precision avoids accumulating rounding errors; round the final monetary result.

12.2

Compounding Frequency and Effective Rates

LEARNING OBJECTIVES

After this section you will be able to:

1. Place the relevant cash flows on a timeline.
2. Calculate the modeled value using a matching periodic rate.
3. Interpret the result and identify the model's assumptions.

REAL-LIFE SITUATION

A student savings timeline

A Philadelphia student uses a hypothetical fixed-rate model to compare cash flows for future school or equipment costs.

A student savings timeline

```
Initial deposit: $1,200.00
Nominal annual rate: 6%; compounded monthly
Monthly rate: 0.5%; periods: 12
One-year value: $1,274.01
```

Figure 12.2 A student savings timeline (hypothetical)

The concept

A nominal annual rate does not directly state the growth over a year when compounding occurs more often than annually. Divide the nominal rate by the number of equal compounding periods per year to find the periodic rate.

Monthly compounding uses m = 12 and n = 12 × years. Do not put the annual rate into every monthly period; doing so greatly overstates growth. A fractional year must correspond to the compounding schedule specified in the model.

The effective annual rate measures one full year of growth after compounding. At a positive nominal rate it exceeds the nominal rate if interest compounds more than once a year. Compare effective rates only after also checking fees, access restrictions, and risk.

All rates and cash flows are hypothetical classroom assumptions, not investment advice or promised returns. Retain full precision through intermediate calculations and round final money to cents; for a minimum savings target, round the deposit upward instead. Draw the timeline before calculating and explain what the result means.

Monthly compounding

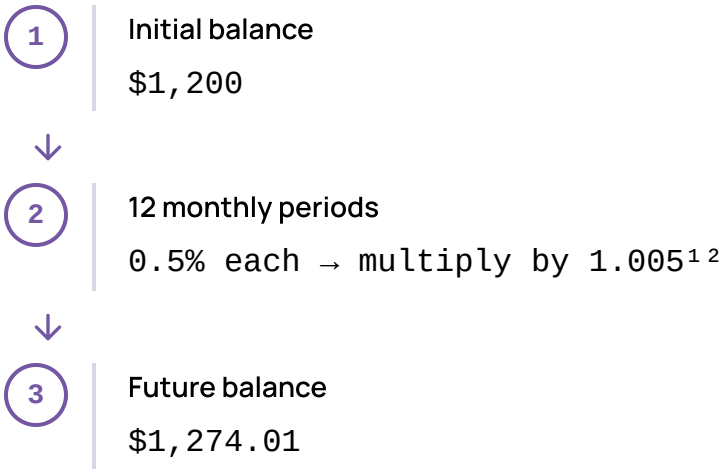


Figure 12.2.V Monthly compounding. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

PRINCIPAL
The original amount of money borrowed or invested, before interest.

INTEREST RATE
The percent used to calculate how much interest is earned or charged.

INVESTMENT
Money put into something with the hope it will grow or earn income later.

KEY TERMS

Nominal annual rate
An annual quoted rate before within-year compounding.

Frequency
The number of compounding periods each year.

Effective annual rate
One-year proportional growth after compounding.

Period count
Total number of compounding intervals.

FORMULAS & RULES

Compounding Frequency and Effective Rates

$$FV = P \times (1 + r \div m)^{n}$$

Related relationship

$$\text{Effective annual rate} = (1 + r \div m)^{m} - 1$$

Worked examples

Example 1 The timeline in the figure

Find the one-year value of $1,200.00 at 6% nominal annual interest compounded monthly.

SOLUTION

1. Use principal $1,200.00 and nominal r = 0.06.
2. Monthly i = 0.06 ÷ 12 = 0.005; n = 12.
3. FV = 1200 × 1.005^{12} = $1,274.01.
4. Effective annual rate = $1.005^{12} - 1 \approx 6.17\%$.

> **Answer:** $1,274.01

Example 2 A different amount

Find the one-year value of $2,400.00 at 6% nominal annual interest compounded monthly.

SOLUTION

1. Use principal $2,400.00 and nominal r = 0.06.
2. Monthly i = 0.06 ÷ 12 = 0.005; n = 12.
3. FV = $2400 \times 1.005^{12} = \$2{,}548.03$.
4. Effective annual rate = $1.005^{12} - 1 \approx 6.17\%$.

> **Answer:** $2,548.03

Practice exercises 12.2

1. Find the one-year value of $1,200.00 at 6% nominal annual interest compounded monthly.
2. Find the one-year value of $2,400.00 at 6% nominal annual interest compounded monthly.
3. Find the one-year value of $600.00 at 6% nominal annual interest compounded monthly.
4. Find the one-year value of $3,600.00 at 6% nominal annual interest compounded monthly.
5. Find the one-year value of $4,800.00 at 6% nominal annual interest compounded monthly.

RECAP QUIZ 12.2

1. Find the one-year value of $1,200.00 at 6% nominal annual interest compounded monthly.

- ○ a. $1,274.01
- ○ b. $1,200.00
- ○ c. $1,080.00
- ○ d. $1,800.00

2. Which practice is correct?

- ○ a. Use an annual rate in every month
- ○ b. Match the rate period to the timeline
- ○ c. Round every growth factor to two decimals
- ○ d. Treat an assumed return as guaranteed

3. How should intermediate growth factors be handled?

- ○ a. Round to whole numbers
- ○ b. Always replace them with 1
- ○ c. Retain full precision
- ○ d. Ignore them

SECTION 12.2 RECAP

- Draw the timeline and match the rate to each interval.
- Keep intermediate precision; distinguish modeled values from guaranteed outcomes.

Answer key with explanations: Section 12.2

PRACTICE EXERCISES

1. Use principal $1,200.00 and nominal r = 0.06. Monthly i = 0.06 ÷ 12 = 0.005; n = 12. FV = $1200 \times 1.005^{12} = \$1,274.01$. Effective annual rate = $1.005^{12} - 1 \approx 6.17\%$.
2. Use principal $2,400.00 and nominal r = 0.06. Monthly i = 0.06 ÷ 12 = 0.005; n = 12. FV = $2400 \times 1.005^{12} = \$2,548.03$. Effective annual rate = $1.005^{12} - 1 \approx 6.17\%$.
3. Use principal $600.00 and nominal r = 0.06. Monthly i = 0.06 ÷ 12 = 0.005; n = 12. FV = $600.0 \times 1.005^{12} = \$637.01$. Effective annual rate = $1.005^{12} - 1 \approx 6.17\%$.

4. Use principal $3,600.00 and nominal r = 0.06. Monthly i = 0.06 ÷ 12 = 0.005; n = 12. FV = 3600 × 1.005^{12} = $3,822.04. Effective annual rate = $1.005^{12} - 1 \approx 6.17\%$.
5. Use principal $4,800.00 and nominal r = 0.06. Monthly i = 0.06 ÷ 12 = 0.005; n = 12. FV = 4800 × 1.005^{12} = $5,096.05. Effective annual rate = $1.005^{12} - 1 \approx 6.17\%$.

RECAP QUIZ

1. **$1,274.01**: Use principal $1,200.00 and nominal r = 0.06. Monthly i = 0.06 ÷ 12 = 0.005; n = 12. FV = 1200 × 1.005^{12} = $1,274.01. Effective annual rate = $1.005^{12} - 1 \approx 6.17\%$.
2. **Match the rate period to the timeline**: The periodic rate and period count must describe the same interval; assumptions are not guarantees.
3. **Retain full precision**: Retaining calculator precision avoids accumulating rounding errors; round the final monetary result.

12.3

Present Value and Financial Equivalence

LEARNING OBJECTIVES

After this section you will be able to:

1. Place the relevant cash flows on a timeline.
2. Calculate the modeled value using a matching periodic rate.
3. Interpret the result and identify the model's assumptions.

REAL-LIFE SITUATION

A student savings timeline

A Philadelphia student uses a hypothetical fixed-rate model to compare cash flows for future school or equipment costs.

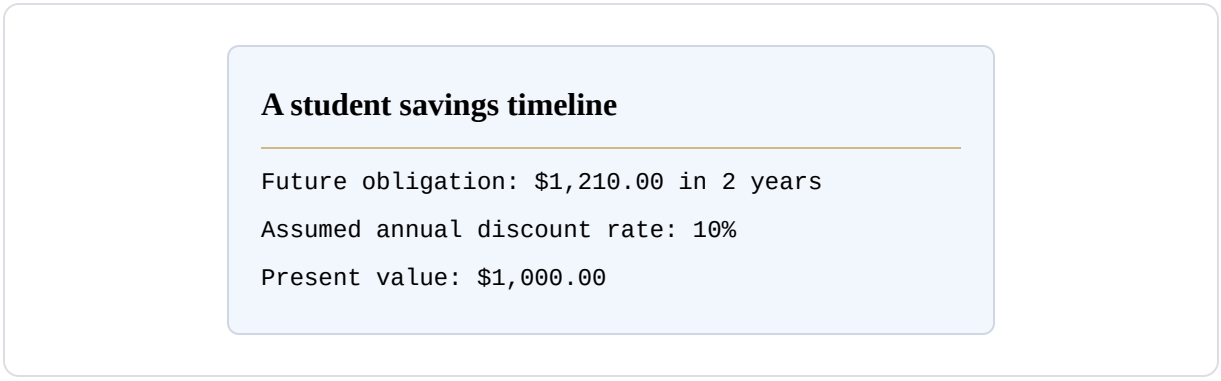


Figure 12.3 A student savings timeline (hypothetical)

The concept

Present value asks how much today is mathematically equivalent to a specified future sum under a stated rate model. It reverses accumulation: divide by the growth factor instead of multiplying.

Place the future payment on a timeline and count the compounding intervals back to today. Use the rate per interval. Discounting at a positive rate makes present value smaller than future value; this is a useful reasonableness check.

A discount rate is an assumption, not a promised return. Financial equivalence in this model does not establish that an account can safely produce the assumed rate. Fees, inflation, taxes, and uncertainty are excluded unless specifically modeled.

All rates and cash flows are hypothetical classroom assumptions, not investment advice or promised returns. Retain full precision through intermediate calculations and round final money to cents; for a minimum savings target, round the deposit upward instead. Draw the timeline before calculating and explain what the result means.

Equivalent amounts at different dates

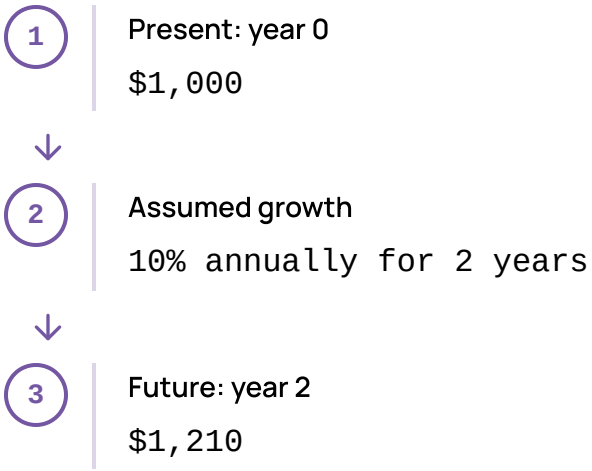


Figure 12.3.V Equivalent amounts at different dates. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

RETURN
The money or gain an investment produces, often shown as a percent.

ANNUITY
A series of equal payments made at regular intervals, such as monthly.

INFLATION
A general rise in prices over time, which makes money buy less.

KEY TERMS

Present value
The equivalent amount at the start of a timeline.

Discounting
Moving a future amount backward using a rate model.

Discount rate
The assumed rate used to value later cash flows today.

Financial equivalence
Equality of values at a common date under specified assumptions.

FORMULAS & RULES

Present Value and Financial Equivalence

$$PV = FV \div (1 + i)^n$$

Related relationship

$$FV = PV \times (1 + i)^n$$

Worked examples

Example 1 The timeline in the figure

Find present value of $1,210.00 due in 2 years using 10% annually compounded.

SOLUTION

1. Future value is $1,210.00; i = 0.10; n = 2.
2. Accumulation factor = $1.10^2 = 1.21$.
3. PV = 1210 ÷ 1.21 = $1,000.00.
4. Check by multiplying the unrounded PV by 1.21 to recover the future amount.

Answer: $1,000.00

Example 2 A different amount

Find present value of $2,420.00 due in 2 years using 10% annually compounded.

SOLUTION

1. Future value is $2,420.00; i = 0.10; n = 2.
2. Accumulation factor = $1.10^2 = 1.21$.
3. PV = 2420 ÷ 1.21 = $2,000.00.
4. Check by multiplying the unrounded PV by 1.21 to recover the future amount.

> **Answer:** $2,000.00

Practice exercises 12.3

1. Find present value of $1,210.00 due in 2 years using 10% annually compounded.
2. Find present value of $2,420.00 due in 2 years using 10% annually compounded.
3. Find present value of $605.00 due in 2 years using 10% annually compounded.
4. Find present value of $3,630.00 due in 2 years using 10% annually compounded.
5. Find present value of $4,840.00 due in 2 years using 10% annually compounded.

RECAP QUIZ 12.3

1. Find present value of $1,210.00 due in 2 years using 10% annually compounded.

- ○ a. $1,000.00
- ○ b. $1,210.00
- ○ c. $1,089.00
- ○ d. $1,815.00

2. Which practice is correct?

- ○ a. Use an annual rate in every month
- ○ b. Match the rate period to the timeline
- ○ c. Round every growth factor to two decimals
- ○ d. Treat an assumed return as guaranteed

3. How should intermediate growth factors be handled?

- ○ a. Round to whole numbers
- ○ b. Always replace them with 1
- ○ c. Retain full precision
- ○ d. Ignore them

SECTION 12.3 RECAP

- Draw the timeline and match the rate to each interval.
- Keep intermediate precision; distinguish modeled values from guaranteed outcomes.

Answer key with explanations: Section 12.3

PRACTICE EXERCISES

1. Future value is \$1,210.00; i = 0.10; n = 2. Accumulation factor = 1.10^2 = 1.21. PV = 1210 ÷ 1.21 = \$1,000.00. Check by multiplying the unrounded PV by 1.21 to recover the future amount.
2. Future value is \$2,420.00; i = 0.10; n = 2. Accumulation factor = 1.10^2 = 1.21. PV = 2420 ÷ 1.21 = \$2,000.00. Check by multiplying the unrounded PV by 1.21 to recover the future amount.
3. Future value is \$605.00; i = 0.10; n = 2. Accumulation factor = 1.10^2 = 1.21. PV = 605.0 ÷ 1.21 = \$500.00. Check by multiplying the unrounded PV by 1.21 to recover the future amount.
4. Future value is \$3,630.00; i = 0.10; n = 2. Accumulation factor = 1.10^2 = 1.21. PV = 3630 ÷ 1.21 = \$3,000.00. Check by multiplying the unrounded PV by 1.21 to recover

the future amount.

5. Future value is $4,840.00; i = 0.10; n = 2. Accumulation factor = $1.10^2 = 1.21$. PV = 4840 ÷ 1.21 = $4,000.00. Check by multiplying the unrounded PV by 1.21 to recover the future amount.

RECAP QUIZ

1. **$1,000.00**: Future value is $1,210.00; i = 0.10; n = 2. Accumulation factor = $1.10^2 = 1.21$. PV = 1210 ÷ 1.21 = $1,000.00. Check by multiplying the unrounded PV by 1.21 to recover the future amount.
2. **Match the rate period to the timeline**: The periodic rate and period count must describe the same interval; assumptions are not guarantees.
3. **Retain full precision**: Retaining calculator precision avoids accumulating rounding errors; round the final monetary result.

CHAPTER 13

Annuities, Sinking Funds, and Long-Term Savings

A cash amount means more when its date is clear. This chapter builds timeline models for future expenses and savings, using hypothetical people, rates, and amounts rather than promised returns. Money is rounded to cents at the end unless a minimum-goal deposit must be rounded upward.

13.1

Ordinary Annuities and Deposit Timing

LEARNING OBJECTIVES

After this section you will be able to:

1. Place the relevant cash flows on a timeline.
2. Calculate the modeled value using a matching periodic rate.
3. Interpret the result and identify the model's assumptions.

REAL-LIFE SITUATION

A student savings timeline

A Philadelphia student uses a hypothetical fixed-rate model to compare cash flows for future school or equipment costs.

A student savings timeline

```
End-of-month deposit: $100.00 for 3 months
Assumed monthly rate: 1%
At month 3: $102.01 + $101.00 + $100.00 =
$303.01
```

Figure 13.1 A student savings timeline (hypothetical)

The concept

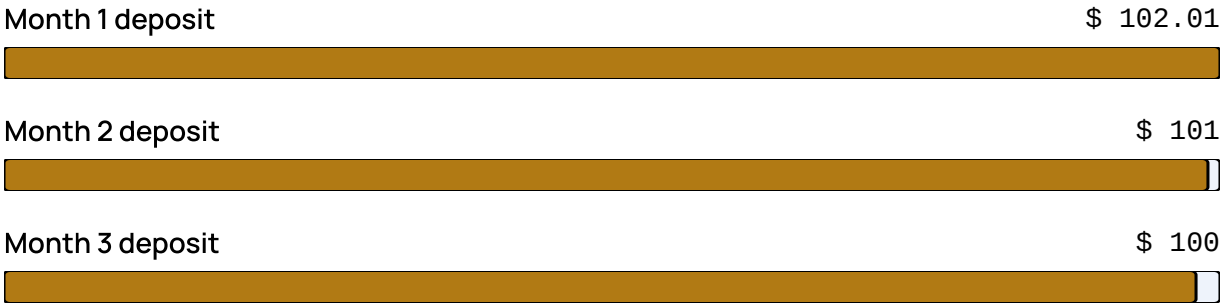
An annuity is a sequence of equal payments at equally spaced times. An ordinary annuity places each payment at the end of its period. Deposits from a part-time job can be modeled this way when both the amounts and timing are regular.

Earlier deposits earn interest longer. At the third deposit date, the first of three end-of-month deposits has grown for two periods, the second for one, and the last for zero. The last deposit is included but earns no interest yet.

The ordinary-annuity factor sums these growth factors. Match the deposit interval to the interest period; these examples assume they match. At zero interest the factor is simply n, because the formula with division by i cannot be used directly.

All rates and cash flows are hypothetical classroom assumptions, not investment advice or promised returns. Retain full precision through intermediate calculations and round final money to cents; for a minimum savings target, round the deposit upward instead. Draw the timeline before calculating and explain what the result means.

Value at month 3 of each $100 deposit



Bars share a zero baseline.

Figure 13.1.V Value at month 3 of each $100 deposit. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

ANNUITY
A series of equal payments made at regular intervals, such as monthly.

SINKING FUND
Money set aside regularly so a future obligation can be paid without borrowing.

FUTURE VALUE
What an amount of money today will be worth at a later date with interest.

KEY TERMS

Annuity
Equal payments at equally spaced intervals.

Ordinary annuity
Payments occur at each period-end.

Annuity due
Payments occur at each period-beginning.

Sinking fund
Regular savings designated for a future obligation.

Valuation date
The date at which cash-flow values are compared.

FORMULAS & RULES

Ordinary Annuities and Deposit Timing

$$FV = R \times ((1 + i)^n - 1) \div i$$

Related relationship

$$\text{Total contributions} = R \times n$$

Worked examples

Example 1 The timeline in the figure

Deposit $100.00 at each month-end for 3 months at an assumed 1% monthly rate. Find value just after deposit 3.

SOLUTION

1. R = $100.00; i = 0.01; n = 3.
2. Factor = $1.01^2 + 1.01 + 1 = 3.0301$.
3. FV = $100 \times 3.0301 = \$303.01$.
4. Contributions = $3 \times 100 = \$300.00$; interest = $3.01.

Answer: $303.01

Example 2 A different amount

Deposit $200.00 at each month-end for 3 months at an assumed 1% monthly rate. Find value just after deposit 3.

SOLUTION

1. R = $200.00; i = 0.01; n = 3.
2. Factor = $1.01^2 + 1.01 + 1 = 3.0301$.
3. FV = 200 × 3.0301 = $606.02.
4. Contributions = 3 × 200 = $600.00; interest = $6.02.

> **Answer:** $606.02

Practice exercises 13.1

1. Deposit $100.00 at each month-end for 3 months at an assumed 1% monthly rate. Find value just after deposit 3.
2. Deposit $200.00 at each month-end for 3 months at an assumed 1% monthly rate. Find value just after deposit 3.
3. Deposit $50.00 at each month-end for 3 months at an assumed 1% monthly rate. Find value just after deposit 3.
4. Deposit $300.00 at each month-end for 3 months at an assumed 1% monthly rate. Find value just after deposit 3.
5. Deposit $400.00 at each month-end for 3 months at an assumed 1% monthly rate. Find value just after deposit 3.

RECAP QUIZ 13.1

1. Deposit $100.00 at each month-end for 3 months at an assumed 1% monthly rate. Find value just after deposit 3.

- ○ a. $303.01
- ○ b. $100.00
- ○ c. $90.00
- ○ d. $150.00

2. Which practice is correct?

- ○ a. Use an annual rate in every month
- ○ b. Match the rate period to the timeline
- ○ c. Round every growth factor to two decimals
- ○ d. Treat an assumed return as guaranteed

3. How should intermediate growth factors be handled?

- ○ a. Round to whole numbers
- ○ b. Always replace them with 1
- ○ c. Retain full precision
- ○ d. Ignore them

SECTION 13.1 RECAP

- Draw the timeline and match the rate to each interval.
- Keep intermediate precision; distinguish modeled values from guaranteed outcomes.

Answer key with explanations: Section 13.1

PRACTICE EXERCISES

1. R = $100.00; i = 0.01; n = 3. Factor = $1.01^2 + 1.01 + 1 = 3.0301$. FV = 100 × 3.0301 = $303.01. Contributions = 3 × 100 = $300.00; interest = $3.01.
2. R = $200.00; i = 0.01; n = 3. Factor = $1.01^2 + 1.01 + 1 = 3.0301$. FV = 200 × 3.0301 = $606.02. Contributions = 3 × 200 = $600.00; interest = $6.02.

3. R = \$50.00; i = 0.01; n = 3. Factor = $1.01^2 + 1.01 + 1 = 3.0301$. FV = 50.0 × 3.0301 = \$151.51. Contributions = 3 × 50.0 = \$150.00; interest = \$1.51.
4. R = \$300.00; i = 0.01; n = 3. Factor = $1.01^2 + 1.01 + 1 = 3.0301$. FV = 300 × 3.0301 = \$909.03. Contributions = 3 × 300 = \$900.00; interest = \$9.03.
5. R = \$400.00; i = 0.01; n = 3. Factor = $1.01^2 + 1.01 + 1 = 3.0301$. FV = 400 × 3.0301 = \$1,212.04. Contributions = 3 × 400 = \$1,200.00; interest = \$12.04.

RECAP QUIZ

1. **\$303.01**: R = \$100.00; i = 0.01; n = 3. Factor = $1.01^2 + 1.01 + 1 = 3.0301$. FV = 100 × 3.0301 = \$303.01. Contributions = 3 × 100 = \$300.00; interest = \$3.01.
2. **Match the rate period to the timeline**: The periodic rate and period count must describe the same interval; assumptions are not guarantees.
3. **Retain full precision**: Retaining calculator precision avoids accumulating rounding errors; round the final monetary result.

13.2

Annuities Due and Present Value

LEARNING OBJECTIVES

After this section you will be able to:

1. Place the relevant cash flows on a timeline.
2. Calculate the modeled value using a matching periodic rate.
3. Interpret the result and identify the model's assumptions.

REAL-LIFE SITUATION

A student savings timeline

A Philadelphia student uses a hypothetical fixed-rate model to compare cash flows for future school or equipment costs.

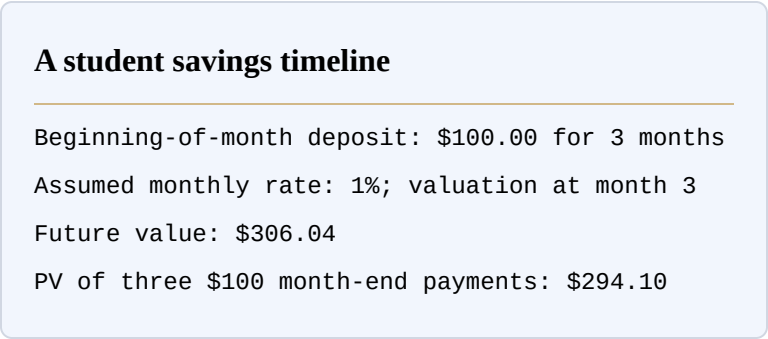


Figure 13.2 A student savings timeline (hypothetical)

The concept

An annuity due makes payments at the beginning of each period. Relative to an ordinary annuity with the same number of payments and the same final valuation date, every deposit earns one extra period of interest. Multiply ordinary-annuity future value by $1 + i$.

Timing must be explicit: three beginning-of-month deposits occur at times 0, 1, and 2, with valuation at time 3. Three month-end deposits occur at times 1, 2, and 3. Confusing these schedules changes the answer even when payment amounts match.

Present value of an ordinary annuity discounts each future payment to today and adds the results. For three \$100 month-end payments at 1% per month, $PV = 100 \div 1.01 + 100 \div 1.01^2 + 100 \div 1.01^3 = \$294.10$. For a due annuity, multiply this ordinary present value by 1.01, giving \$297.04.

All rates and cash flows are hypothetical classroom assumptions, not investment advice or promised returns. Retain full precision through intermediate calculations and round final money to cents; for a minimum savings target, round the deposit upward instead. Draw the timeline before calculating and explain what the result means.

Timing changes deposit value

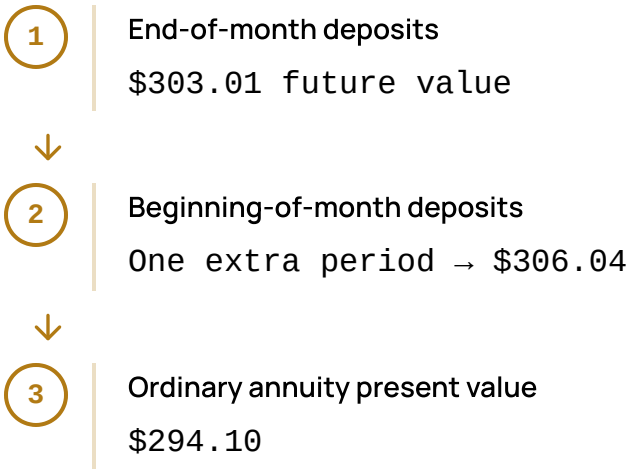


Figure 13.2.V Timing changes deposit value. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

PRESENT VALUE
What a future amount of money is worth today at a given interest rate.

COMPOUND INTEREST
Interest earned on both the original amount and the interest already earned.

DEPOSIT
Money placed into a bank account.

KEY TERMS

Annuity
Equal payments at equally spaced intervals.

Ordinary annuity

Payments occur at each period-end.

Annuity due
Payments occur at each period-beginning.

Sinking fund
Regular savings designated for a future obligation.

Valuation date
The date at which cash-flow values are compared.

FORMULAS & RULES

Annuities Due and Present Value

$$FV\ due = R \times ((1 + i)^{n} - 1) \div i \times (1 + i)$$

Related relationship

$$PV\ ordinary = R \times (1 - (1 + i)^{-n}) \div i$$

Worked examples

Example 1 The timeline in the figure

Find value at month 3 of three beginning-of-month deposits of $100.00 at an assumed 1% monthly rate.

SOLUTION

1. R = $100.00, i = 0.01, n = 3; deposits occur at times 0, 1, and 2.
2. Ordinary future-value factor is 3.0301.
3. Due factor = 3.0301 × 1.01 = 3.060401.
4. FV at time 3 = 100 × 3.060401 = $306.04.

Answer: $306.04

Example 2 A different amount

Find value at month 3 of three beginning-of-month deposits of $200.00 at an assumed 1% monthly rate.

SOLUTION

1. R = $200.00, i = 0.01, n = 3; deposits occur at times 0, 1, and 2.
2. Ordinary future-value factor is 3.0301.
3. Due factor = 3.0301 × 1.01 = 3.060401.
4. FV at time 3 = 200 × 3.060401 = $612.08.

Answer: $612.08

Practice exercises 13.2

1. Find value at month 3 of three beginning-of-month deposits of $100.00 at an assumed 1% monthly rate.
2. Find value at month 3 of three beginning-of-month deposits of $200.00 at an assumed 1% monthly rate.
3. Find value at month 3 of three beginning-of-month deposits of $50.00 at an assumed 1% monthly rate.
4. Find the present value of three $100 month-end payments at 1% per month.
5. Find present value of three $100 beginning-of-month payments at 1% per month.

RECAP QUIZ 13.2

1. Find value at month 3 of three beginning-of-month deposits of $100.00 at an assumed 1% monthly rate.

- ○ a. $306.04
- ○ b. $100.00
- ○ c. $90.00
- ○ d. $150.00

2. Which practice is correct?

- ○ a. Use an annual rate in every month
- ○ b. Match the rate period to the timeline
- ○ c. Round every growth factor to two decimals
- ○ d. Treat an assumed return as guaranteed

3. How should intermediate growth factors be handled?

- ○ a. Round to whole numbers
- ○ b. Always replace them with 1
- ○ c. Retain full precision
- ○ d. Ignore them

SECTION 13.2 RECAP

- Draw the timeline and match the rate to each interval.
- Keep intermediate precision; distinguish modeled values from guaranteed outcomes.

Answer key with explanations: Section 13.2

PRACTICE EXERCISES

1. R = $100.00, i = 0.01, n = 3; deposits occur at times 0, 1, and 2. Ordinary future-value factor is 3.0301. Due factor = 3.0301 × 1.01 = 3.060401. FV at time 3 = 100 × 3.060401 = $306.04.

2. R = $200.00, i = 0.01, n = 3; deposits occur at times 0, 1, and 2. Ordinary future-value factor is 3.0301. Due factor = 3.0301 × 1.01 = 3.060401. FV at time 3 = 200 × 3.060401 = $612.08.
3. R = $50.00, i = 0.01, n = 3; deposits occur at times 0, 1, and 2. Ordinary future-value factor is 3.0301. Due factor = 3.0301 × 1.01 = 3.060401. FV at time 3 = 50.0 × 3.060401 = $153.02.
4. PV = $100 \times (1 - 1.01^{-3}) \div 0.01$ = $294.10, rounded at the end.
5. PV due = unrounded ordinary PV × 1.01 = $297.04. The first payment is at time 0, so it is not discounted.

RECAP QUIZ

1. **$306.04**: R = $100.00, i = 0.01, n = 3; deposits occur at times 0, 1, and 2. Ordinary future-value factor is 3.0301. Due factor = 3.0301 × 1.01 = 3.060401. FV at time 3 = 100 × 3.060401 = $306.04.
2. **Match the rate period to the timeline**: The periodic rate and period count must describe the same interval; assumptions are not guarantees.
3. **Retain full precision**: Retaining calculator precision avoids accumulating rounding errors; round the final monetary result.

13.3

Sinking Funds and Savings Goals

LEARNING OBJECTIVES

After this section you will be able to:

1. Place the relevant cash flows on a timeline.
2. Calculate the modeled value using a matching periodic rate.
3. Interpret the result and identify the model's assumptions.

REAL-LIFE SITUATION

A student savings timeline

A Philadelphia student uses a hypothetical fixed-rate model to compare cash flows for future school or equipment costs.

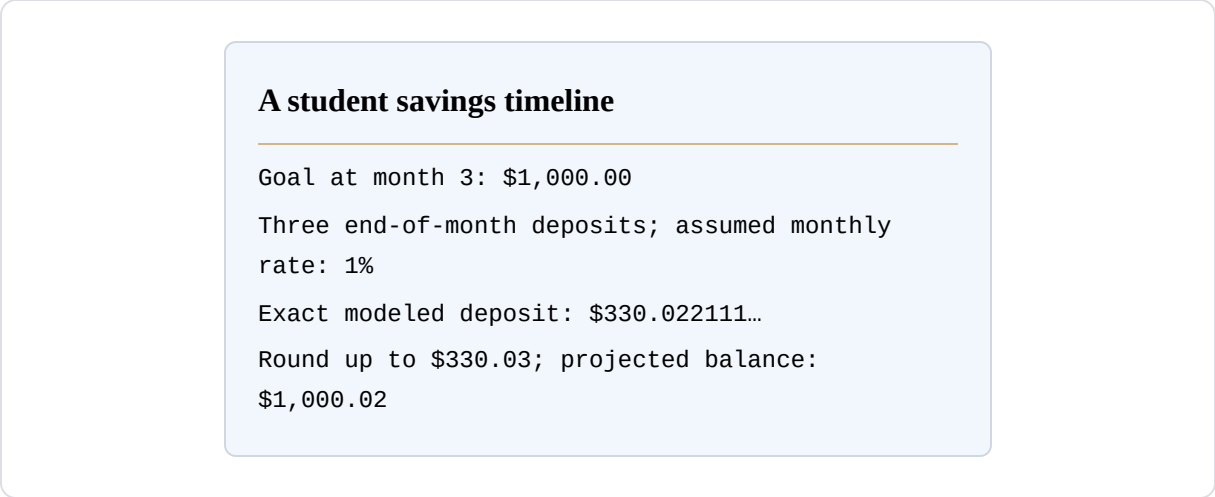


Figure 13.3 A student savings timeline (hypothetical)

The concept

A sinking fund sets aside regular deposits for a known future expense. A student might model equipment replacement or a course fee rather than wait until the entire bill arrives. Solving the ordinary-annuity formula for R finds the deposit needed for a target.

Divide the target by the accumulation factor. The result may require fractions of a cent, which cannot be deposited as ordinary cash. For a minimum-goal problem, round the required deposit up to the next cent and check the resulting future value.

A long-term plan is a model, not a guarantee. A lower realized rate, missed deposit, fee, or higher future price can create a shortfall. At a zero rate the required deposit equals target divided by the number of deposits; regular reviews allow the plan to adapt.

All rates and cash flows are hypothetical classroom assumptions, not investment advice or promised returns. Retain full precision through

intermediate calculations and round final money to cents; for a minimum savings target, round the deposit upward instead. Draw the timeline before calculating and explain what the result means.

Build a sinking fund

1. Goal
   `$1,000 at month 3`
2. Rounded deposit
   `$330.03 at each month-end`
3. Projected balance
   `$1,000.02`

Figure 13.3.V Build a sinking fund. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

INTEREST RATE
The percent used to calculate how much interest is earned or charged.

INFLATION
A general rise in prices over time, which makes money buy less.

RETURN
The money or gain an investment produces, often shown as a percent.

KEY TERMS

Annuity
Equal payments at equally spaced intervals.

Ordinary annuity
Payments occur at each period-end.

Annuity due
Payments occur at each period-beginning.

Sinking fund
Regular savings designated for a future obligation.

Valuation date
The date at which cash-flow values are compared.

FORMULAS & RULES

Sinking Funds and Savings Goals

$$R = \text{target FV} \times i \div ((1 + i)^n - 1)$$

Related relationship

$$\text{Shortfall} = \text{target} - \text{available balance}$$

Worked examples

Example 1 The timeline in the figure

Find equal month-end deposits to reach at least $1,000.00 after 3 deposits at an assumed 1% monthly rate.

SOLUTION

1. Target = $1,000.00; i = 0.01; n = 3.
2. Accumulation factor = $(1.01^3 - 1) \div 0.01 = 3.0301$.
3. Exact R = 1000 ÷ 3.0301 = 330.022111481… .

4 Round up to cents: $330.03.

5 Check: rounded deposit × 3.0301 = $1,000.02.

> **Answer:** $330.03

Example 2 A different amount

Find equal month-end deposits to reach at least $2,000.00 after 3 deposits at an assumed 1% monthly rate.

SOLUTION

1 Target = $2,000.00; i = 0.01; n = 3.

2 Accumulation factor = $(1.01^3 - 1) \div 0.01 = 3.0301$.

3 Exact R = 2000 ÷ 3.0301 = 660.044222962… .

4 Round up to cents: $660.05.

5 Check: rounded deposit × 3.0301 = $2,000.02.

> **Answer:** Required deposit rounded upward: $660.05

Practice exercises 13.3

1. Find equal month-end deposits to reach at least $1,000.00 after 3 deposits at an assumed 1% monthly rate.
2. Find equal month-end deposits to reach at least $2,000.00 after 3 deposits at an assumed 1% monthly rate.

3. Find equal month-end deposits to reach at least $500.00 after 3 deposits at an assumed 1% monthly rate.
4. With no interest, what month-end deposit reaches $900 in 6 deposits?
5. A $1,200 target has only $1,050 available. Find the shortfall and a six-month zero-interest catch-up amount.

RECAP QUIZ 13.3

1. Find equal month-end deposits to reach at least $1,000.00 after 3 deposits at an assumed 1% monthly rate.

- ○ a. $330.03
- ○ b. $1,000.00
- ○ c. $900.00
- ○ d. $1,500.00

2. Which practice is correct?

- ○ a. Use an annual rate in every month
- ○ b. Match the rate period to the timeline
- ○ c. Round every growth factor to two decimals
- ○ d. Treat an assumed return as guaranteed

3. How should intermediate growth factors be handled?

- ○ a. Round to whole numbers
- ○ b. Always replace them with 1
- ○ c. Retain full precision
- ○ d. Ignore them

SECTION 13.3 RECAP

- Draw the timeline and match the rate to each interval.

- Keep intermediate precision; distinguish modeled values from guaranteed outcomes.

Answer key with explanations: Section 13.3

PRACTICE EXERCISES

1. Target = $1,000.00; i = 0.01; n = 3. Accumulation factor = $(1.01^3 - 1) \div 0.01 = 3.0301$. Exact R = 1000 ÷ 3.0301 = 330.022111481… . Round up to cents: $330.03. Check: rounded deposit × 3.0301 = $1,000.02.
2. Target = $2,000.00; i = 0.01; n = 3. Accumulation factor = $(1.01^3 - 1) \div 0.01 = 3.0301$. Exact R = 2000 ÷ 3.0301 = 660.044222962… . Round up to cents: $660.05. Check: rounded deposit × 3.0301 = $2,000.02.
3. Target = $500.00; i = 0.01; n = 3. Accumulation factor = $(1.01^3 - 1) \div 0.01 = 3.0301$. Exact R = 500.0 ÷ 3.0301 = 165.011055740… . Round up to cents: $165.02. Check: rounded deposit × 3.0301 = $500.03.
4. At i = 0 use target ÷ n: 900 ÷ 6 = $150.00.
5. Shortfall = 1,200 − 1,050 = $150.00; 150 ÷ 6 = $25.00 per month, assuming no other changes.

RECAP QUIZ

1. **$330.03**: Target = $1,000.00; i = 0.01; n = 3. Accumulation factor = $(1.01^3 - 1) \div 0.01 = 3.0301$. Exact R = 1000 ÷ 3.0301 = 330.022111481… . Round up to cents: $330.03. Check: rounded deposit × 3.0301 = $1,000.02.
2. **Match the rate period to the timeline**: The periodic rate and period count must describe the same interval; assumptions are not guarantees.
3. **Retain full precision**: Retaining calculator precision avoids accumulating rounding errors; round the final monetary result.

CHAPTER 14

Installment Purchases, Credit Cards, and Consumer Debt

A small monthly payment can hide a surprisingly large total cost. Learn to compare installment offers, calculate credit card charges, and explain how payments reduce debt. All people, prices, rates, and contract terms in this chapter are hypothetical; round money to the nearest cent.

14.1

The Real Cost of an Installment Purchase

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate the amount financed after a down payment.
2. Find total payments, finance charges, and total purchase cost.
3. Compare offers using both affordability and total cost.

REAL-LIFE SITUATION

A computer for school and creative work

A Philadelphia student compares two payment plans for a $900 computer. Both fit the student's monthly budget, but the smaller advertised payment is not enough information to choose.

Offer	Cash price	Down payment	Monthly payment	Months
A	$900	$180	$65	12
B	$900	$100	$88	10

Figure 14.1 Hypothetical installment offers; no tax or additional fees

The concept

An installment purchase spreads payment across a stated number of scheduled payments. The cash price is the price paid without financing. A down payment reduces the amount that must be financed; it does not disappear from your total spending.

Multiply the regular payment by the number of payments to find the scheduled payment total. When all borrowing charges are included in those payments, subtract the amount financed to find the finance charge. Add the down payment to the scheduled payments to find total purchase cost.

For these simplified examples, there are no taxes, insurance products, balloon payments, or separate fees unless stated. Real disclosures may include these items, so identify every required payment before comparing offers. Do not add a fee twice if it is already included in the payment schedule.

A lower monthly payment may simply mean a longer repayment period. Compare total cost and timing as well as monthly affordability. Finance charge divided by amount financed is a cost percentage for the whole contract, not the annual percentage rate, because principal is repaid during the term.

The Real Cost of an Installment Purchase: solution map

1. Identify the question

   Use Figure 14.1 to find Offer A's amount financed, finance charge, and total purchase cost.

2. Calculation 1

   Amount financed: $900 − $180 = $720.

3. Calculation 2

   Scheduled payments: 12 × $65 = $780.

4. Calculation 3

   Finance charge: $780 − $720 = $60.

5. Calculation 4

   Total purchase cost: $180 + $780 = $960.

6. Interpret and check

   The student borrows $720, pays a $60 finance charge, and spends $960 altogether.

Figure 14.1.V The Real Cost of an Installment Purchase: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

CREDIT CARD

A card that lets you borrow up to a limit and repay over time, usually with interest.

APR

Annual percentage rate — the yearly cost of borrowing, including interest and some fees, shown as a percent.

MINIMUM PAYMENT

The smallest amount due on a credit card bill each month; paying only this keeps you in debt longer.

KEY TERMS

Cash price
The price of an item before borrowing charges.

Down payment
Money paid at the start toward the purchase price.

Amount financed
The amount borrowed to complete the purchase.

Finance charge
The dollar cost of borrowing under the stated terms.

Installment
One scheduled payment in a series.

FORMULAS & RULES

Amount financed

```
Amount financed = cash price − down payment
```

Payment total

```
Payment total = payment amount × number of payments
```

Finance charge

```
Finance charge = payment total − amount financed
```

Assumes all borrowing charges are in the scheduled payments.

Total purchase cost

```
Total purchase cost = down payment + payment total +
separate required fees
```

Worked examples

Example 1 Unpack Offer A

Use Figure 14.1 to find Offer A's amount financed, finance charge, and total purchase cost.

SOLUTION

1. Amount financed: $900 − $180 = $720.
2. Scheduled payments: 12 × $65 = $780.
3. Finance charge: $780 − $720 = $60.
4. Total purchase cost: $180 + $780 = $960.

> **Answer:** The student borrows $720, pays a $60 finance charge, and spends $960 altogether.

Example 2 Compare the second offer

Find Offer B's total cost and compare it with Offer A.

SOLUTION

1. Amount financed: $900 − $100 = $800.

2. Scheduled payments: 10 × $88 = $880.
3. Finance charge: $880 − $800 = $80.
4. Total cost: $100 + $880 = $980.
5. Difference: $980 − $960 = $20; Offer B also requires the larger monthly payment.

> **Answer:** Offer A costs $20 less; Offer B requires $80 less upfront but $23 more each month.

Practice exercises 14.1

1. What percent of the $900 cash price is Offer A's $180 down payment?
2. A genuinely fee-free, zero-interest plan finances $720 over 12 equal payments. Find each payment.
3. Offer A also requires a separate $25 setup fee not included in its payments. Find its revised total cost.
4. Find $60 ÷ $720 as a percent. Explain why it is not Offer A's APR.
5. Your monthly payment limit is $70. Which original offer meets it, and what else should you check?

RECAP QUIZ 14.1

1. A $900 item has a $180 down payment. How much is financed?

- ○ a. $180
- ○ b. $720
- ○ c. $900
- ○ d. $1,080

2. Twelve payments of $65 total:

- ○ a. $720
- ○ b. $745
- ○ c. $780
- ○ d. $960

3. Which comparison best reveals overall purchase spending?

- ○ a. Monthly payment alone
- ○ b. Down payment alone
- ○ c. Number of payments alone
- ○ d. Down payment plus all required payments and separate fees

SECTION 14.1 RECAP

- Separate the cash price, amount financed, finance charge, and total purchase cost.
- A manageable payment is necessary, but it does not prove an offer is inexpensive.

Answer key with explanations: Section 14.1

PRACTICE EXERCISES

1. $180 ÷ $900 × 100 = 20%. The down payment covers one-fifth of the cash price.
2. $720 ÷ 12 = $60. With no finance charge, all $720 of payments repay principal.
3. $180 + 12 × $65 + $25 = $985. Count the separate fee once.
4. $60 ÷ $720 × 100 ≈ 8.33%. This compares the full contract charge with the initial amount financed; it ignores the timing of declining balances, so it is not APR.
5. Offer A's $65 is at or below $70; Offer B's $88 exceeds it by $18. Also check the $180 down payment, total cost of $960, and any terms or fees not shown.

RECAP QUIZ

1. **$720**: $900 − $180 = $720.
2. **$780**: 12 × $65 = $780; the down payment is separate.

3. **Down payment plus all required payments and separate fees**: Every required dollar belongs in total purchase cost.

14.2

Credit Card Balances and Finance Charges

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate a weighted average daily balance.
2. Estimate interest using a stated daily periodic rate.
3. Reconcile purchases, payments, interest, and ending balance.

REAL-LIFE SITUATION

Two balances during one billing cycle

A hypothetical card statement has a $600 balance for the first 10 days and a $900 balance for the next 20 days. A $100 payment posts at the cycle's end, after the daily balance measurement used here.

Period or item	Amount or duration
First balance	`$600 for 10 days`
After a purchase	`$900 for 20 days`
New purchase	`$300`
End-of-cycle payment	`$100`
Daily interest convention	`APR ÷ 365; no daily compounding`

Figure 14.2 Simplified 30-day card cycle at 24% APR

The concept

A credit card balance changes when purchases, payments, credits, fees, and interest post. A statement summarizes activity over a billing cycle, while a current online balance may include later transactions. Always match the dates before comparing the two.

An average daily balance weights each balance by the number of days it applied. Multiply each balance by its days, add the products, and divide by the cycle's total days. Simply averaging the highest and lowest balances works only when they lasted equally long.

This lesson uses a simplified interest method: average daily balance × APR ÷ 365 × cycle days, with no daily compounding. Convert APR from percent to decimal before calculating and round the final charge to cents. Actual card agreements can use other conventions and separate rates for different balance types.

A grace period may allow eligible purchases to avoid interest when the required statement balance is paid by its due date. Eligibility and exclusions depend on the agreement; cash advances may be treated differently. Our interest examples assume no grace-period benefit applies.

Credit Card Balances and Finance Charges: solution map

1. Identify the question

   Find the average daily balance and interest for Figure 14.2.

2. Calculation 1

   First period: $600 × 10 = 6,000 dollar-days.

3. Calculation 2

   Second period: $900 × 20 = 18,000 dollar-days.

4. Calculation 3

   Average daily balance: (6,000 + 18,000) ÷ 30 = $800.

5. Calculation 4

   Daily rate: 0.24 ÷ 365; retain full precision.

6. Calculation 5

   Interest: $800 × (0.24 ÷ 365) × 30 ≈ $15.78.

7. Interpret and check

   Average daily balance is $800; the simplified cycle interest is $15.78.

Figure 14.2.V Credit Card Balances and Finance Charges: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

INSTALLMENT PLAN
Paying for something in a set number of equal payments over time.

LATE FEE
An extra charge when a payment arrives after its due date.

GRACE PERIOD
A short time after the due date before a fee or interest charge applies.

KEY TERMS

Billing cycle
The period covered by one statement.

APR
Annual percentage rate; an annualized borrowing-rate disclosure.

Average daily balance
The sum of daily balances divided by the number of days.

Daily periodic rate
The daily interest rate specified by the agreement.

Grace period
A conditional interval in which eligible purchases may avoid interest.

FORMULAS & RULES

Weighted balance

```
Average daily balance = sum of (balance × days) ÷ cycle days
```

Daily rate

```
Daily rate = APR as a decimal ÷ 365
```

The convention assumed in this section.

Cycle interest

```
Interest = average daily balance × daily rate × cycle days
```

Statement reconciliation

```
Ending balance = beginning balance + purchases + fees + interest − payments − credits
```

Worked examples

Example 1 Weight balances by time

Find the average daily balance and interest for Figure 14.2.

SOLUTION

1. First period: $600 × 10 = 6,000 dollar-days.
2. Second period: $900 × 20 = 18,000 dollar-days.
3. Average daily balance: (6,000 + 18,000) ÷ 30 = $800.
4. Daily rate: 0.24 ÷ 365; retain full precision.
5. Interest: $800 × (0.24 ÷ 365) × 30 ≈ $15.78.

> **Answer:** Average daily balance is $800; the simplified cycle interest is $15.78.

Example 2 Reconcile the statement

Use a $600 beginning balance, $300 purchase, $15.78 interest, $100 payment, and no fees or credits.

SOLUTION

1. Before interest or payment: $600 + $300 = $900.
2. Add interest: $900 + $15.78 = $915.78.
3. Subtract the payment: $915.78 − $100 = $815.78.

> **Answer:** The ending statement balance is $815.78.

Practice exercises 14.2

1. A $900 balance lasts all 30 days. What is its average daily balance?
2. For a separate simplified monthly model, estimate one month's interest on $900 at 24% APR using APR ÷ 12.
3. A statement has $900 owed on a $3,000 limit. Find utilization.
4. Why is ($600 + $900) ÷ 2 = $750 not the average balance in Figure 14.2?
5. A $100 payment posts earlier instead of at the end. With all else unchanged, would interest under the daily-balance method rise or fall?

RECAP QUIZ 14.2

1. Which APR decimal represents 24%?

- ○ a. 24
- ○ b. 2.4
- ○ c. 0.24
- ○ d. 0.024

2. Which transaction usually reduces the balance?

- ○ a. Purchase
- ○ b. Payment
- ○ c. Interest charge
- ○ d. Late fee

3. Why weight daily balances?

- ○ a. Every month has 365 days
- ○ b. Larger balances always last longer
- ○ c. Payments are ignored
- ○ d. Balances can apply for different lengths of time

SECTION 14.2 RECAP

- Weight balances by days and use the interest convention actually stated.
- A statement balance is a dated accounting total, not necessarily today's balance.

Answer key with explanations: Section 14.2

PRACTICE EXERCISES

1. ($900 × 30) ÷ 30 = $900. With no changes, the average equals the constant balance.
2. Monthly rate = 0.24 ÷ 12 = 0.02. Interest = $900 × 0.02 = $18. This monthly convention differs from the daily convention.
3. $900 ÷ $3,000 × 100 = 30%. Utilization compares the balance with the available credit limit, not with income.
4. The $900 balance lasted 20 days, twice as long as $600. Weighting gives (600 × 10 + 900 × 20) ÷ 30 = $800, not $750.
5. It would fall: each affected daily balance is $100 lower. The reduction equals $100 × affected days × (0.24 ÷ 365), rounded under the agreement's rules.

RECAP QUIZ

1. **0.24**: 24 ÷ 100 = 0.24.
2. **Payment**: A payment is subtracted when reconciling the balance.
3. **Balances can apply for different lengths of time**: Dollar-days reflect both the balance and its duration.

14.3

Building a Consumer Debt Repayment Plan

LEARNING OBJECTIVES

After this section you will be able to:

1. Separate interest from principal reduction.
2. Compare ordinary and larger payments.
3. Measure scheduled debt payments relative to income.

REAL-LIFE SITUATION

Make progress, not just payments

A part-time worker has a $1,000 balance at a hypothetical monthly rate of 2%. There are no new purchases or fees, and interest is added before the month-end payment.

Same debt, different payments

```
Starting balance: $1,000.00
Monthly interest: 2% × $1,000 = $20.00
Pay $120: principal falls $100; ending balance $900
Pay $220: principal falls $200; ending balance $800
```

Figure 14.3 Two hypothetical first-month repayment choices

The concept

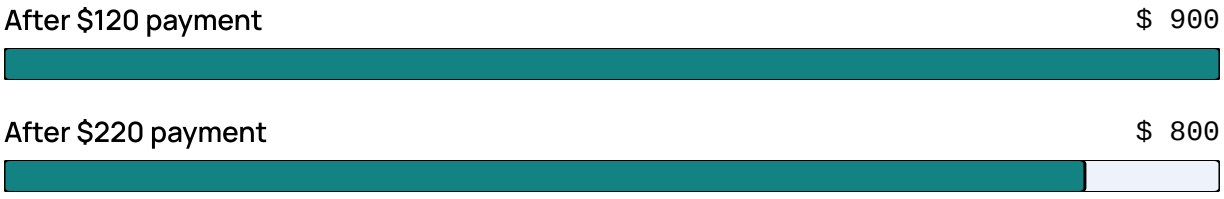
A payment does not necessarily reduce principal by its full amount. Under this section's model, interest is added first, then the payment covers that interest and reduces the remaining debt. If a payment is less than the interest charged, the balance grows even without new purchases.

A repayment forecast needs explicit assumptions: the rate, payment date, payment amount, fees, and whether borrowing continues. With a fixed positive payment and no new charges, a falling balance usually produces a smaller next interest charge when the rate stays constant.

For several debts, list each balance, rate, minimum payment, and due date. Keep required payments in the plan before deciding where any extra money goes. Directing extra money to a higher-rate balance generally reduces interest under comparable terms, while a smallest-balance approach emphasizes eliminating accounts; fees and special terms can change the comparison.

A payment-to-income ratio measures how much income is already committed to debt payments. State whether the denominator is gross or take-home income. The ratio is a budgeting indicator, not an approval guarantee or a universal safety cutoff; housing, food, transportation, and emergency needs still matter.

Ending debt after different payments



Bars share a zero baseline.

Figure 14.3.V Ending debt after different payments. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

CREDIT LIMIT

The most you can borrow on a credit card or credit line.

STATEMENT BALANCE

The amount owed on a credit account as of the statement date.

CREDIT SCORE

A number that estimates how reliably someone repays debts.

KEY TERMS

Principal
The unpaid borrowed amount, excluding newly calculated interest in this model.

Principal reduction
The portion of a payment that lowers the debt after interest.

Minimum payment
The smallest payment required by the agreement for that period.

Negative amortization
Growth in a balance because payments do not cover added interest.

Payment-to-income ratio
Scheduled debt payments divided by a stated income measure.

FORMULAS & RULES

Monthly interest

```
Interest = starting balance × monthly rate
```

Principal reduction

```
Principal reduction = payment − interest
```

Assumes no other charges.

Next balance

```
Next balance = starting balance + interest − payment
```

Payment burden

```
Payment-to-income percent = monthly debt payments ÷
monthly income × 100
```

Worked examples

Example 1 Track an ordinary payment

Find the balance after paying $120 on the $1,000 debt, then find the next month's interest.

SOLUTION

1. First-month interest: $1,000 × 0.02 = $20.
2. Principal reduction: $120 − $20 = $100.
3. Ending balance: $1,000 − $100 = $900.
4. Next month's interest: $900 × 0.02 = $18.

> **Answer:** The balance falls to $900; the following month's interest is $18 before its payment.

Example 2 Add money to the payment

Instead pay $220 in the first month. Compare the ending balance with the $120 plan.

SOLUTION

1. Interest is still $1,000 × 0.02 = $20 because the payment occurs at month-end.
2. Principal reduction: $220 − $20 = $200.
3. Ending balance: $1,000 − $200 = $800.
4. Compared with $900, the ending balance is $100 lower.

> **Answer:** The extra $100 payment lowers principal by an extra $100; it also lowers later interest.

Practice exercises 14.3

1. What payment lowers the first month's balance from $1,000 to $850 at 2% monthly interest?
2. What happens if the first payment is only $15?
3. Find the payment-to-take-home-income ratio for $120 in debt payments and $1,500 take-home pay.
4. Find next month's interest after the larger-payment plan leaves an $800 balance.
5. A plan predicts a $900 ending balance, but you add a $50 purchase after interest is calculated. Recompute the ending balance.

RECAP QUIZ 14.3

1. A $120 payment includes $20 interest. Principal reduction is:

- ○ a. $20
- ○ b. $100
- ○ c. $120
- ○ d. $140

2. Which assumption is essential to the illustrated declining-balance forecast?

- ○ a. New purchases continue
- ○ b. The rate doubles
- ○ c. No new purchases or fees
- ○ d. Payments are skipped

3. If interest is $20 and payment is $15, the balance:

- ○ a. Falls $35
- ○ b. Falls $5
- ○ c. Stays unchanged
- ○ d. Rises $5

SECTION 14.3 RECAP

- Subtract interest from a payment to see actual principal reduction.
- A useful repayment plan includes required payments, living costs, and clear assumptions.

Answer key with explanations: Section 14.3

PRACTICE EXERCISES

1. Interest is $20, so the amount before payment is $1,020. Required payment = $1,020 − $850 = $170.
2. Interest is $20. Ending balance = $1,000 + $20 − $15 = $1,005, a $5 increase.
3. $120 ÷ $1,500 × 100 = 8%. This leaves 92% before considering other spending, not necessarily 92% available to save.
4. $800 × 0.02 = $16. Compared with $18 on $900, this saves $2 in the next month.
5. $1,000 + $20 + $50 − $120 = $950. The original $900 forecast assumed no new purchases.

RECAP QUIZ

1. **$100**: $120 − $20 = $100.
2. **No new purchases or fees**: New charges or changed terms require a new forecast.

3. **Rises $5**: $20 − $15 = $5 of unpaid interest increases the balance.

CHAPTER 15

Homeownership, Mortgages, and Amortization

A home's price is only one part of the cost of owning it. This chapter connects down payments, monthly housing budgets, and mortgage amortization so you can read a loan illustration with confidence. All properties, rates, taxes, insurance costs, and financing terms are hypothetical examples, not current law or offers; round money to cents.

15.1

Down Payments and Cash Needed at Closing

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate a down payment and initial mortgage principal.
2. Distinguish loan principal from cash needed at closing.
3. Calculate loan-to-value and initial equity.

REAL-LIFE SITUATION

A first look at a Philadelphia rowhouse

A hypothetical rowhouse costs $200,000. The buyer plans a 10% down payment, closing costs equal to 3% of the price, and a separate $2,400

prepaid reserve deposit.

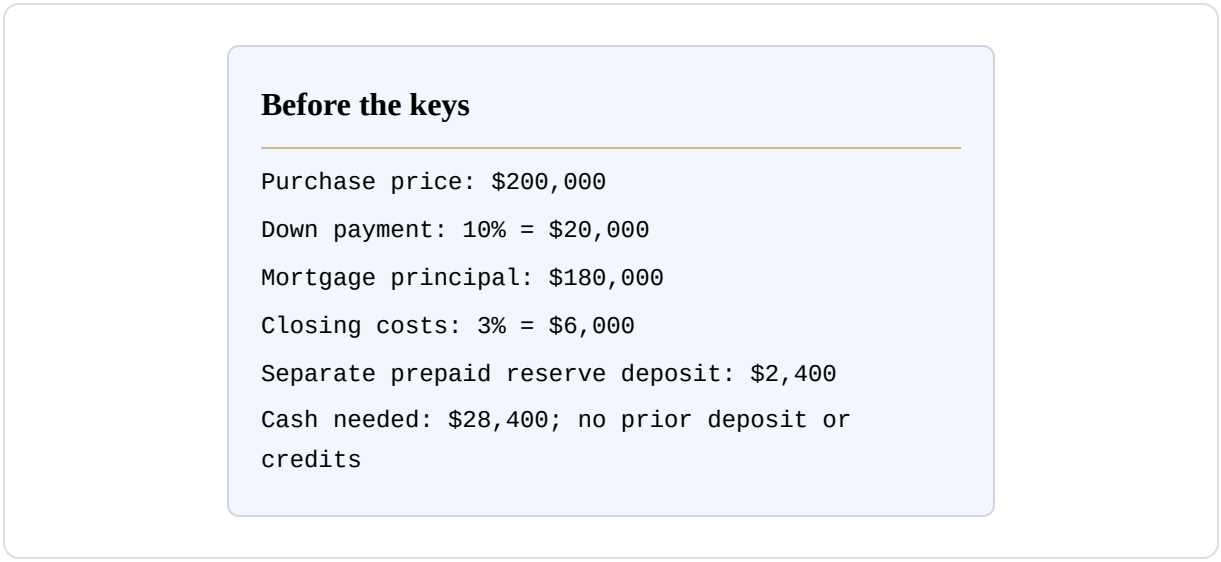


Figure 15.1 Simplified hypothetical purchase worksheet

The concept

The down payment is the part of the purchase price paid without the mortgage. Subtract it from the price to find the initial mortgage principal when no other costs are financed. A percentage down payment is always applied to its stated base, here the purchase price.

Cash needed at closing can exceed the down payment. Closing costs pay for transaction-related services or charges, while prepaid items and initial reserve deposits fund specified future costs. Our worksheet treats these as separate categories so nothing is counted twice.

Actual cash due also depends on amounts already deposited, seller or lender credits, prorations, and any financed charges. A prior deposit reduces cash still due; it does not reduce the purchase's overall cost a second time. Read the final disclosure rather than assuming every worksheet is complete.

Loan-to-value compares the loan with the property's stated value. Equity is value minus outstanding secured debt. At purchase, equity equals the down payment only under simplifying assumptions such as price equaling value

and having no other secured debt; transaction costs are not automatically equity.

Cash needed at closing

1. Down payment
   `$20,000`
2. Closing costs + reserve
   `$6,000 + $2,400`
3. Cash needed
   `$28,400; mortgage principal separately: $180,000`

Figure 15.1.V Cash needed at closing. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

MORTGAGE
A long-term loan used to buy a home, with the property as security.

DOWN PAYMENT
Money paid upfront when financing a purchase, which reduces the amount borrowed.

AMORTIZATION
Spreading a loan into scheduled payments so that early payments are mostly interest and later payments are mostly principal.

KEY TERMS

Mortgage principal
The amount borrowed and still unpaid on a home loan.

Closing costs
Transaction charges separate from the down payment in this worksheet.

Prepaid reserve deposit
Money deposited in advance for specified future housing bills.

Loan-to-value
Loan balance divided by the property's stated value.

Equity
Property value minus outstanding debt secured by it.

FORMULAS & RULES

Down payment

```
Down payment = purchase price × down payment rate
```

Initial mortgage

```
Mortgage principal = purchase price − down payment
```

No financed costs or other loans in this model.

Cash due

```
Cash due = down payment + closing costs + separate
prepaids − prior deposits − credits
```

Loan-to-value

```
LTV percent = loan balance ÷ property value × 100
```

Worked examples

Example 1 Find the mortgage and cash requirement

Complete the purchase worksheet in Figure 15.1.

SOLUTION

1. Down payment: $200,000 × 0.10 = $20,000.
2. Mortgage principal: $200,000 − $20,000 = $180,000.
3. Closing costs: $200,000 × 0.03 = $6,000.
4. Cash needed: $20,000 + $6,000 + $2,400 = $28,400.

Answer: The mortgage is $180,000, but the buyer needs $28,400 in cash under these assumptions.

Example 2 Change the down payment

Use a 20% down payment on the same price. Find principal, LTV, and initial equity, assuming value equals price.

SOLUTION

1. Down payment: $200,000 × 0.20 = $40,000.
2. Mortgage: $200,000 − $40,000 = $160,000.
3. LTV: $160,000 ÷ $200,000 × 100 = 80%.
4. Equity: $200,000 − $160,000 = $40,000.

Answer: Principal is $160,000, LTV is 80%, and initial equity is $40,000.

Practice exercises 15.1

1. Find the LTV for the original $180,000 mortgage and $200,000 value.
2. The original buyer already paid a $3,000 deposit credited toward closing. How much cash remains due?
3. A $1,000 transaction credit also applies, in addition to the $3,000 deposit. Find cash still due.
4. Without changing other costs, how much more cash is needed for 20% down instead of 10%?
5. Does paying $6,000 in closing costs create $6,000 of additional equity?

RECAP QUIZ 15.1

1. Ten percent of $200,000 is:

- ○ a. $2,000
- ○ b. $20,000
- ○ c. $180,000
- ○ d. $220,000

2. Which reduces cash still due without being a new reduction in total purchase cost?

- ○ a. A prior credited deposit
- ○ b. An added fee
- ○ c. A higher price
- ○ d. An increased reserve deposit

3. Equity is:

- ○ a. Price plus loan
- ○ b. Monthly payment
- ○ c. Value minus secured debt
- ○ d. Closing costs alone

SECTION 15.1 RECAP

- Mortgage principal and cash needed at closing answer different questions.
- Avoid double-counting deposits, credits, prepaid items, and transaction costs.

Answer key with explanations: Section 15.1

PRACTICE EXERCISES

1. $180,000 ÷ $200,000 × 100 = 90%.
2. $28,400 − $3,000 = $25,400. The deposit is part of the original cash requirement, not an extra discount.
3. $28,400 − $3,000 − $1,000 = $24,400, assuming the credit is usable for the listed charges.
4. $40,000 − $20,000 = $20,000 more. The unchanged $8,400 in closing costs and prepaids makes the new total $48,400.
5. No. With value $200,000 and debt $180,000, equity remains $200,000 − $180,000 = $20,000. Transaction spending is not automatically property value.

RECAP QUIZ

1. **$20,000**: $200,000 × 0.10 = $20,000.
2. **A prior credited deposit**: The buyer has already supplied that portion of the required cash.
3. **Value minus secured debt**: Equity measures the owner's residual interest after secured debt.

15.2

The Full Monthly Housing Budget

LEARNING OBJECTIVES

After this section you will be able to:

1. Build a payment including principal, interest, taxes, and insurance.
2. Convert annual housing bills to monthly budget amounts.
3. Include maintenance and other ownership costs outside the loan.

REAL-LIFE SITUATION

The payment is not the whole budget

A hypothetical mortgage has a $1,079.19 monthly principal-and-interest payment. Annual property taxes are $3,600 and annual homeowners insurance is $1,200; these are illustrative amounts, not current tax rules or quotes.

Category	Monthly amount
Principal and interest	$1,079.19
Property taxes	$300.00
Homeowners insurance	$100.00
Maintenance reserve	$150.00
Utilities	$180.00
Total	$1,809.19

Figure 15.2 Hypothetical monthly ownership budget

The concept

A quoted mortgage payment may include only principal and interest, often shortened to P&I. A broader payment estimate adds property taxes and homeowners insurance, producing PITI. Always ask what an advertised payment includes before comparing it with rent or take-home pay.

When a lender collects money for taxes and insurance through escrow, those amounts help fund later bills; they are not extra mortgage principal. Escrow estimates can change when the underlying bills change. Paying a bill through escrow does not mean the lender pays it for free.

Divide an annual cost by 12 to create a monthly budget allowance. This is a planning average, not a claim that every bill arrives monthly. A separate maintenance reserve helps prepare for uneven repair spending, but no fixed reserve guarantees that all repairs are covered.

Ownership can also involve utilities, association dues, mortgage insurance, and special assessments, depending on the property and financing. Our illustration has no association dues or mortgage insurance. A responsible comparison lists excluded costs explicitly and does not treat the entire payment as equity building.

The Full Monthly Housing Budget: solution map

1. Identify the question

   `Use $1,079.19 P&I, $3,600 annual taxes, and $1,200 annual insurance.`

2. Calculation 1

   `Monthly tax allowance: $3,600 ÷ 12 = $300.`

3. Calculation 2

   `Monthly insurance allowance: $1,200 ÷ 12 = $100.`

4. Calculation 3

   `PITI: $1,079.19 + $300 + $100 = $1,479.19.`

5. Interpret and check

   `Estimated PITI is $1,479.19 per month.`

Figure 15.2.V The Full Monthly Housing Budget: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

EQUITY

What is left of an asset after debts are subtracted; in a home, value minus what you still owe.

ESCROW

Money held by a third party, such as property tax collected along with a mortgage payment.

CLOSING COSTS

Fees paid when a home or property purchase is finalized, such as appraisal and title fees.

KEY TERMS

P&I
The principal-and-interest portion of a mortgage payment.

PITI
Principal, interest, property taxes, and homeowners insurance combined.

Escrow
An account used to hold money for specified bills such as taxes and insurance.

Maintenance reserve
Money budgeted for upkeep and repairs.

Ownership budget
A plan for loan payments and other costs of owning a home.

FORMULAS & RULES

Monthly allowance

```
Monthly allowance = annual cost ÷ 12
```

PITI

```
PITI = principal and interest + monthly taxes + monthly insurance
```

Housing budget

```
Housing budget = PITI + maintenance + utilities + other
applicable costs
```

Worked examples

Example 1 Build PITI

Use $1,079.19 P&I, $3,600 annual taxes, and $1,200 annual insurance.

SOLUTION

1. Monthly tax allowance: $3,600 ÷ 12 = $300.
2. Monthly insurance allowance: $1,200 ÷ 12 = $100.
3. PITI: $1,079.19 + $300 + $100 = $1,479.19.

> **Answer:** Estimated PITI is $1,479.19 per month.

Example 2 Budget beyond the lender payment

Add a $150 maintenance reserve and $180 utilities allowance. Find three months of the full budget.

SOLUTION

1. Costs outside PITI: $150 + $180 = $330.
2. Full monthly budget: $1,479.19 + $330 = $1,809.19.
3. Three months: $1,809.19 × 3 = $5,427.57.

> **Answer:** The monthly estimate is $1,809.19; three months total $5,427.57,

> without guaranteeing future costs.

Practice exercises 15.2

1. Taxes rise by a hypothetical $600 annually. What is the new monthly tax allowance?
2. Insurance rises from $1,200 to $1,440 annually. Find the new monthly allowance.
3. Apply both increases above. Find the revised full monthly budget.
4. An additional $90 monthly association fee applies. Using the original budget, find the new total.
5. How much of a $150 monthly maintenance reserve accumulates over a year before any repairs?

RECAP QUIZ 15.2

1. Annual taxes of $3,600 equal what monthly allowance?

- ○ a. $30
- ○ b. $300
- ○ c. $360
- ○ d. $3,600

2. Which is outside the PITI definition?

- ○ a. Principal
- ○ b. Interest
- ○ c. Property taxes
- ○ d. Maintenance reserve

3. An escrow payment for taxes:

- ○ a. Automatically reduces mortgage principal
- ○ b. Eliminates the tax bill
- ○ c. Funds a future tax bill
- ○ d. Guarantees taxes cannot change

SECTION 15.2 RECAP

- Separate P&I, PITI, and the full ownership budget.
- Annual bills and irregular repairs still require monthly planning.

Answer key with explanations: Section 15.2

PRACTICE EXERCISES

1. New annual taxes = $3,600 + $600 = $4,200. Divide by 12 to get $350, an increase of $50 monthly.
2. $1,440 ÷ 12 = $120, which is $20 above the original $100.
3. $1,809.19 + $50 + $20 = $1,879.19, with all other costs unchanged.
4. $1,809.19 + $90 = $1,899.19. Association fees were not included in the original PITI or other costs.
5. $150 × 12 = $1,800. Actual repairs reduce that amount; the contribution is a budget allowance, not a repair-price guarantee.

RECAP QUIZ

1. **$300**: $3,600 ÷ 12 = $300.
2. **Maintenance reserve**: Maintenance matters to ownership budgeting but is not part of PITI.
3. **Funds a future tax bill**: Escrow holds funds for specified expenses; it does not erase those expenses.

15.3

Mortgage Payments and Amortization

LEARNING OBJECTIVES

After this section you will be able to:

1. Use the fixed-payment mortgage formula with consistent periods.
2. Split a payment between interest and principal.
3. Explain how extra principal affects later interest.

REAL-LIFE SITUATION

Watch a mortgage shrink

A hypothetical $180,000 fixed-rate loan charges 6% annually with 360 monthly payments. The regular payment is rounded to $1,079.19, and monthly interest is rounded to cents.

Month	Payment	Interest	Principal	Ending balance
1	$1,079.19	$900.00	$179.19	$179,820.81
2	$1,079.19	$899.10	$180.09	$179,640.72

Figure 15.3 First two rounded amortization rows; no fees

The concept

Amortization is repayment through scheduled payments that cover interest and reduce principal. A fixed payment can have a changing composition: as the balance falls, interest usually falls, leaving more of the same payment for principal.

For a fixed-rate, fully amortizing loan with end-of-month payments, the payment formula uses monthly rate r and total monthly payment count n. Here r = 0.06 ÷ 12 = 0.005 and n = 30 × 12 = 360. Mixing an annual rate with a monthly payment count produces an incorrect result.

Each schedule row begins with the previous ending balance. Multiply that balance by the monthly rate, round interest to cents, subtract interest from the payment, and subtract the principal portion from the balance. Carry forward the actual rounded balance rather than restarting from the original loan.

Extra principal lowers the balance used for later interest when the servicer applies it as intended. Our examples assume no prepayment penalty and immediate principal application at the scheduled payment date. Rounding may require an adjusted final payment; multiplying the rounded regular payment by every month is not an exact payoff schedule.

Mortgage Payments and Amortization: solution map

1. **Identify the question**

   Find the payment and first month's principal reduction for $180,000 at 6% over 30 years.

2. **Calculation 1**

   Monthly rate: 0.06 ÷ 12 = 0.005; payment count: 30 × 12 = 360.

3. **Calculation 2**

   $M = 180{,}000 \times 0.005 \div [1 - (1.005)^{-360}] \approx \$1{,}079.19$.

4. **Calculation 3**

   First interest: $180,000 × 0.005 = $900.

5. **Calculation 4**

   Principal: $1,079.19 − $900 = $179.19.

6. **Calculation 5**

   Ending balance: $180,000 − $179.19 = $179,820.81.

7. **Interpret and check**

   The regular payment is $1,079.19; only $179.19 reduces principal in the first month.

Figure 15.3.V Mortgage Payments and Amortization: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

LOAN TERM
How long a loan lasts before it must be fully repaid.

MONTHLY PAYMENT
The amount paid each month toward a loan or installment plan.

PRINCIPAL
The original amount of money borrowed or invested, before interest.

KEY TERMS

Amortization
Gradual loan repayment through interest and principal payments.

Fixed-rate loan
A loan whose stated interest rate stays unchanged during the agreed term.

Periodic rate
The interest rate for one payment period.

Principal portion
The part of a payment that reduces the loan balance.

Extra principal
An additional amount applied directly to unpaid principal.

FORMULAS & RULES

Fixed payment

$$M = P \times r \div [1 - (1 + r)^{-n}]$$

P is principal; r is the positive periodic rate; n is the number of payments. At zero interest, M = P ÷ n.

Periodic interest

$$\text{Interest} = \text{beginning balance} \times \text{periodic rate}$$

Principal paid

```
Principal paid = regular payment − interest + extra principal
```

Ending balance

```
Ending balance = beginning balance − principal paid
```

Worked examples

Example 1 Calculate the payment and first row

Find the payment and first month's principal reduction for $180,000 at 6% over 30 years.

SOLUTION

1. Monthly rate: 0.06 ÷ 12 = 0.005; payment count: 30 × 12 = 360.
2. $M = 180{,}000 \times 0.005 \div [1 - (1.005)^{-360}] \approx \$1{,}079.19$.
3. First interest: $180,000 × 0.005 = $900.
4. Principal: $1,079.19 − $900 = $179.19.
5. Ending balance: $180,000 − $179.19 = $179,820.81.

> **Answer:** The regular payment is $1,079.19; only $179.19 reduces principal in the first month.

Example 2 Apply extra principal

Add $200 of principal to the first payment. Find the ending balance and next month's interest.

SOLUTION

1. Regular first-month principal is $1,079.19 − $900 = $179.19.
2. Total principal reduction: $179.19 + $200 = $379.19.
3. Ending balance: $180,000 − $379.19 = $179,620.81.
4. Next interest: $179,620.81 × 0.005 = $898.10405, rounded to $898.10.
5. Without the extra payment, next interest is $899.10; the difference is $1.

> **Answer:** The balance is $179,620.81, and next month's rounded interest is $898.10.

Practice exercises 15.3

1. Reproduce month 2 in Figure 15.3.
2. Find total principal paid in the first two regular payments.
3. Assuming the home still has a $200,000 value and no other secured debt, find equity after month 2.
4. Why is the second month's principal portion larger than the first?
5. If a loan has zero interest and finances $180,000 over 360 months, find the payment.

RECAP QUIZ 15.3

1. The monthly rate corresponding to 6% nominal annual interest is:

- ○ a. 0.06
- ○ b. 0.005
- ○ c. 0.5
- ○ d. 6

2. First-month interest on $180,000 at 0.5% monthly is:

- ○ a. $179.19
- ○ b. $500
- ○ c. $900
- ○ d. $1,079.19

3. Properly applied extra principal usually:

- ○ a. Raises the balance
- ○ b. Replaces all future required payments
- ○ c. Raises the fixed interest rate
- ○ d. Reduces later interest by lowering the balance

SECTION 15.3 RECAP

- Use matching rate and payment periods, then update the balance row by row.
- Only principal reduction builds equity through repayment; interest pays for borrowing.

Answer key with explanations: Section 15.3

PRACTICE EXERCISES

1. Interest = $179,820.81 × 0.005 = $899.10405 → $899.10. Principal = $1,079.19 − $899.10 = $180.09. Balance = $179,820.81 − $180.09 = $179,640.72.
2. $179.19 + $180.09 = $359.28. This also equals $180,000 − $179,640.72.
3. $200,000 − $179,640.72 = $20,359.28. The $359.28 increase comes from principal reduction, not the entire payments.

4. Interest falls from $900 to $899.10, a $0.90 decrease. With payment fixed, principal rises from $179.19 to $180.09 by the same $0.90.
5. Use the zero-interest case: $180,000 ÷ 360 = $500 per month. The positive-rate formula would otherwise divide by zero.

RECAP QUIZ

1. **0.005**: 0.06 ÷ 12 = 0.005, or 0.5% per month.
2. **$900**: $180,000 × 0.005 = $900.
3. **Reduces later interest by lowering the balance**: Later interest is calculated on a smaller outstanding principal, under the stated model.

CHAPTER 16

Understanding Financial Statements

A busy business can report a profit and still struggle to pay a bill. Learn how income statements, balance sheets, and cash flow statements answer different questions about the same operation. All businesses and amounts are hypothetical; round money to cents and label each statement's date or period.

16.1

Income Statements: From Revenue to Profit

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate gross profit and net income.
2. Distinguish revenue, expenses, and cash receipts.
3. Interpret a net profit margin.

REAL-LIFE SITUATION

A neighborhood food cart closes the month

A hypothetical Philadelphia food cart summarizes one month's earned revenue and expenses. The figures use simplified accrual accounting, so profit is not automatically the amount of cash collected.

Item	Amount
Revenue	$5,000
Cost of goods sold	$1,800
Gross profit	$3,200
Operating expenses	$2,200
Operating income	$1,000
Other expense	$200
Net income	$800

Figure 16.1 Hypothetical monthly income statement

The concept

An income statement reports financial performance over a period, such as a month or year. Revenue records amounts earned from ordinary sales or services under the accounting method used. It is not the same as owner contributions, borrowed money, or necessarily cash collected.

For a seller of goods, cost of goods sold is the assigned cost of inventory actually sold during the period. Subtract it from revenue to find gross profit. Inventory purchased but not yet sold generally remains an asset rather than becoming this period's cost of goods sold.

Operating expenses are the costs of running the operation beyond cost of goods sold. Subtract them from gross profit to find operating income, then include other stated revenues or expenses to find net income. Figure 16.1 omits taxes solely to simplify the example, not because a real business necessarily owes none.

Net profit margin expresses net income as a percentage of revenue. It helps compare different sales levels, but a margin alone does not explain risk, cash shortages, or owner compensation. Use consistent periods and expense categories before comparing businesses.

Income Statements: From Revenue to Profit: solution map

1. Identify the question

   `Calculate the food cart's gross profit, operating income, and net income.`

2. Calculation 1

   `Gross profit: $5,000 − $1,800 = $3,200.`

3. Calculation 2

   `Operating income: $3,200 − $2,200 = $1,000.`

4. Calculation 3

   `Net income: $1,000 − $200 = $800.`

5. Interpret and check

   `Gross profit is $3,200, operating income is $1,000, and net income is $800.`

Figure 16.1.V Income Statements: From Revenue to Profit: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

BALANCE SHEET

A snapshot listing what a business owns (assets) and owes (liabilities) on one date.

INCOME STATEMENT

A report showing revenue, expenses, and profit over a period of time.

CASH FLOW

The movement of money into and out of a business or account.

KEY TERMS

Revenue
Amounts earned from sales or services during the stated period.

Cost of goods sold
The assigned inventory cost of goods sold during the period.

Gross profit
Revenue minus cost of goods sold.

Operating expense
A cost of ordinary operations outside cost of goods sold.

Net income
Revenue and gains minus all expenses and losses included in the statement.

Net profit margin
Net income expressed as a percentage of revenue.

FORMULAS & RULES

Gross profit

```
Gross profit = revenue − cost of goods sold
```

Operating income

```
Operating income = gross profit − operating expenses
```

Net income

```
Net income = operating income + other revenues − other expenses
```

Net margin

```
Net profit margin = net income ÷ revenue × 100
```

Revenue must be nonzero.

Worked examples

Example 1 Follow the income statement

Calculate the food cart's gross profit, operating income, and net income.

SOLUTION

1. Gross profit: $5,000 − $1,800 = $3,200.
2. Operating income: $3,200 − $2,200 = $1,000.
3. Net income: $1,000 − $200 = $800.

> **Answer:** Gross profit is $3,200, operating income is $1,000, and net income is $800.

Example 2 Put profit into perspective

Find the cart's net profit margin and cost-of-goods-sold percentage.

SOLUTION

1. Net income divided by revenue: $800 ÷ $5,000 = 0.16.

2. Convert to percent: 0.16 × 100 = 16%.

3. Cost-of-goods-sold percentage: $1,800 ÷ $5,000 × 100 = 36%.

Answer: Net profit margin is 16%; cost of goods sold is 36% of revenue.

Practice exercises 16.1

1. Operating expenses rise by $150 with everything else unchanged. Find new net income and margin.
2. Find total listed expenses in Figure 16.1.
3. At the same total expense dollars, what revenue would make net income zero?
4. An owner puts $500 into the business. Should it increase the statement's sales revenue?
5. The business earns $300 from a customer who will pay next month. Under the stated accrual approach, what happens now?

RECAP QUIZ 16.1

1. Revenue less cost of goods sold is:

- ○ a. Cash balance
- ○ b. Gross profit
- ○ c. Owner investment
- ○ d. Loan principal

2. An income statement covers:

- ○ a. A period of time
- ○ b. Only one instant
- ○ c. Only cash deposits
- ○ d. Only assets

3. $800 net income on $5,000 revenue gives a margin of:

- ○ a. 6.25%
- ○ b. 8%

○ c. 16%

○ d. 80%

SECTION 16.1 RECAP

- Read down the statement from revenue through gross profit to net income.
- Profit measures performance under an accounting method, not simply cash collected.

Answer key with explanations: Section 16.1

PRACTICE EXERCISES

1. New net income = $800 − $150 = $650. Margin = $650 ÷ $5,000 × 100 = 13%.
2. $1,800 + $2,200 + $200 = $4,200. Revenue of $5,000 less $4,200 equals the stated $800 net income.
3. Revenue must equal $4,200 in expenses. This is a fixed-dollar illustration, not a claim that costs would stay unchanged at every sales level.
4. No. Owner investment is financing, not earned revenue. Revenue stays $5,000; the contribution increases cash and equity by $500.
5. Record $300 revenue and $300 accounts receivable now, with no immediate cash increase. If no associated expenses change, net income rises by $300.

RECAP QUIZ

1. **Gross profit**: Gross profit = $5,000 − $1,800 = $3,200 in the example.
2. **A period of time**: Income statements measure activity during a stated period.
3. **16%**: $800 ÷ $5,000 × 100 = 16%.

16.2

Balance Sheets and Financial Position

LEARNING OBJECTIVES

After this section you will be able to:

1. Apply the accounting equation.
2. Calculate working capital and the current ratio.
3. Explain how transactions affect assets, liabilities, and equity.

REAL-LIFE SITUATION

A repair shop's financial snapshot

A hypothetical neighborhood repair shop lists its resources and obligations at month-end. Equipment is shown at its net book amount, and no other accounts are omitted from this simplified balance sheet.

Classification	Account	Amount
Current asset	Cash	$3,000
Current asset	Accounts receivable	$2,000
Current asset	Inventory	$5,000
Noncurrent asset	Equipment, net	$10,000
Current liability	Accounts payable	$4,000
Noncurrent liability	Long-term loan	$6,000
Equity	Owner's equity	$10,000

Figure 16.2 Hypothetical balance sheet at month-end

The concept

A balance sheet reports financial position at one date. Assets are resources controlled by the business; liabilities are obligations; equity is the residual after liabilities. The accounting equation must balance: assets = liabilities + equity.

Current assets are expected to turn into cash, be sold, or be used within a year or the normal operating cycle if longer. Current liabilities are due in that horizon. Classification matters: a loan's amount due soon belongs in current liabilities even if the original loan was long-term.

Working capital is current assets minus current liabilities. The current ratio divides those amounts instead. Both describe short-term financial position, but neither guarantees bill-paying ability: inventory may sell slowly, and customers may pay late.

Every transaction has effects that preserve the equation. Paying an existing payable reduces cash and liabilities; it does not create a second expense if that expense was already recorded. Borrowing cash increases both assets and liabilities without creating revenue.

Balance Sheets and Financial Position: solution map

1. Identify the question

   `Find total assets, liabilities, and equity from Figure 16.2.`

2. Calculation 1

   `Current assets: $3,000 + $2,000 + $5,000 = $10,000.`

3. Calculation 2

   `Total assets: $10,000 + $10,000 = $20,000.`

4. Calculation 3

   `Total liabilities: $4,000 + $6,000 = $10,000.`

5. Calculation 4

   `Equity: $20,000 − $10,000 = $10,000.`

6. Calculation 5

   `Check: $20,000 = $10,000 + $10,000.`

7. Interpret and check

   `Assets are $20,000, liabilities are $10,000, and equity is $10,000.`

Figure 16.2.V Balance Sheets and Financial Position: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

ASSET

Something valuable that a person or business owns, like cash, equipment, or a building.

LIABILITY

Money or obligations a person or business owes to others.

EQUITY

What is left of an asset after debts are subtracted; in a home, value minus what you still owe.

KEY TERMS

Asset
A resource controlled by a business as a result of past events.

Liability
An obligation the business must settle.

Equity
The residual interest after liabilities are subtracted from assets.

Current asset
An asset expected to be realized, sold, or used in the short-term classification horizon.

Working capital
Current assets minus current liabilities.

Current ratio
Current assets divided by current liabilities.

FORMULAS & RULES

Accounting equation

```
Assets = liabilities + equity
```

Residual equity

```
Equity = assets − liabilities
```

Working capital

```
Working capital = current assets − current liabilities
```

Current ratio

```
Current ratio = current assets ÷ current liabilities
```

Requires nonzero current liabilities.

Worked examples

Example 1 Check the snapshot

Find total assets, liabilities, and equity from Figure 16.2.

SOLUTION

1. Current assets: $3,000 + $2,000 + $5,000 = $10,000.
2. Total assets: $10,000 + $10,000 = $20,000.
3. Total liabilities: $4,000 + $6,000 = $10,000.
4. Equity: $20,000 − $10,000 = $10,000.
5. Check: $20,000 = $10,000 + $10,000.

> **Answer:** Assets are $20,000, liabilities are $10,000, and equity is $10,000.

Example 2 Measure short-term resources

Find working capital and current ratio, then pay $1,000 of accounts payable in cash.

SOLUTION

1. Original working capital: $10,000 − $4,000 = $6,000.
2. Original current ratio: $10,000 ÷ $4,000 = 2.5 to 1.
3. After payment, current assets are $9,000 and current liabilities are $3,000.
4. New working capital: $9,000 − $3,000 = $6,000.
5. New current ratio: $9,000 ÷ $3,000 = 3 to 1.

> **Answer:** Working capital stays $6,000, but the ratio rises from 2.5 to 3; cash nevertheless falls.

Practice exercises 16.2

1. What percent of the original assets is financed by liabilities?
2. Collect $500 of existing accounts receivable. Find new cash, receivables, and total current assets.
3. Buy $600 of inventory on account. Find new total assets and liabilities.
4. Why might $10,000 of current assets fail to cover an immediate $4,000 cash bill?
5. Reclassify $1,000 of the long-term loan as due within a year. Find the current ratio.

RECAP QUIZ 16.2

1. Assets of $20,000 less liabilities of $10,000 imply equity of:

- ○ a. $30,000
- ○ b. $20,000
- ○ c. $10,000
- ○ d. $0

2. A balance sheet is measured:

- ○ a. At a stated date
- ○ b. Only over a full year
- ○ c. Only when cash changes
- ○ d. Without liabilities

3. Collecting an existing receivable generally:

- ○ a. Creates sales a second time
- ○ b. Changes one asset into another
- ○ c. Eliminates equity
- ○ d. Creates a loan

SECTION 16.2 RECAP

- Keep the accounting equation balanced and distinguish totals from classifications.
- Ratios summarize financial position but do not replace a cash-timing analysis.

Answer key with explanations: Section 16.2

PRACTICE EXERCISES

1. $10,000 ÷ $20,000 × 100 = 50%. This is a debt-to-assets percentage, not the current ratio.
2. Cash becomes $3,500 and receivables become $1,500. Their combined amount is unchanged, so current assets remain $10,000.
3. Assets rise to $20,600 and liabilities to $10,600. Equity stays $10,000, so $20,600 = $10,600 + $10,000.
4. Only $3,000 is cash; the other $7,000 is receivables and inventory. The shop could be $1,000 short immediately despite $6,000 positive working capital.

5. Current liabilities become $4,000 + $1,000 = $5,000. Current ratio = $10,000 ÷ $5,000 = 2. Total liabilities do not change.

RECAP QUIZ

1. **$10,000**: Equity = $20,000 − $10,000 = $10,000.
2. **At a stated date**: It is a snapshot of financial position.
3. **Changes one asset into another**: Cash rises while accounts receivable falls by the same amount.

16.3

Cash Flow and the Connections Between Statements

LEARNING OBJECTIVES

After this section you will be able to:

1. Classify operating, investing, and financing cash flows.
2. Reconcile a simplified net income amount to operating cash flow.
3. Calculate the change in cash and ending cash balance.

REAL-LIFE SITUATION

Profit and cash tell different stories

A hypothetical workshop reports $800 net income. Depreciation is $200, receivables increase $500, and operating accounts payable increase $300; all other reconciliation adjustments are zero.

Follow the cash

```
Net income: $800
Add depreciation: $200
Subtract receivables increase: $500
Add operating payables increase: $300
Cash equipment purchase: $1,200
Cash borrowed: $600
Beginning cash: $3,000
```

Figure 16.3 Simplified monthly cash flow information

The concept

A cash flow statement explains the change in cash over a period. Operating activities relate to the business's main earning activities. Investing activities include buying and selling long-lived assets. Financing activities include borrowing, repaying loan principal, and owner capital transactions.

Under a simplified indirect reconciliation, begin with net income and adjust for noncash items and timing differences. Depreciation reduces accounting income without being a current-period cash payment, so add it back. This does not mean equipment was free; its purchase cash flow is shown separately.

An increase in receivables means some recorded revenue has not yet been collected, so subtract that increase. An increase in operating payables means some recorded operating costs have not yet been paid, so add it. These sign rules require consistent account definitions and can be complicated by transactions omitted from a beginner model.

The three statements connect: net income affects equity, the balance sheet reports ending cash, and the cash flow statement explains how cash changed. Borrowing can raise cash without raising profit, while profitable sales on account can raise profit without immediate cash. No single statement tells the entire story.

Distinguish profit from cash movements

1. Starting information
   `$3,000 beginning cash; $800 net income`
2. Investing cash movement
   `$1,200 equipment purchase`
3. Financing cash movement
   `$600 borrowed; apply operating adjustments in the example`

Figure 16.3.V Distinguish profit from cash movements. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

REVENUE
The total money a business brings in from sales, before any costs are paid.

EXPENSE
Money a business spends to run, such as rent, supplies, or wages.

NET PROFIT
The final profit after every cost is paid — sometimes called the bottom line.

KEY TERMS

Operating cash flow
Cash generated or used by the main operating activities.

Investing cash flow

Cash flows from long-lived assets and other investing activities.

Financing cash flow
Cash flows involving borrowing and owner capital.

Noncash expense
An expense not requiring a cash payment in the same period.

Indirect reconciliation
Adjusting net income to explain operating cash flow.

FORMULAS & RULES

Simplified operating cash

```
Operating cash = net income + depreciation −
receivables increase + operating payables increase
```

All other adjustments are zero in this model.

Net cash change

```
Cash change = operating cash + investing cash +
financing cash
```

Ending cash

```
Ending cash = beginning cash + cash change
```

Worked examples

Example 1 Reconcile profit to operating cash

Use the reconciliation data in Figure 16.3.

SOLUTION

1 Begin with $800 net income.

2. Add noncash depreciation: $800 + $200 = $1,000.
3. Subtract the receivables increase: $1,000 − $500 = $500.
4. Add the operating payables increase: $500 + $300 = $800.

> **Answer:** Operating cash flow is $800; matching net income here is a coincidence of the adjustments.

Example 2 Find ending cash

Include a $1,200 cash equipment purchase, $600 cash borrowing, and $3,000 beginning cash.

SOLUTION

1. Operating cash flow is +$800.
2. Investing cash flow is −$1,200 for equipment.
3. Financing cash flow is +$600 from borrowing.
4. Net cash change: $800 − $1,200 + $600 = $200.
5. Ending cash: $3,000 + $200 = $3,200.

> **Answer:** Cash rises $200 to $3,200, even though net income is $800.

Practice exercises 16.3

1. Receivables increase $700 instead of $500. Recompute operating cash flow.
2. Classify a $400 cash repayment of loan principal and show its effect on the original ending cash.

3. If there were no equipment purchase, find net cash change using the original other figures.
4. Why is depreciation added back rather than treated as extra earned cash?
5. An owner contributes $250 cash after the original transactions. Find financing cash flow and ending cash.

RECAP QUIZ 16.3

1. A cash equipment purchase is generally:

- ○ a. An investing outflow
- ○ b. Sales revenue
- ○ c. A financing inflow
- ○ d. A noncash receipt

2. Why subtract an increase in receivables from net income?

- ○ a. It is always a loss
- ○ b. Some recorded revenue has not been collected
- ○ c. It repays a loan
- ○ d. It is depreciation

3. Beginning cash $3,000 plus net cash change $200 equals:

- ○ a. $800
- ○ b. $2,800
- ○ c. $3,000
- ○ d. $3,200

SECTION 16.3 RECAP

- Separate performance, financial position, and cash movement.
- Cash borrowing is not profit, and depreciation is not a current cash payment.

Answer key with explanations: Section 16.3

PRACTICE EXERCISES

1. $800 + $200 − $700 + $300 = $600. The additional $200 uncollected revenue lowers operating cash by $200.
2. Loan principal repayment is a financing outflow. Original ending cash $3,200 − $400 = $2,800; principal repayment is not itself an income-statement expense.
3. $800 + $0 + $600 = $1,400. Ending cash would be $3,000 + $1,400 = $4,400.
4. Net income already subtracts the $200 noncash expense. Adding $200 reverses that subtraction in the cash reconciliation; it does not create a new receipt.
5. Financing cash = $600 + $250 = $850. Total cash change = $800 − $1,200 + $850 = $450, so ending cash is $3,450.

RECAP QUIZ

1. **An investing outflow**: Cash paid for a long-lived operating asset belongs in investing activities in this model.
2. **Some recorded revenue has not been collected**: Accrual revenue can precede the related cash receipt.
3. **$3,200**: $3,000 + $200 = $3,200.

CHAPTER 17

Depreciation and Business Assets

Equipment can help a business for years even when the purchase is paid for today. Depreciation allocates that cost across the periods or output receiving the benefit; it does not predict resale prices. All assets, businesses, estimates, and amounts are hypothetical, and the methods here are accounting examples rather than current tax rules; round money to cents.

17.1

Asset Cost and Straight-Line Depreciation

LEARNING OBJECTIVES

After this section you will be able to:

1. Determine an asset's depreciable cost basis.
2. Calculate straight-line depreciation and book value.
3. Distinguish accounting allocation from market value.

REAL-LIFE SITUATION

A shared workshop buys equipment

A hypothetical Philadelphia workshop buys a machine for $9,000, pays $600 delivery and $400 installation, and expects five years of service with a $1,000 residual value. It is ready for use at the start of the first year.

Machine record

```
Purchase price: $9,000
Delivery: $600; installation: $400
Recorded cost: $10,000
Estimated residual value: $1,000
Estimated useful life: 5 years
Straight-line depreciation: $1,800 per full year
```

Figure 17.1 Hypothetical asset cost and depreciation estimates

The concept

A long-lived business asset helps operations beyond the current period. Its recorded cost generally includes the purchase price and necessary costs to bring it to the location and condition required for use. This illustration capitalizes delivery and installation; routine later maintenance is treated separately.

Depreciable amount is recorded cost minus estimated residual value. Residual value is the expected amount remaining at the end of the useful life under the estimates used. Neither the life nor the residual is a promise; actual outcomes may differ.

Straight-line depreciation allocates the same depreciable amount to each full year. Accumulated depreciation adds all depreciation recorded so far, while book value equals cost minus that accumulated amount. Under unchanged estimates, the scheduled ending book value is the residual value, not necessarily zero.

Depreciation is an allocation of cost, not an appraisal and not a cash savings account. A machine's market value may rise or fall differently from its book value. For partial years, this section uses an explicitly stated months-in-service fraction; real policies and tax rules may use different conventions.

Spread depreciable cost over useful life

1. Asset cost
   `$9,000 + $600 + $400 = $10,000`
2. Depreciable base
   `$10,000 − $1,000 residual = $9,000`
3. Annual expense
   `$9,000 ÷ 5 years = $1,800`

Figure 17.1.V Spread depreciable cost over useful life. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

DEPRECIATION
The way a costly item, such as a machine, loses value and its cost is spread over its useful life.

ASSET
Something valuable that a person or business owns, like cash, equipment, or a building.

LIABILITY
Money or obligations a person or business owes to others.

KEY TERMS

Capitalized cost
An amount recorded as part of an asset rather than expensed immediately.

Useful life

The estimated period over which an asset benefits the business.

Residual value
The estimated value remaining at the end of the useful life.

Depreciable amount
Recorded cost minus estimated residual value.

Accumulated depreciation
Total depreciation recorded on an asset to date.

Book value
Recorded asset cost minus accumulated depreciation in this simplified model.

FORMULAS & RULES

Recorded cost

```
Cost = purchase price + necessary delivery + necessary installation
```

Straight-line expense

```
Annual depreciation = (cost − residual value) ÷ useful life
```

Book value

```
Book value = cost − accumulated depreciation
```

Partial-year expense

```
Partial-year depreciation = annual depreciation × months in service ÷ 12
```

The convention used in these examples.

Worked examples

Example 1 Build the annual expense

Find the workshop machine's recorded cost and annual straight-line depreciation.

SOLUTION

1. Recorded cost: $9,000 + $600 + $400 = $10,000.
2. Depreciable amount: $10,000 − $1,000 = $9,000.
3. Annual expense: $9,000 ÷ 5 = $1,800.

> **Answer:** Record the machine at $10,000 and allocate $1,800 per full year.

Example 2 Read book value after three years

Find accumulated depreciation and book value after three full years.

SOLUTION

1. Annual straight-line depreciation is $1,800.
2. Accumulated depreciation: 3 × $1,800 = $5,400.
3. Book value: $10,000 − $5,400 = $4,600.
4. Remaining depreciable amount: $4,600 − $1,000 = $3,600.

> **Answer:** Accumulated depreciation is $5,400 and book value is $4,600, leaving two $1,800 full-year allocations.

Practice exercises 17.1

1. Find book value after two full years.
2. The machine is first used halfway through a year. Under the stated six-month convention, find first-year depreciation.
3. Find accumulated depreciation and book value after five full years.
4. What percent of recorded cost is the $1,000 residual estimate?
5. A routine $200 maintenance bill occurs in year 2. Under the stated policy, does it increase the machine's recorded cost?

RECAP QUIZ 17.1

1. The machine's depreciable amount is:

- ○ a. $1,000
- ○ b. $9,000
- ○ c. $10,000
- ○ d. $11,000

2. Straight-line depreciation on $9,000 over five years is:

- ○ a. $900
- ○ b. $1,000
- ○ c. $1,800
- ○ d. $2,000

3. Book value necessarily equals:

- ○ a. Current resale price
- ○ b. Cash saved for replacement
- ○ c. Original purchase price forever
- ○ d. Cost minus accumulated depreciation in this model

SECTION 17.1 RECAP

- Depreciate cost minus residual, not automatically the entire cost.
- Book value, market value, and cash on hand are different quantities.

Answer key with explanations: Section 17.1

PRACTICE EXERCISES

1. Accumulated depreciation = 2 × $1,800 = $3,600. Book value = $10,000 − $3,600 = $6,400.
2. $1,800 × 6 ÷ 12 = $900. Book value at that year-end is $10,000 − $900 = $9,100.
3. Accumulated depreciation = 5 × $1,800 = $9,000. Book value = $10,000 − $9,000 = $1,000, the estimated residual.
4. $1,000 ÷ $10,000 × 100 = 10%. The remaining 90%, or $9,000, is depreciable.
5. No. Record the $200 as maintenance expense under this simplified policy; cost stays $10,000 and scheduled annual depreciation stays $1,800.

RECAP QUIZ

1. **$9,000**: $10,000 cost − $1,000 residual = $9,000.
2. **$1,800**: $9,000 ÷ 5 = $1,800.
3. **Cost minus accumulated depreciation in this model**: Book value is an accounting amount, not a market-price estimate.

17.2

Units-of-Production Depreciation

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate depreciation per unit of expected output.
2. Allocate expense according to actual production.
3. Track cumulative depreciation without crossing residual value.

REAL-LIFE SITUATION

A machine works harder in some years

Use the same $10,000 machine and $1,000 residual value, but assume expected lifetime output is 45,000 units. A units-of-production method connects expense to actual use rather than equal calendar years.

Period	Units	Depreciation	Ending book value
Year 1	12000	$2,400	$7,600
Year 2	15000	$3,000	$4,600
Remaining expected output	18000	$3,600 remaining	$1,000 after full allocation

Figure 17.2 Hypothetical output-based depreciation record

The concept

Units-of-production depreciation is useful when wear or benefit is better represented by output than by elapsed time. The activity measure might be units produced, machine hours, or another consistent measure. Estimated lifetime activity must use the same unit as actual activity.

Divide depreciable amount by total expected activity to get a per-unit allocation. Multiply that rate by the period's actual activity. Keep enough rate precision for the schedule; premature rounding can distort total expense when expected output is large.

The method can produce unequal annual expenses without changing the original total depreciable amount. A busier year records more depreciation under this method, while straight-line records equal full-year amounts

under unchanged estimates. Neither method is automatically a prediction of selling price.

Track cumulative activity and accumulated depreciation. Under unchanged estimates, total depreciation must not exceed cost minus residual. Unexpected additional use does not justify pushing book value below the residual in this simplified schedule; revised estimates require a separate, clearly explained calculation.

Units-of-Production Depreciation: solution map

1. Identify the question

   `Find the depreciation rate per unit and the first year's expense for 12,000 units.`

2. Calculation 1

   `Depreciable amount: $10,000 − $1,000 = $9,000.`

3. Calculation 2

   `Per-unit amount: $9,000 ÷ 45,000 = $0.20.`

4. Calculation 3

   `First-year depreciation: 12,000 × $0.20 = $2,400.`

5. Calculation 4

   `Ending book value: $10,000 − $2,400 = $7,600.`

6. Interpret and check

   `Allocate $0.20 per unit; year 1 expense is $2,400 and book value is $7,600.`

Figure 17.2.V Units-of-Production Depreciation: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

EQUITY
What is left of an asset after debts are subtracted; in a home, value minus what you still owe.

BALANCE SHEET
A snapshot listing what a business owns (assets) and owes (liabilities) on one date.

CASH FLOW
The movement of money into and out of a business or account.

KEY TERMS

Units-of-production method
Depreciation based on actual activity relative to estimated lifetime activity.

Activity base
The measurable use, such as units produced or machine hours.

Per-unit depreciation
Depreciable amount allocated to one unit of activity.

Cumulative output
Total units produced from the start through the current period.

Remaining depreciable amount
Cost minus residual minus depreciation already recorded.

FORMULAS & RULES

Per-unit allocation

```
Depreciation per unit = (cost − residual value) ÷
estimated total units
```

Period expense

```
Period depreciation = actual units × depreciation per unit
```

Cap expense at the remaining depreciable amount.

Remaining allocation

```
Remaining depreciable amount = cost − residual value − accumulated depreciation
```

Worked examples

Example 1 Allocate the first year's use

Find the depreciation rate per unit and the first year's expense for 12,000 units.

SOLUTION

1. Depreciable amount: $10,000 − $1,000 = $9,000.
2. Per-unit amount: $9,000 ÷ 45,000 = $0.20.
3. First-year depreciation: 12,000 × $0.20 = $2,400.
4. Ending book value: $10,000 − $2,400 = $7,600.

> **Answer:** Allocate $0.20 per unit; year 1 expense is $2,400 and book value is $7,600.

Example 2 Continue the output record

The machine produces 15,000 units in year 2. Find expense, book value, and remaining expected units.

SOLUTION

1. Year 2 depreciation: 15,000 × $0.20 = $3,000.
2. Accumulated depreciation: $2,400 + $3,000 = $5,400.
3. Book value: $10,000 − $5,400 = $4,600.
4. Cumulative output: 12,000 + 15,000 = 27,000 units.
5. Remaining expected output: 45,000 − 27,000 = 18,000 units.

> **Answer:** Year 2 expense is $3,000, book value is $4,600, and 18,000 expected units remain.

Practice exercises 17.2

1. Find year 1's remaining expected units after 12,000 units.
2. A separate year has 8,000 units of output with sufficient depreciable amount remaining. Find expense.
3. After the first two years, find the remaining depreciable amount.
4. Compare first-year units-of-production expense with the $1,800 straight-line expense.
5. After the first two years, actual additional output is 20,000 units. Under unchanged estimates, apply the depreciation cap.

RECAP QUIZ 17.2

1. $9,000 depreciable amount over 45,000 units gives:

- ○ a. $0.02 per unit
- ○ b. $0.20 per unit
- ○ c. $2 per unit
- ○ d. $5 per unit

2. At $0.20 per unit, 15,000 units produce expense of:

- ○ a. $750
- ○ b. $1,500
- ○ c. $3,000
- ○ d. $30,000

3. With unchanged estimates, total depreciation is limited to:

- ○ a. Revenue
- ○ b. Cash collected
- ○ c. Cost plus residual
- ○ d. Cost minus residual

SECTION 17.2 RECAP

- Match the activity measure in the rate with the activity actually recorded.
- Track accumulated depreciation so the allocation does not exceed its cap.

Answer key with explanations: Section 17.2

PRACTICE EXERCISES

1. 45,000 − 12,000 = 33,000 units remain under the original lifetime estimate.
2. 8,000 × $0.20 = $1,600.
3. $10,000 − $1,000 − $5,400 = $3,600. This also equals 18,000 expected units × $0.20.
4. $2,400 − $1,800 = $600 more under units of production. The difference changes timing, not the original $9,000 total depreciable amount.
5. Uncapped expense = 20,000 × $0.20 = $4,000. Only $3,600 remains depreciable, so record $3,600 and leave book value at $1,000.

RECAP QUIZ

1. **$0.20 per unit**: $9,000 ÷ 45,000 = $0.20 per unit.
2. **$3,000**: 15,000 × $0.20 = $3,000.

3. **Cost minus residual**: The method allocates the depreciable amount, leaving the estimated residual.

17.3

Declining Balance and Asset Disposal

LEARNING OBJECTIVES

After this section you will be able to:

1. Construct a capped double-declining-balance schedule.
2. Compare accelerated and straight-line expense timing.
3. Calculate a gain or loss when an asset is sold.

REAL-LIFE SITUATION

More expense early, less later

For a separate method comparison, depreciate the $10,000 machine using double-declining balance over five years. Residual remains $1,000; this illustration uses no switch to straight-line and adjusts the last year's expense to reach residual.

Year	Beginning book value	Expense	Ending book value
1	$10,000	$4,000	$6,000
2	$6,000	$2,400	$3,600
3	$3,600	$1,440	$2,160
4	$2,160	$864	$1,296
5	$1,296	$296 adjusted	$1,000

Figure 17.3 Hypothetical double-declining-balance schedule

The concept

An accelerated depreciation method records more expense early than later under typical conditions. Double-declining balance starts with twice the straight-line rate, then applies that rate to beginning book value rather than repeatedly to original cost.

For a five-year life, the straight-line rate is 1 ÷ 5 = 20%, so the double-declining rate is 40%. Do not subtract residual before multiplying book value by this rate. Instead, residual acts as a floor that limits the expense allowed.

A pure declining-balance calculation may not land exactly on residual at the end of the estimated life. Figure 17.3 therefore states a specific final-year adjustment to allocate the remaining amount. Other schedules may switch to straight-line earlier; identifying the convention is part of solving the problem.

When an asset is sold, compare net proceeds with its book value on the sale date after recording depreciation through that date. Proceeds above book value create an accounting gain; proceeds below it create a loss. This gain or loss does not equal the total lifetime cash profit from owning the asset, and any tax consequences depend on rules outside this illustration.

Declining Balance and Asset Disposal: solution map

1. Identify the question

   Find the rate and the first two years of double-declining depreciation.

2. Calculation 1

   Rate: 2 ÷ 5 = 0.40, or 40%.

3. Calculation 2

   Year 1 expense: $10,000 × 0.40 = $4,000.

4. Calculation 3

   Year 1 ending book value: $10,000 − $4,000 = $6,000.

5. Calculation 4

   Year 2 expense: $6,000 × 0.40 = $2,400.

6. Calculation 5

   Year 2 ending book value: $6,000 − $2,400 = $3,600.

7. Interpret and check

   The rate is 40%; expenses are $4,000 and $2,400, leaving $3,600 after year 2.

Figure 17.3.V Declining Balance and Asset Disposal: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

COST

The amount paid to make or buy an item before it is sold.

PROFIT

What is left when expenses are subtracted from revenue.

OVERHEAD

Ongoing business costs such as rent, utilities, and insurance that do not change with each sale.

KEY TERMS

Accelerated depreciation
A method that generally allocates more depreciation in earlier periods.

Double-declining balance
A method applying twice the straight-line rate to beginning book value.

Residual floor
The minimum scheduled book value under the stated unchanged estimates.

Net sale proceeds
Sale receipts minus direct selling costs.

Gain on disposal
Net proceeds exceeding book value at disposal.

Loss on disposal
Book value exceeding net proceeds at disposal.

FORMULAS & RULES

Double-declining rate

```
Double-declining rate = 2 ÷ useful life
```

Ordinary period calculation

```
Depreciation = beginning book value × double-declining rate
```

Apply the residual floor and the stated final-year convention.

Final-year adjustment

```
Final-year depreciation = beginning book value − residual value
```

Used in this chapter's no-switch schedule.

Disposal result

```
Gain or loss = net sale proceeds − book value
```

Positive is a gain; negative is a loss.

Worked examples

Example 1 Begin an accelerated schedule

Find the rate and the first two years of double-declining depreciation.

SOLUTION

1. Rate: 2 ÷ 5 = 0.40, or 40%.
2. Year 1 expense: $10,000 × 0.40 = $4,000.
3. Year 1 ending book value: $10,000 − $4,000 = $6,000.
4. Year 2 expense: $6,000 × 0.40 = $2,400.
5. Year 2 ending book value: $6,000 − $2,400 = $3,600.

Answer: The rate is 40%; expenses are $4,000 and $2,400, leaving $3,600 after year 2.

Example 2 Sell an asset at a gain or loss

For a separate disposal example, a machine has $10,000 cost and $5,400 accumulated depreciation at sale. Compare net proceeds of $5,000 and $4,000.

SOLUTION

1. Book value: $10,000 − $5,400 = $4,600.
2. At $5,000 net proceeds: $5,000 − $4,600 = $400.
3. The positive $400 result is an accounting gain.
4. At $4,000 net proceeds: $4,000 − $4,600 = −$600, an accounting loss.

Answer: $5,000 net proceeds create a $400 gain; $4,000 net proceeds create a $600 loss.

Practice exercises 17.3

1. Calculate year 3 of the double-declining schedule.
2. Calculate year 4 of the schedule.
3. Find the final-year adjusted expense and explain why ordinary multiplication is not used.
4. Compare first-year double-declining expense with first-year straight-line expense.
5. An asset with $4,600 book value sells for $5,000, but selling costs are $200. Find the gain or loss.

RECAP QUIZ 17.3

1. The double-declining rate for a five-year life is:

- ○ a. 10%
- ○ b. 20%
- ○ c. 40%
- ○ d. 50%

2. Ordinary double-declining expense uses:

- ○ a. Beginning book value
- ○ b. Sales revenue
- ○ c. Original cost every year
- ○ d. Residual value alone

3. $4,000 net proceeds minus $4,600 book value gives:

- ○ a. A $600 gain
- ○ b. A $600 loss
- ○ c. A $4,000 gain
- ○ d. No result

SECTION 17.3 RECAP

- Accelerated depreciation changes expense timing, not the original depreciable total.
- Compare net proceeds with updated book value to determine a disposal gain or loss.

Answer key with explanations: Section 17.3

PRACTICE EXERCISES

1. Expense = $3,600 × 0.40 = $1,440. Ending book value = $3,600 − $1,440 = $2,160.
2. Expense = $2,160 × 0.40 = $864. Ending book value = $2,160 − $864 = $1,296.

3. Final expense = $1,296 − $1,000 = $296. Ordinary 40% expense would be $518.40 and push book value to $777.60, below the residual floor.
4. $4,000 − $1,800 = $2,200 more expense under double-declining balance. Both schedules still allocate $9,000 in total under the stated full-life conventions.
5. Net proceeds = $5,000 − $200 = $4,800. Gain = $4,800 − $4,600 = $200.

RECAP QUIZ

1. **40%**: 2 ÷ 5 = 0.40 = 40%.
2. **Beginning book value**: The rate is applied to the declining beginning book value.
3. **A $600 loss**: $4,000 − $4,600 = −$600, so the disposal produces a $600 loss.

CHAPTER 18

Inventory, Overhead, and Operating Costs

A busy shop can sell plenty and still lose money if stock disappears or costs are overlooked. Learn to track inventory, share overhead, and test an operating plan. All businesses, people, prices, and cost assumptions in this chapter are hypothetical; round monetary answers to cents.

18.1

Inventory Counts, Cost, and Turnover

LEARNING OBJECTIVES

After this section you will be able to:

1. Reconcile goods available with ending inventory.
2. Compute cost of goods sold and shrinkage.
3. Use average inventory to calculate turnover.

REAL-LIFE SITUATION

Counting a neighborhood pop-up

A hypothetical Philadelphia student cooperative sells plain tote bags. Its stock sheet separates goods sold from bags still on the shelf; every bag has the same $8 cost.

Item	Amount or quantity
Beginning inventory	`100 bags at $8`
Purchases	`150 bags at $8`
Recorded sales	`180 bags`
Physical ending count	`65 bags`

Figure 18.1 Tote-bag stock sheet (hypothetical)

The concept

Inventory is merchandise held for sale. A physical count tells a business what it actually has; a stock record tells it what it expects to have. Compare both in units before assigning dollar values. In the figure, 100 + 150 − 180 = 70 bags should remain, but only 65 are counted.

For this simplified model, goods available at cost equals beginning inventory plus net purchases. Net purchases means purchases after returns and discounts, including any assigned freight-in. Cost of goods sold, or COGS, is the cost assigned to merchandise sold, not the selling price charged to customers.

A shortage between expected and counted inventory is shrinkage. Here 70 − 65 = 5 bags are missing, a $40 cost. Businesses investigate counting errors, damage, and theft rather than assuming every shortage is theft. Keep shrinkage separate from recorded sales cost in this lesson; subtracting physical ending inventory from goods available would combine both.

Turnover compares a period's COGS with average inventory at cost. Using the simple mean of beginning and ending inventory gives a useful estimate, but seasonal businesses may need monthly averages. Higher turnover can mean efficient selling or dangerously low stock, so compare similar businesses over the same period. Round money to cents and turnover to two decimal places.

Inventory Counts, Cost, and Turnover: solution map

1. Identify the question

   Use the figure to find recorded COGS, physical ending value, and shrinkage.

2. Calculation 1

   Available units = 100 + 150 = 250; available cost = 250 × $8 = $2,000.00.

3. Calculation 2

   Recorded COGS = 180 × $8 = $1,440.00.

4. Calculation 3

   Expected ending units = 250 − 180 = 70; physical ending cost = 65 × $8 = $520.00.

5. Calculation 4

   Shrinkage = (70 − 65) × $8 = $40.00.

6. Calculation 5

   Check: $1,440 + $520 + $40 = $2,000.

7. Interpret and check

   Recorded COGS is $1,440.00, ending inventory is $520.00, and shrinkage is $40.00.

Figure 18.1.V Inventory Counts, Cost, and Turnover: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

INVENTORY

The stock of goods a business holds ready to sell.

OVERHEAD

Ongoing business costs such as rent, utilities, and insurance that do not change with each sale.

FIXED COST

A cost that stays the same no matter how many items are sold, like rent.

KEY TERMS

Inventory
Goods held for sale, measured in units or at cost.

Net purchases
Purchase cost after returns and discounts, plus assigned freight-in.

Cost of goods sold
Inventory cost assigned to goods actually sold.

Shrinkage
Expected inventory minus actual inventory, when positive.

Inventory turnover
Period COGS divided by average inventory at cost.

FORMULAS & RULES

Goods available

```
Available cost = beginning inventory cost + net purchases
```

Recorded sales cost

```
COGS = units sold × cost per unit (equal-cost goods)
```

Shrinkage

```
Shrinkage cost = (expected units − counted units) × unit cost
```

Turnover

```
Turnover = COGS ÷ [(beginning inventory + ending inventory) ÷ 2]
```

Worked examples

Example 1 Reconcile the tote bags

Use the figure to find recorded COGS, physical ending value, and shrinkage.

SOLUTION

1. Available units = 100 + 150 = 250; available cost = 250 × $8 = $2,000.00.
2. Recorded COGS = 180 × $8 = $1,440.00.
3. Expected ending units = 250 − 180 = 70; physical ending cost = 65 × $8 = $520.00.
4. Shrinkage = (70 − 65) × $8 = $40.00.
5. Check: $1,440 + $520 + $40 = $2,000.

> **Answer:** Recorded COGS is $1,440.00, ending inventory is $520.00, and shrinkage is $40.00.

Example 2 Measure turnover

A shop has $3,000 beginning inventory, $5,000 ending inventory, and $24,000 annual COGS. Find annual turnover.

SOLUTION

1. Average inventory = ($3,000 + $5,000) ÷ 2 = $4,000.00.
2. Turnover = $24,000 ÷ $4,000 = 6.
3. The annual COGS is six times the average inventory investment.

Answer: Annual inventory turnover is 6.00 times.

Practice exercises 18.1

1. A booth begins with 40 hats, buys 75, and sells 90. How many should remain?
2. The booth counts 22 hats instead of 25. Each cost $6. Find shrinkage.
3. Beginning inventory costs $700; net purchases cost $1,100. Ending inventory costs $500, with no shrinkage. Find COGS.
4. Beginning and ending inventory are $2,000 and $4,000. Annual COGS is $15,000. Find turnover.
5. Ten equal-cost cases cost $240 total; three remain with no loss. Find unit cost and COGS.

RECAP QUIZ 18.1

1. Which amount values goods at cost rather than selling price?

- ○ a. COGS
- ○ b. Sales revenue
- ○ c. Sales tax collected
- ○ d. Customer payments

2. Expected stock is 48 units; the count is 45. Each costs $9. Shrinkage is:

- ○ a. $3
- ○ b. $27
- ○ c. $405
- ○ d. $432

3. COGS is $18,000 and average inventory is $3,000. Turnover is:

- ○ a. 0.17 times
- ○ b. 3 times
- ○ c. 6 times
- ○ d. 15 times

SECTION 18.1 RECAP

- Reconcile units first, then multiply by cost; keep shrinkage separate from sales.
- Turnover uses COGS and average inventory at cost for the same period.

Answer key with explanations: Section 18.1

PRACTICE EXERCISES

1. Available = 40 + 75 = 115 hats; expected ending = 115 − 90 = 25 hats.
2. Missing units = 25 − 22 = 3; shrinkage cost = 3 × $6 = $18.00.
3. Available cost = $700 + $1,100 = $1,800; COGS = $1,800 − $500 = $1,300.00.
4. Average = ($2,000 + $4,000) ÷ 2 = $3,000; turnover = $15,000 ÷ $3,000 = 5.00 times per year.
5. Unit cost = $240 ÷ 10 = $24; units sold = 10 − 3 = 7; COGS = 7 × $24 = $168.00.

RECAP QUIZ

1. **COGS**: COGS assigns acquisition cost to the merchandise sold.
2. **$27**: (48 − 45) × $9 = $27.00.
3. **6 times**: $18,000 ÷ $3,000 = 6.00 times for the stated period.

18.2

Overhead and Cost Allocation

LEARNING OBJECTIVES

After this section you will be able to:

1. Distinguish direct costs from overhead.
2. Allocate shared overhead using a stated activity base.
3. Calculate a full unit cost without double-counting.

REAL-LIFE SITUATION

Sharing a kitchen

Two hypothetical food stalls share a Philadelphia preparation kitchen. They allocate $1,200 of monthly kitchen overhead by preparation hours, not by the number of meals sold.

Item	Amount or quantity
Stall A preparation hours	80
Stall B preparation hours	40
Shared kitchen overhead	$1,200.00
Stall A meals	400
Stall A direct cost per meal	$4.00

Figure 18.2 Monthly kitchen allocation (hypothetical)

The concept

Direct costs can be traced to a chosen product or job, such as ingredients used for a meal. Overhead supports operations but cannot conveniently be

traced to a single unit, such as shared kitchen rent. The same cost can be direct for one purpose and indirect for another; always state the cost object.

An allocation base is a measurable activity used to divide a shared cost. Preparation hours may be reasonable for a kitchen; floor area might fit rent, and machine hours might fit equipment costs. A fair-looking base is not automatically a perfect measure of cost caused.

Divide the overhead pool by total activity to get an overhead rate. Then multiply each job's activity by that rate. Allocations must add back to the full overhead pool, aside from any disclosed rounding difference. Use a single consistent base within an allocation.

Full unit cost combines direct unit cost with allocated overhead per unit. This is useful for planning prices, but allocating fixed rent to a meal does not mean the rent disappears if one fewer meal is made. Do not add allocated overhead again after already including it in full cost. Round final money to cents; retain the unrounded allocation rate when necessary.

Overhead and Cost Allocation: solution map

1. Identify the question

   `Use the figure to allocate the $1,200 pool.`

2. Calculation 1

   `Total preparation time = 80 + 40 = 120 hours.`

3. Calculation 2

   `Overhead rate = $1,200 ÷ 120 = $10.00 per hour.`

4. Calculation 3

   `Stall A receives 80 × $10 = $800.00.`

5. Calculation 4

   `Stall B receives 40 × $10 = $400.00; check $800 + $400 = $1,200.`

6. Interpret and check

   `Allocate $800.00 to A and $400.00 to B.`

Figure 18.2.V Overhead and Cost Allocation: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

VARIABLE COST

A cost that rises or falls with the number of items made or sold.

COST

The amount paid to make or buy an item before it is sold.

REVENUE

The total money a business brings in from sales, before any costs are paid.

KEY TERMS

Direct cost
A cost traceable to the product or job being measured.

Overhead
Indirect operating costs shared by products or jobs.

Cost pool
The total shared cost to be allocated.

Allocation base
The activity measure used to divide a cost pool.

Full unit cost
Direct unit cost plus allocated overhead per unit.

FORMULAS & RULES

Overhead rate

```
Rate = overhead pool ÷ total allocation-base units
```

Allocated overhead

```
Job overhead = rate × job activity
```

Full unit cost

```
Full unit cost = direct unit cost + (allocated overhead
÷ units made)
```

Worked examples

Example 1 Split the shared kitchen bill

Use the figure to allocate the $1,200 pool.

SOLUTION

1. Total preparation time = 80 + 40 = 120 hours.
2. Overhead rate = $1,200 ÷ 120 = $10.00 per hour.
3. Stall A receives 80 × $10 = $800.00.
4. Stall B receives 40 × $10 = $400.00; check $800 + $400 = $1,200.

> **Answer:** Allocate $800.00 to A and $400.00 to B.

Example 2 Price information, not a price rule

Find Stall A's full cost for each of 400 meals, using $4 direct cost per meal and $800 allocated overhead.

SOLUTION

1. Total direct cost = 400 × $4 = $1,600.00.
2. Overhead per meal = $800 ÷ 400 = $2.00.
3. Full unit cost = $4 + $2 = $6.00.

4 Check: 400 × $6 = $2,400 = $1,600 + $800.

> **Answer:** The full unit cost is $6.00; this alone does not determine an appropriate selling price.

Practice exercises 18.2

1. Allocate $900 of overhead over 300 machine hours. What is the rate?
2. At $3 per machine hour, how much overhead goes to a 45-hour job?
3. A batch has $250 direct cost and $150 allocated overhead for 50 units. Find full unit cost.
4. Allocate $600 rent between rooms of 200 and 400 square feet.
5. A seller counts $5 direct cost and $2 allocated overhead per item, then adds the same $2 overhead again. Correct the cost.

RECAP QUIZ 18.2

1. A $2,400 pool covers 600 labor hours. The overhead rate is:

- ○ a. $0.25 per hour
- ○ b. $4 per hour
- ○ c. $600 per hour
- ○ d. $1,800 per hour

2. Which is a plausible base for allocating shared floor rent?

- ○ a. Shoe size
- ○ b. Floor area occupied
- ○ c. Customer age
- ○ d. Invoice number

3. Direct unit cost is $7 and overhead per unit is $3. Full unit cost is:

- ○ a. $4
- ○ b. $7
- ○ c. $10
- ○ d. $21

SECTION 18.2 RECAP

- Allocate the whole pool with an explicit, reasonable base.
- Full cost informs planning; allocated fixed costs are not necessarily avoidable.

Answer key with explanations: Section 18.2

PRACTICE EXERCISES

1. Rate = $900 ÷ 300 = $3.00 per machine hour.
2. Allocated overhead = 45 × $3 = $135.00.
3. Full batch cost = $250 + $150 = $400; unit cost = $400 ÷ 50 = $8.00.
4. Total area = 200 + 400 = 600 square feet; rate = $600 ÷ 600 = $1 per square foot. Allocations are $200.00 and $400.00, totaling $600.00.
5. Correct full cost = $5 + $2 = $7.00, not $9.00. The extra $2 duplicates the allocation.

RECAP QUIZ

1. **$4 per hour**: $2,400 ÷ 600 = $4.00 per hour.
2. **Floor area occupied**: Floor area measures use of the shared space.
3. **$10**: $7 + $3 = $10.00, counting overhead once.

18.3

Operating Profit and Break-Even Planning

LEARNING OBJECTIVES

After this section you will be able to:

1. Separate fixed and variable operating costs.
2. Compute contribution margin and operating profit.
3. Find whole-unit break-even and target-profit sales.

REAL-LIFE SITUATION

A weekend print booth

A hypothetical booth near a Philadelphia community event sells posters. The simplified monthly model assumes constant selling price and variable cost, enough capacity, and no sales tax in revenue.

Item	Amount or quantity
Price per poster	$20.00
Variable cost per poster	$8.00
Monthly fixed costs	$600.00
Planned sales	80

Figure 18.3 Poster booth monthly plan (hypothetical)

The concept

Operating profit is revenue minus the costs of running the business. In a simplified cost-volume-profit model, variable costs change in proportion to units sold, while total fixed costs remain unchanged within the relevant

activity range. A monthly booth fee can stay fixed even when no posters sell.

Contribution margin per unit is price minus variable cost per unit. It first covers fixed costs; only after those costs are covered does additional contribution become operating profit. It is not the same as net income, which may also reflect interest, income taxes, and other items excluded here.

Break-even occurs when total contribution equals fixed costs. Divide fixed costs by contribution margin per unit. If products can only be sold whole, round the required unit count up, never to the nearest integer. A nonpositive contribution margin cannot cover positive fixed costs in this model.

To plan for a target operating profit, add that target to fixed costs before dividing. A plan is not a guarantee: price cuts, overtime, waste, or capacity limits can change the inputs. Use consistent time periods and round money to cents; do not round contribution margin before calculating required units.

Operating Profit and Break-Even Planning: solution map

1. Identify the question

   Use the figure to find break-even units and operating profit at 80 posters.

2. Calculation 1

   Contribution per poster = $20 − $8 = $12.00.

3. Calculation 2

   Break-even = $600 ÷ $12 = 50 posters.

4. Calculation 3

   At 80 posters, contribution = 80 × $12 = $960.00.

5. Calculation 4

   Operating profit = $960 − $600 = $360.00.

6. Interpret and check

   Break-even is 50 posters; operating profit at 80 is $360.00.

Figure 18.3.V Operating Profit and Break-Even Planning: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

PROFIT MARGIN

Profit shown as a percent of revenue; a 10% margin means $10 of profit per $100 of sales.

BREAK-EVEN POINT

The sales level where revenue exactly equals total cost, so there is no profit and no loss.

NET PROFIT

The final profit after every cost is paid — sometimes called the bottom line.

KEY TERMS

Fixed cost
A total cost unchanged within the stated activity range and period.

Variable cost
A cost whose total changes with the number of units.

Contribution margin
Revenue minus variable costs, available to cover fixed costs and profit.

Break-even point
Sales volume at which operating profit is zero.

Operating profit
Revenue less operating costs in the defined model.

FORMULAS & RULES

Unit contribution

```
Contribution per unit = price − variable cost per unit
```

Operating profit

```
Profit = units × contribution per unit − fixed costs
```

Break-even units

```
Break-even units = fixed costs ÷ contribution per unit;
```

```
round up
```

Target-profit units

```
Required units = (fixed costs + target profit) ÷
contribution per unit; round up
```

Worked examples

Example 1 Can the booth cover its costs?

Use the figure to find break-even units and operating profit at 80 posters.

SOLUTION

1. Contribution per poster = $20 − $8 = $12.00.
2. Break-even = $600 ÷ $12 = 50 posters.
3. At 80 posters, contribution = 80 × $12 = $960.00.
4. Operating profit = $960 − $600 = $360.00.

> **Answer:** Break-even is 50 posters; operating profit at 80 is $360.00.

Example 2 Round up to meet a goal

A seller earns $7 contribution per item and has $500 fixed costs. Find sales needed for at least $200 operating profit.

SOLUTION

1. Required contribution = $500 + $200 = $700.00.
2. Required units = $700 ÷ $7 = 100.

3 Check: 100 × $7 − $500 = $200.00.

Answer: Sell at least 100 items.

Practice exercises 18.3

1. A snack sells for $5 and costs $2 in variable costs. Find contribution per snack.
2. With $250 fixed costs and $3 contribution per snack, find whole-unit break-even sales.
3. A booth sells 120 items at $10 each, variable cost $6 each, with $300 fixed cost. Find profit.
4. Find units needed for $450 profit with $350 fixed costs and $8 contribution per unit.
5. Price is $9, variable cost is $10, and fixed cost is $100. Can greater sales produce break-even?

RECAP QUIZ 18.3

1. A $15 item has $9 variable cost. Unit contribution is:

- ○ a. $24
- ○ b. $9
- ○ c. $6
- ○ d. $1.67

2. Fixed cost is $101 and contribution is $10 per unit. Minimum whole-unit sales to avoid a loss?

- ○ a. 10
- ○ b. 10.1
- ○ c. 11
- ○ d. 101

3. At 70 units, $5 contribution each and $200 fixed costs, profit is:

- ○ a. $150
- ○ b. $200

○ c. $350

○ d. $550

SECTION 18.3 RECAP

- Contribution covers fixed costs before it produces profit.
- Round required whole-unit sales upward and check the model assumptions.

Answer key with explanations: Section 18.3

PRACTICE EXERCISES

1. Contribution = $5 − $2 = $3.00 per snack.
2. $250 ÷ $3 = 83.333...; round up to 84 snacks. Profit at 84 = 84 × $3 − $250 = $2.00, the first nonnegative whole-unit result.
3. Revenue = 120 × $10 = $1,200; variable costs = 120 × $6 = $720; profit = $1,200 − $720 − $300 = $180.00.
4. Required contribution = $350 + $450 = $800; units = $800 ÷ $8 = 100.
5. Unit contribution = $9 − $10 = −$1. Profit = units × (−$1) − $100, which remains negative for all nonnegative sales. No break-even is possible without changing the inputs.

RECAP QUIZ

1. **$6**: $15 − $9 = $6.00.
2. **11**: $101 ÷ $10 = 10.1; round up to 11. Ten units leave a $1 loss.
3. **$150**: 70 × $5 − $200 = $150.00.

CHAPTER 19

Sales Taxes, Property Taxes, and Business Taxation

A tax calculation starts with deciding which amount is taxed, not just finding a percent. Practice sales-tax invoices, property assessments, and simplified business income taxes. All people, businesses, prices, rates, brackets, exemptions, and legal rules are hypothetical classroom assumptions, not current law or tax advice; round money to cents.

19.1

Sales Tax and Tax-Inclusive Prices

LEARNING OBJECTIVES

After this section you will be able to:

1. Apply a hypothetical tax rate to a taxable subtotal.
2. Distinguish taxable from exempt items under stated rules.
3. Recover a pretax price from a tax-inclusive total.

REAL-LIFE SITUATION

Checking a supply invoice

A hypothetical Philadelphia club buys supplies. For this exercise only, notebooks and markers are taxable at 8%, while the separately listed

donated-food purchase is exempt. These rules are invented, not current law.

Item	Amount or quantity
Notebooks	`$30.00 taxable`
Markers	`$20.00 taxable`
Food	`$15.00 exempt`
Illustrative sales tax rate	8%

Figure 19.1 Hypothetical taxable and exempt purchases (hypothetical)

The concept

Sales tax is computed on the taxable selling amount, not automatically on every dollar of a purchase. A tax rate written as a percent must be divided by 100 before multiplication. The rules in this section are hypothetical teaching rules, not statements of Philadelphia or any other jurisdiction's law.

First classify the items using the stated assumptions. Add taxable items, apply any eligible discount, and then compute tax on the resulting taxable subtotal. In these exercises, round the total tax once to the nearest cent, rather than rounding tax separately on each item. Actual invoicing rules can differ.

The customer's total includes both the taxable and exempt amounts plus tax. A retailer generally treats tax collected for the government as a liability to remit, not as sales revenue. The money passing through the cash drawer is therefore not all business income.

If a fully taxable price already includes tax, undo multiplication by dividing by one plus the decimal tax rate. Simply subtracting the tax percent of the total removes too much. Reverse-tax calculations assume one uniform rate and may have cent-level ambiguities; the examples are chosen to reconcile after rounding.

Sales Tax and Tax-Inclusive Prices: solution map

1. Identify the question

   `Find the club's total using the figure.`

2. Calculation 1

   `Taxable subtotal = $30 + $20 = $50.00.`

3. Calculation 2

   `Exempt subtotal = $15.00; pretax total = $50 + $15 = $65.00.`

4. Calculation 3

   `Tax = $50 × 0.08 = $4.00.`

5. Calculation 4

   `Total due = $65 + $4 = $69.00.`

6. Interpret and check

   `The club owes $69.00, including $4.00 tax.`

Figure 19.1.V Sales Tax and Tax-Inclusive Prices: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

SALES TAX

A percentage added by the government to the price of many purchases.

INCOME STATEMENT

A report showing revenue, expenses, and profit over a period of time.

EXPENSE

Money a business spends to run, such as rent, supplies, or wages.

KEY TERMS

Taxable subtotal
The portion of a purchase subject to the stated tax.

Exempt item
An item not taxed under the stated rules.

Sales tax rate
The percent applied to the taxable amount.

Tax-inclusive price
A price already containing sales tax.

Tax liability
Collected tax owed to the taxing authority rather than earned revenue.

FORMULAS & RULES

Sales tax

```
Tax = taxable subtotal × decimal tax rate
```

Customer total

```
Total = taxable subtotal + exempt subtotal + rounded
tax
```

Reverse tax

```
Pretax price = tax-inclusive price ÷ (1 + decimal tax
```

```
rate)
```

Worked examples

Example 1 Add tax only where it belongs

Find the club's total using the figure.

SOLUTION

1. Taxable subtotal = $30 + $20 = $50.00.
2. Exempt subtotal = $15.00; pretax total = $50 + $15 = $65.00.
3. Tax = $50 × 0.08 = $4.00.
4. Total due = $65 + $4 = $69.00.

> **Answer:** The club owes $69.00, including $4.00 tax.

Example 2 Work backward from a total

An entirely taxable item totals $54.00 with hypothetical 8% tax included. Find the pretax price and tax.

SOLUTION

1. Convert 8% to 0.08; the inclusive multiplier is 1 + 0.08 = 1.08.
2. Pretax price = $54 ÷ 1.08 = $50.00.
3. Tax = $54 − $50 = $4.00.
4. Check: $50 × 0.08 = $4.00.

Answer: Pretax price is $50.00 and tax is $4.00.

Practice exercises 19.1

1. Under a hypothetical 6% tax, find tax and total on $25 of taxable goods.
2. At hypothetical 5% tax, a bill has $40 taxable goods and $12 exempt goods. Find the total.
3. A $100 taxable item has a 20% discount before hypothetical 8% tax. Find total.
4. A fully taxable total is $63.60 at hypothetical 6% tax. Recover the base and tax.
5. Taxable goods cost $19.95 and the hypothetical rate is 7%. Round tax once.

RECAP QUIZ 19.1

1. At hypothetical 8% tax, tax on a $75 taxable subtotal is:

- ○ a. $0.60
- ○ b. $6.00
- ○ c. $8.00
- ○ d. $81.00

2. To recover a pretax price from a fully taxable total at 5%, divide by:

- ○ a. 0.05
- ○ b. 0.95
- ○ c. 1.05
- ○ d. 5

3. A retailer collects $108, including $8 tax. Sales revenue before expenses is:

- ○ a. $8
- ○ b. $100
- ○ c. $108
- ○ d. $116

SECTION 19.1 RECAP

- Classify items and apply stated discounts before computing tax.
- Divide by 1 + rate to undo tax; collected tax is not retailer revenue.

Answer key with explanations: Section 19.1

PRACTICE EXERCISES

1. Tax = $25 × 0.06 = $1.50; total = $25 + $1.50 = $26.50.
2. Tax = $40 × 0.05 = $2.00; total = $40 + $12 + $2 = $54.00.
3. Discount = $100 × 0.20 = $20; taxable price = $80; tax = $80 × 0.08 = $6.40; total = $86.40.
4. Base = $63.60 ÷ 1.06 = $60.00; tax = $63.60 − $60 = $3.60.
5. Unrounded tax = $19.95 × 0.07 = $1.3965; rounded tax = $1.40; total = $19.95 + $1.40 = $21.35.

RECAP QUIZ

1. **$6.00**: $75 × 0.08 = $6.00; $81.00 would be the total.
2. **1.05**: Total = pretax price × 1.05, so division reverses it.
3. **$100**: $108 − $8 = $100.00 revenue; the $8 is a tax liability.

19.2

Property Assessment and Millage

LEARNING OBJECTIVES

After this section you will be able to:

1. Convert market value to assessed value under a stated ratio.
2. Apply exemptions and millage in the correct order.
3. Compare annual tax with a monthly budget allowance.

REAL-LIFE SITUATION

Budgeting a small workspace

A hypothetical workshop owner estimates a property-tax bill. The assessment ratio, exemption, and millage below are invented classroom assumptions, not current law or an actual assessment.

Item	Amount or quantity
Market value	`$200,000.00`
Assessment ratio	`80%`
Exemption from assessed value	`$10,000.00`
Rate	`20 mills`

Figure 19.2 Illustrative annual property-tax worksheet (hypothetical)

The concept

Property taxes commonly use an assessed value, which may differ from market value. In this model, multiply market value by the stated assessment ratio. The model does not imply that any actual jurisdiction assesses property in exactly this way.

A mill means one dollar of tax per thousand dollars of taxable assessed value. Thus 20 mills is 20 ÷ 1,000 = 0.020, or 2%, not 20%. Expressing rates in different units is a frequent source of large errors.

For the problems here, subtract the exemption from assessed value before applying the millage rate. Taxable assessed value cannot fall below zero. An exemption reduces the tax base; a tax credit instead reduces the calculated tax bill. They are not interchangeable.

Dividing an annual bill by 12 creates a monthly budget estimate, not necessarily the actual payment schedule. Extra fees, reassessments, due dates, appeals, and relief programs are outside this simplified model. Always verify the applicable rules before using a real estimate, and round money to cents.

Property Assessment and Millage: solution map

1. Identify the question
   `Use the figure to find annual tax and a monthly allowance.`
2. Calculation 1
   `Assessed value = $200,000 × 0.80 = $160,000.00.`
3. Calculation 2
   `Taxable assessed value = $160,000 − $10,000 = $150,000.00.`
4. Calculation 3
   `Annual tax = $150,000 × 20 ÷ 1,000 = $3,000.00.`
5. Calculation 4
   `Monthly allowance = $3,000 ÷ 12 = $250.00.`
6. Interpret and check
   `Estimated annual tax is $3,000.00; budget $250.00 per month.`

Figure 19.2.V Property Assessment and Millage: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

INVOICE

A document listing goods or services sold and the amount owed.

PAYMENT TERMS

The rules in an agreement about when a bill must be paid and any discounts for paying early.

BUSINESS

An organization that makes or sells goods or services to earn money.

KEY TERMS

Market value
An estimated exchange price for property.

Assessed value
The valuation used in the tax system before specified exemptions.

Assessment ratio
The assumed fraction of market value used for assessment.

Exemption
An amount excluded from the tax base under stated rules.

Mill
One dollar of tax per $1,000 of taxable value.

FORMULAS & RULES

Assessment

```
Assessed value = market value × assessment ratio
```

Tax base

```
Taxable assessed value = maximum of 0 and (assessed value − exemption)
```

Millage tax

```
Annual tax = taxable assessed value × mills ÷ 1,000
```

Budget allowance

```
Monthly allowance = annual tax ÷ 12
```

Worked examples

Example 1 Build the workshop tax estimate

Use the figure to find annual tax and a monthly allowance.

SOLUTION

1. Assessed value = $200,000 × 0.80 = $160,000.00.
2. Taxable assessed value = $160,000 − $10,000 = $150,000.00.
3. Annual tax = $150,000 × 20 ÷ 1,000 = $3,000.00.
4. Monthly allowance = $3,000 ÷ 12 = $250.00.

> **Answer:** Estimated annual tax is $3,000.00; budget $250.00 per month.

Example 2 Translate a rate

A hypothetical property has $120,000 taxable assessed value and a 15-mill rate. Find the equivalent percent and annual tax.

SOLUTION

1. Decimal rate = 15 ÷ 1,000 = 0.015.
2. Percent rate = 0.015 × 100 = 1.5%.
3. Tax = $120,000 × 0.015 = $1,800.00.

Answer: 15 mills equals 1.5%, giving $1,800.00 annual tax.

Practice exercises 19.2

1. At a hypothetical 75% assessment ratio, assess a $160,000 property.
2. A $90,000 assessment has a $15,000 exemption from assessed value. Find taxable value.
3. Apply hypothetical 24 mills to a $75,000 taxable value.
4. Convert a hypothetical 1.8% tax rate to mills.
5. Annual tax is $2,460. Find a monthly allowance and explain its limitation.

RECAP QUIZ 19.2

1. Ten mills is equivalent to:

- ○ a. 10%
- ○ b. 1%
- ○ c. 0.1%
- ○ d. 100%

2. Assessment is $80,000 and an exemption removes $5,000 of assessed value. The tax base is:

- ○ a. $5,000
- ○ b. $75,000
- ○ c. $80,000
- ○ d. $85,000

3. At hypothetical 12 mills, annual tax on $100,000 taxable value is:

- ○ a. $12
- ○ b. $120
- ○ c. $1,200
- ○ d. $12,000

SECTION 19.2 RECAP

- Millage applies to the stated taxable assessed value, not automatically to market value.
- An exemption changes the base; a monthly allowance is only a budgeting tool.

Answer key with explanations: Section 19.2

PRACTICE EXERCISES

1. Assessed value = $160,000 × 0.75 = $120,000.00.
2. Taxable value = $90,000 − $15,000 = $75,000.00.
3. Decimal rate = 24 ÷ 1,000 = 0.024; tax = $75,000 × 0.024 = $1,800.00.
4. 1.8% = 0.018; mills = 0.018 × 1,000 = 18 mills.
5. $2,460 ÷ 12 = $205.00 per month. This is a savings allowance; it does not establish the legal due dates.

RECAP QUIZ

1. **1%**: 10 ÷ 1,000 = 0.010 = 1%.
2. **$75,000**: $80,000 − $5,000 = $75,000.00.
3. **$1,200**: $100,000 × 12 ÷ 1,000 = $1,200.00.

19.3

Business Income, Marginal Rates, and Estimated Payments

LEARNING OBJECTIVES

After this section you will be able to:

1. Build taxable profit from stated business records.
2. Apply hypothetical marginal tax brackets.
3. Distinguish deductions, credits, and estimated payments.

REAL-LIFE SITUATION

A creator keeps tax records

A hypothetical student creator tracks a year of earnings. This simplified exercise assumes all listed expenses are deductible and uses invented brackets; real business, payroll, and self-employment rules are not modeled.

Item	Amount or quantity
Gross revenue	$30,000.00
Deductible expenses	$10,000.00
First $10,000 of taxable profit	10%
Taxable profit above $10,000	20%
Credit against calculated tax	$200.00
Estimated payments already made	$2,400.00

Figure 19.3 Illustrative annual creator tax record (hypothetical)

The concept

Gross revenue is not the amount a business necessarily pays income tax on. In the simplified model, subtract allowed expenses to obtain taxable profit.

Actual law can treat depreciation, losses, deductions, and entity types differently; the invented rules here are only arithmetic practice.

A marginal rate applies only to the slice of taxable income inside that bracket. Moving into a higher bracket does not cause all earlier income to be taxed at the higher rate. Add the tax on each slice to find total tax before credits.

A deduction reduces taxable income, while a credit reduces calculated tax. In this lesson, credits are nonrefundable: they cannot reduce tax below zero. The effective tax rate is final tax after credits divided by taxable profit, and it can be lower than the top marginal rate.

Estimated payments and withholding are prepayments, not extra deductions from profit. Subtract them from final tax to determine the remaining balance; an excess is a possible refund in this model. Quarterly installments here mean four equal planning payments, not a claim about real deadlines, penalties, or safe-harbor requirements. Round money to cents.

Business Income, Marginal Rates, and Estimated Payments: solution map

1. Identify the question

   Use the figure to calculate final tax and the remaining balance.

2. Calculation 1

   Taxable profit = $30,000 − $10,000 = $20,000.00.

3. Calculation 2

   First bracket tax = $10,000 × 0.10 = $1,000.00.

4. Calculation 3

   Remaining income = $20,000 − $10,000 = $10,000; second bracket tax = $10,000 × 0.20 = $2,000.00.

5. Calculation 4

   Tax before credit = $1,000 + $2,000 = $3,000; final tax = $3,000 − $200 = $2,800.00.

6. Calculation 5

   Balance due = $2,800 − $2,400 = $400.00.

7. Interpret and check

   Final tax is $2,800.00, leaving $400.00 due.

Figure 19.3.V Business Income, Marginal Rates, and Estimated Payments: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

LIABILITY

Money or obligations a person or business owes to others.

EQUITY

What is left of an asset after debts are subtracted; in a home, value minus what you still owe.

BUDGET

A plan that shows expected income and how it will be spent.

KEY TERMS

Taxable profit
Revenue minus expenses and deductions allowed in the stated model.

Marginal rate
Rate applied to the next dollar in a specified income bracket.

Effective tax rate
Final tax divided by the defined income base.

Deduction
An allowed subtraction from taxable income.

Tax credit
An allowed subtraction from calculated tax.

Estimated payment
A tax prepayment credited against the eventual bill.

FORMULAS & RULES

Taxable profit

```
Taxable profit = revenue − allowed expenses
```

Bracket tax

```
Tax before credits = sum of (income in each bracket × bracket rate)
```

Final tax

```
Final tax = maximum of 0 and (tax before credits − nonrefundable credits)
```

Balance

```
Balance due = final tax − prepayments
```

Worked examples

Example 1 Tax the slices

Use the figure to calculate final tax and the remaining balance.

SOLUTION

1. Taxable profit = $30,000 − $10,000 = $20,000.00.
2. First bracket tax = $10,000 × 0.10 = $1,000.00.
3. Remaining income = $20,000 − $10,000 = $10,000; second bracket tax = $10,000 × 0.20 = $2,000.00.
4. Tax before credit = $1,000 + $2,000 = $3,000; final tax = $3,000 − $200 = $2,800.00.
5. Balance due = $2,800 − $2,400 = $400.00.

> **Answer:** Final tax is $2,800.00, leaving $400.00 due.

Example 2 Compare a deduction with a credit

Under a hypothetical flat 20% rate and $5,000 initial taxable profit, compare a $500 deduction with a separate $500 nonrefundable credit. Assume no other changes.

SOLUTION

1. Initial tax = $5,000 × 0.20 = $1,000.00.
2. With only the deduction, taxable profit = $5,000 − $500 = $4,500; tax = $4,500 × 0.20 = $900.00.
3. Deduction tax saving = $1,000 − $900 = $100.00.
4. With only the credit, final tax = $1,000 − $500 = $500.00; saving = $500.00.

Answer: The deduction saves $100.00; the credit saves $500.00 in this model.

Practice exercises 19.3

1. A hypothetical business earns $18,000 revenue and has $6,000 allowed expenses. Find taxable profit.
2. Tax $12,000 profit at hypothetical 10% on the first $10,000 and 20% above that.
3. Final tax is $1,400 on $12,000 taxable profit. Find the effective rate to two decimal percent places.
4. Final tax is $3,200 and estimated payments total $3,500. Find balance or refund.
5. A hypothetical next-year tax estimate is $4,800 with no withholding. Find four equal planning installments.

RECAP QUIZ 19.3

1. A $100 deduction in a hypothetical 20% bracket saves, if entirely in that bracket:

- a. $20
- b. $80
- c. $100
- d. $120

2. A $2,000 final tax bill and $1,700 prepayments leave:

- a. $300 due
- b. $300 refund
- c. $1,700 due
- d. $3,700 due

3. An invented schedule taxes the first $1,000 at 10% and the next $1,000 at 20%. Tax on $2,000 is:

- a. $100
- b. $200
- c. $300
- d. $400

SECTION 19.3 RECAP

- Apply each marginal rate to its own slice, not to all income.
- Deductions change income, credits change tax, and prepayments settle the bill.

Answer key with explanations: Section 19.3

PRACTICE EXERCISES

1. Taxable profit = $18,000 − $6,000 = $12,000.00.
2. First slice = $10,000 × 0.10 = $1,000; remaining $2,000 × 0.20 = $400; total tax = $1,400.00.
3. Effective rate = $1,400 ÷ $12,000 × 100 = 11.666...%, or 11.67%; this is not the 20% marginal rate.
4. Balance = $3,200 − $3,500 = −$300.00, so the model gives a $300.00 refund rather than an additional bill.

5. $4,800 ÷ 4 = $1,200.00 per installment. Actual deadlines and required amounts would need separate verification.

RECAP QUIZ

1. **$20**: Tax reduction = $100 × 0.20 = $20.00.
2. **$300 due**: $2,000 − $1,700 = $300.00 due.
3. **$300**: $1,000 × 0.10 + $1,000 × 0.20 = $100 + $200 = $300.00.

CHAPTER 20

Insurance, Risk, and Financial Protection

Insurance math connects everyday premiums with potentially large losses. Learn to read simplified coverage terms, split health costs, and compare expected losses with financial capacity. All people, policies, prices, and probabilities are hypothetical; these models are not coverage advice, and money is rounded to cents.

20.1

Premiums, Deductibles, and Coverage Limits

LEARNING OBJECTIVES

After this section you will be able to:

1. Compute annual premiums.
2. Apply a deductible before an insurer-payment cap.
3. Separate premiums from claim costs.

REAL-LIFE SITUATION

Protecting a bicycle

A hypothetical Philadelphia rider compares a covered theft loss with a policy summary. All claims in this section are eligible; the limit caps

insurer payment after the deductible.

Item	Amount or quantity
Monthly premium	$15.00
Per-claim deductible	$200.00
Insurer-payment limit	$1,500.00
Covered loss	$1,800.00

Figure 20.1 Simplified bicycle policy (hypothetical)

The concept

Insurance transfers specified financial risks in exchange for a premium. A premium pays for coverage during a period; it is not a deposit automatically returned when no claim occurs. Policy terms determine which events and property qualify.

A deductible is the part of an eligible loss the policyholder must absorb before payment begins. For these exercises there is one deductible per claim, and the insurer-payment limit applies after subtracting it. Other contracts can define limits differently.

Compute the covered loss above the deductible, never less than zero, then apply the insurer-payment cap. The policyholder bears the remainder of the loss, which can exceed the deductible when a limit applies. Excluded losses would receive no payment.

Annual cash cost in a claim year includes premiums plus retained loss. Keep the premium separate from the claim settlement to avoid subtracting it twice. These simplified policies omit depreciation, sublimits, and claim disputes; round money to cents.

Premiums, Deductibles, and Coverage Limits: solution map

1. Identify the question

   `Use the figure to find payment and retained loss.`

2. Calculation 1

   `Loss above deductible = $1,800 − $200 = $1,600.00.`

3. Calculation 2

   `Payment = minimum of $1,600 and $1,500 = $1,500.00.`

4. Calculation 3

   `Retained loss = $1,800 − $1,500 = $300.00.`

5. Interpret and check

   `The insurer pays $1,500.00; the rider retains $300.00.`

Figure 20.1.V Premiums, Deductibles, and Coverage Limits: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

INSURANCE PREMIUM

The amount paid, often monthly, to keep an insurance policy active.

DEDUCTIBLE

The amount you pay yourself before an insurance policy starts paying.

RISK

The chance that an investment or decision will turn out worse than expected.

KEY TERMS

Premium
Price paid to maintain coverage.

Deductible
Eligible loss retained before insurer payment begins.

Coverage limit
Maximum insurer payment under the stated model.

Exclusion
An event or loss the contract does not cover.

Retained loss
Loss not reimbursed by insurance.

FORMULAS & RULES

Annual premium

```
Annual premium = monthly premium × 12
```

Claim payment

```
Payment = minimum of limit and maximum of 0 and
(covered loss − deductible)
```

Retained loss

```
Retained loss = covered loss − payment
```

Annual cash cost

```
Cash cost = annual premiums + retained losses
```

Worked examples

Example 1 A loss above the limit

Use the figure to find payment and retained loss.

SOLUTION

1. Loss above deductible = $1,800 − $200 = $1,600.00.
2. Payment = minimum of $1,600 and $1,500 = $1,500.00.
3. Retained loss = $1,800 − $1,500 = $300.00.

> **Answer:** The insurer pays $1,500.00; the rider retains $300.00.

Example 2 Add the cost of coverage

Using the same policy and one claim from Example 1, find annual cash cost.

SOLUTION

1. Annual premiums = 12 × $15 = $180.00.
2. Retained claim loss = $300.00.
3. Annual cash cost = $180 + $300 = $480.00.

> **Answer:** Annual cash cost is $480.00.

Practice exercises 20.1

1. Premium is $22 monthly. Find its annual cost.

2. An eligible $150 loss has a $200 deductible. Find payment.
3. An eligible $900 loss has a $250 deductible and $2,000 payment limit. Find payment and retained loss.
4. An eligible $3,000 loss has a $500 deductible and $2,000 payment cap. Find payment and retained loss.
5. Annual premiums are $240 and retained loss is $125. Find annual cash cost.

RECAP QUIZ 20.1

1. An eligible $700 loss with a $200 deductible and adequate limit yields payment of:

- ○ a. $200
- ○ b. $500
- ○ c. $700
- ○ d. $900

2. A $10 monthly premium costs annually:

- ○ a. $10
- ○ b. $100
- ○ c. $120
- ○ d. $1,200

3. If a loss is entirely excluded, the simplified insurer payment is:

- ○ a. The deductible
- ○ b. The full loss
- ○ c. $0
- ○ d. The premium

SECTION 20.1 RECAP

- Read the deductible and the limit before calculating payment.
- Total cash cost includes both premiums and unreimbursed losses.

Answer key with explanations: Section 20.1

PRACTICE EXERCISES

1. 12 × $22 = $264.00.
2. Maximum of 0 and ($150 − $200) is $0.00; the policyholder retains $150.00.
3. Above deductible = $900 − $250 = $650; payment = $650.00; retained loss = $250.00.
4. Above deductible = $2,500; payment capped at $2,000.00; retained loss = $3,000 − $2,000 = $1,000.00.
5. $240 + $125 = $365.00; do not subtract the premiums from the insurer payment.

RECAP QUIZ

1. **$500**: $700 − $200 = $500.00.
2. **$120**: 12 × $10 = $120.00.
3. **$0**: Excluded losses are outside coverage.

20.2

Health Cost Sharing and Out-of-Pocket Maximums

LEARNING OBJECTIVES

After this section you will be able to:

1. Apply a deductible and coinsurance sequentially.
2. Use a remaining out-of-pocket maximum.
3. Distinguish billed and allowed amounts.

REAL-LIFE SITUATION

Reading a medical benefit summary

A hypothetical student has one covered in-network bill. The plan uses an allowed amount, then an annual deductible and 20% patient coinsurance; all patient cost sharing here counts toward the annual maximum.

Item	Amount or quantity
Allowed amount	$5,000.00
Deductible still owed	$1,000.00
Patient coinsurance	20% after deductible
Remaining out-of-pocket maximum	$1,500.00

Figure 20.2 Simplified health-plan claim (hypothetical)

The concept

The billed charge is not necessarily the amount used to compute a covered claim. These exercises start with the allowed amount, and assume no balance billing. The plan pays the remainder of that allowed amount after patient cost sharing.

Apply the remaining deductible first. Coinsurance then assigns the stated percentage of the remaining allowed amount to the patient. A 20% patient coinsurance rate means the plan generally pays 80% of that post-deductible amount, not 80% of the original claim.

A remaining out-of-pocket maximum limits the patient cost sharing that counts toward it. Compute ordinary deductible plus coinsurance and then cap that amount at the remaining maximum. Premiums and excluded services do not count toward the maximum in this simplified model.

Real plans can have copays, separate networks, family deductibles, and services with different rules. Do not treat this illustration as a universal benefits formula. Round money to cents and keep the remaining deductible and remaining maximum distinct.

Health Cost Sharing and Out-of-Pocket Maximums: solution map

1. Identify the question

   `Use the figure to find patient and plan payments.`

2. Calculation 1

   `Apply the $1,000 remaining deductible to the $5,000 allowed amount.`

3. Calculation 2

   `Post-deductible amount = $5,000 − $1,000 = $4,000.00.`

4. Calculation 3

   `Coinsurance = $4,000 × 0.20 = $800; ordinary share = $1,800.00.`

5. Calculation 4

   `Patient share is capped at $1,500; plan pays $5,000 − $1,500 = $3,500.00.`

6. Interpret and check

   `Patient pays $1,500.00 and plan pays $3,500.00.`

Figure 20.2.V Health Cost Sharing and Out-of-Pocket Maximums: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

LIABILITY

Money or obligations a person or business owes to others.

COLLATERAL

Property a borrower promises to a lender if the loan is not repaid, such as a car.

ASSET

Something valuable that a person or business owns, like cash, equipment, or a building.

KEY TERMS

Allowed amount
The claim amount used for the covered-service calculation.

Coinsurance
A percentage share of covered cost after a deductible.

Remaining deductible
The annual deductible not yet satisfied.

Out-of-pocket maximum
An annual cap on specified patient cost sharing.

Copay
A stated fixed patient charge for a service, not used in this model.

FORMULAS & RULES

Deductible applied

```
Applied deductible = minimum of allowed amount and
remaining deductible
```

Ordinary patient share

```
Ordinary share = applied deductible + (allowed amount −
applied deductible) × patient coinsurance
```

Capped share

```
Patient share = minimum of ordinary share and remaining
out-of-pocket maximum
```

Plan payment

```
Plan payment = allowed amount − patient share
```

Worked examples

Example 1 A capped claim

Use the figure to find patient and plan payments.

SOLUTION

1. Apply the $1,000 remaining deductible to the $5,000 allowed amount.
2. Post-deductible amount = $5,000 − $1,000 = $4,000.00.
3. Coinsurance = $4,000 × 0.20 = $800; ordinary share = $1,800.00.
4. Patient share is capped at $1,500; plan pays $5,000 − $1,500 = $3,500.00.

> **Answer:** Patient pays $1,500.00 and plan pays $3,500.00.

Example 2 Deductible already met

A $600 allowed claim has no remaining deductible, 20% patient coinsurance, and $1,000 remaining maximum. Find the split.

SOLUTION

1. Applied deductible = $0.00.

2 Ordinary patient share = $600 × 0.20 = $120.00.

3 The $1,000 remaining maximum does not bind; plan payment = $600 − $120 = $480.00.

Answer: Patient pays $120.00; plan pays $480.00.

Practice exercises 20.2

1. A $400 allowed claim is below the $700 remaining deductible. Find the split with an ample maximum.
2. With the deductible met, a $1,000 allowed claim has 10% patient coinsurance and ample maximum. Find patient cost.
3. Ordinary patient cost is $600 but the remaining maximum is $250. Find capped patient cost.
4. A $2,000 allowed claim gives $300 patient cost. Find plan payment.
5. A person pays $1,200 annual premiums and $800 covered cost sharing. Find combined spending; do premiums satisfy the maximum here?

RECAP QUIZ 20.2

1. After a $500 deductible on $1,500 allowed cost, 20% coinsurance is applied to:

- ○ a. $500
- ○ b. $1,000
- ○ c. $1,500
- ○ d. $2,000

2. Ordinary patient share is $900; remaining maximum is $400. Patient pays:

- ○ a. $0
- ○ b. $400
- ○ c. $500
- ○ d. $900

3. Which does not count toward the maximum in this model?

- ○ a. Covered deductible
- ○ b. Covered coinsurance
- ○ c. Monthly premium
- ○ d. Eligible cost sharing

SECTION 20.2 RECAP

- Deductible comes before coinsurance in this model.
- Cap eligible patient cost sharing, not premiums, at the remaining maximum.

Answer key with explanations: Section 20.2

PRACTICE EXERCISES

1. Applied deductible = $400; post-deductible amount = $0; patient pays $400.00 and plan pays $0.00.
2. $1,000 × 0.10 = $100.00 patient cost; the plan pays $900.00.
3. Minimum of $600 and $250 = $250.00; the maximum reduces patient cost by $350.00.
4. $2,000 − $300 = $1,700.00 plan payment.
5. Combined spending = $1,200 + $800 = $2,000.00. Only the $800 cost sharing counts in this model, not premiums.

RECAP QUIZ

1. **$1,000**: $1,500 − $500 = $1,000.00 remains.
2. **$400**: The share is capped at $400.00.
3. **Monthly premium**: Premiums buy coverage and are excluded from the cost-sharing maximum.

20.3

Expected Loss and Financial Protection

LEARNING OBJECTIVES

After this section you will be able to:

1. Compute expected annual loss.
2. Compare a premium saving with extra retained risk.
3. Build a reserve for an uninsured exposure.

REAL-LIFE SITUATION

Protecting a mobile work kit

A hypothetical student estimates a 5% annual chance of one $2,000 covered equipment loss and otherwise no loss. A policy costs $140 annually, has a $300 deductible, and has enough coverage for this loss.

Item	Amount or quantity
Loss probability	5%
Loss if event occurs	$2,000.00
Annual premium	$140.00
Deductible	$300.00

Figure 20.3 One-year risk model (hypothetical)

The concept

Expected loss is a probability-weighted average, not a prediction of the next year. In a two-outcome model, multiply the event probability by its loss and include the no-loss outcome with zero cost. A household may experience zero loss or the entire loss, rather than the average.

Insurance often costs more than expected claims because administration, reserves, and other expenses also require funding. Comparing average costs alone misses the value of avoiding a large unaffordable loss. Risk capacity concerns what a person can afford; risk tolerance concerns comfort with uncertainty.

A higher deductible usually shifts more risk to the policyholder. Compare the annual premium saving with the possible extra retained cost and the expected extra cost under explicit probabilities. The comparison is a model, not a recommendation, and estimates may be wrong.

A reserve can cover deductibles and excluded losses without assuming an insurer will pay everything. Subtract any money already set aside from the target, then divide the remaining amount needed by the months available, ignoring interest; if the reserve is already fully funded, no new saving is required. Insurance contracts, emergency savings, and loss prevention address different parts of financial protection; none guarantees that every loss is affordable. Round money to cents.

Expected Loss and Financial Protection: solution map

1. Identify the question

   Compare expected annual costs with and without the figure's policy.

2. Calculation 1

   Uninsured expected loss = 0.05 × $2,000 + 0.95 × $0 = $100.00.

3. Calculation 2

   If insured, retained loss in the event is $300.00.

4. Calculation 3

   Expected retained loss = 0.05 × $300 = $15.00.

5. Calculation 4

   Expected insured cost = $140 + $15 = $155.00.

6. Interpret and check

   Expected insured cost is $155.00 versus $100.00 uninsured, but insurance reduces the modeled event loss retained from $2,000.00 to $300.00.

Figure 20.3.V Expected Loss and Financial Protection: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

DEFAULT

Failing to repay a loan as agreed, which can harm credit and lead to collection.

DEPOSIT

Money placed into a bank account.

BALANCE

The amount of money in an account at a given moment.

KEY TERMS

Expected loss
Probability-weighted average of possible losses.

Risk capacity
Ability to absorb a financial loss.

Risk tolerance
Comfort with uncertain financial outcomes.

Reserve
Money set aside for possible future costs.

Risk transfer
Shifting specified financial exposure through a contract.

FORMULAS & RULES

Expected loss

```
Expected loss = sum of (outcome probability × outcome loss)
```

Expected insured cost

```
Expected cost = annual premium + expected retained loss
```

Reserve saving

```
Monthly saving = (reserve target − starting reserve) ÷
```

```
months available
```

Worked examples

Example 1 Average cost versus large-loss protection

Compare expected annual costs with and without the figure's policy.

SOLUTION

1. Uninsured expected loss = 0.05 × $2,000 + 0.95 × $0 = $100.00.
2. If insured, retained loss in the event is $300.00.
3. Expected retained loss = 0.05 × $300 = $15.00.
4. Expected insured cost = $140 + $15 = $155.00.

> **Answer:** Expected insured cost is $155.00 versus $100.00 uninsured, but insurance reduces the modeled event loss retained from $2,000.00 to $300.00.

Example 2 Fund a deductible reserve

A household wants a $900 reserve in 12 months, starting with $300 and ignoring interest. Find the monthly saving.

SOLUTION

1. Amount still needed = $900 − $300 = $600.00.
2. Monthly saving = $600 ÷ 12 = $50.00.
3. Check: $300 + 12 × $50 = $900.00.

Answer: Save $50.00 monthly.

Practice exercises 20.3

1. A 2% chance of a $5,000 loss otherwise means no loss. Find expected loss.
2. A higher deductible saves $60 annually but adds $400 retained cost in a claim. With a 10% chance of one claim, find expected extra retained cost and expected net saving.
3. Build a $600 reserve over 8 months from zero with no interest.
4. An annual premium is $240 and expected retained loss is $35. Find expected insured cost.
5. A model has expected loss $80. Does it mean the next loss will be $80?

RECAP QUIZ 20.3

1. A 4% chance of $1,000 loss gives expected loss of:

- ○ a. $4
- ○ b. $40
- ○ c. $400
- ○ d. $1,040

2. Saving $40 monthly for 10 months with no interest builds:

- ○ a. $4
- ○ b. $40
- ○ c. $400
- ○ d. $4,000

3. A lower expected cost proves a choice is affordable in the worst case.

- ○ a. Always
- ○ b. Only with a low premium
- ○ c. Only with a deductible
- ○ d. No

SECTION 20.3 RECAP

- Expected value is an average, not a guarantee or a complete risk decision.
- Compare retained exposure as well as premiums, and plan reserves.

Answer key with explanations: Section 20.3

PRACTICE EXERCISES

1. 0.02 × $5,000 + 0.98 × $0 = $100.00.
2. Expected extra cost = 0.10 × $400 = $40; expected net saving = $60 − $40 = $20.00, but a claim year adds $400 before the premium saving.
3. $600 ÷ 8 = $75.00 per month.
4. $240 + $35 = $275.00.
5. No. Expected loss is the probability-weighted average; for example, 0.04 × $2,000 = $80 while individual outcomes can be $0 or $2,000.

RECAP QUIZ

1. **$40**: 0.04 × $1,000 = $40.00.
2. **$400**: $40 × 10 = $400.00.
3. **No**: An average does not measure whether the large-loss outcome is affordable.

CHAPTER 21

Stocks, Bonds, Mutual Funds, and Investment Mathematics

Investing apps show prices, yields, and percentages all day. This chapter teaches the math behind those numbers so you can read them accurately. Every company, fund, price, and rate here is hypothetical. Nothing in this chapter is investment advice, and no return is ever guaranteed.

21.1

Stocks: Dividends, Yield, and Total Return

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate the cost of a stock purchase and annual dividend income.
2. Find dividend yield and the price-to-earnings ratio.
3. Compute total return, including both price change and dividends.

REAL-LIFE SITUATION

A first brokerage account

A 19-year-old in Philadelphia buys 50 shares of a hypothetical company, Riverside Goods, at $40 per share. The company pays a $0.30 dividend every quarter. A year later the share price is $44.

Item	Value
Shares	50
Purchase price	$40.00
Quarterly dividend per share	$0.30
Price one year later	$44.00
Earnings per share (EPS)	$2.50

Figure 21.1 Hypothetical stock position after one year

The concept

A share of stock is a small piece of ownership in a company. The cost of a purchase is the number of shares times the price per share, plus any trading fee. Many apps now charge no commission, but always check.

Some companies pay part of their profits to shareholders as dividends, often quarterly. Dividend yield compares the yearly dividend to the current share price. Dividends are not promised; a company can cut or stop them.

Total return measures everything you earned: the change in price plus any dividends received, divided by what you paid. A stock can rise in price and still disappoint, or fall in price while dividends soften the loss.

The price-to-earnings (P/E) ratio divides the share price by earnings per share. It shows how many dollars investors pay for each $1 of yearly profit. A high or low P/E is a starting question, not a buy or sell signal.

Stocks: Dividends, Yield, and Total Return: solution map

1. Identify the question

   `Using Figure 21.1, find the purchase cost (no fee), the year's dividend income, and the dividend yield at the $40 purchase price.`

2. Calculation 1

   `Cost: 50 × $40 = $2,000.`

3. Calculation 2

   `Annual dividend per share: 4 × $0.30 = $1.20.`

4. Calculation 3

   `Dividend income: 50 × $1.20 = $60.`

5. Calculation 4

   `Yield: $1.20 ÷ $40 = 0.03 = 3%.`

6. Interpret and check

   `Cost $2,000; dividends $60; yield 3%`

Figure 21.1.V Stocks: Dividends, Yield, and Total Return: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

STOCK

A share of ownership in a company.

BOND

A loan to a government or company that pays interest and repays the face amount at maturity.

MUTUAL FUND

A pool of money from many investors used to buy a mix of stocks, bonds, or both.

KEY TERMS

Share
One unit of ownership in a company.

Dividend
A payment of company profits to shareholders.

Dividend yield
Annual dividend per share ÷ price per share.

Total return
Price change plus income, as a percent of the amount invested.

Earnings per share (EPS)
A company's yearly profit divided by its number of shares.

FORMULAS & RULES

Cost of purchase

```
Cost = shares × price per share + fees
```

Dividend yield

```
Yield = annual dividend per share ÷ price × 100%
```

Total return

```
Total return = (ending price − starting price +
dividends) ÷ starting price × 100%
```

P/E ratio

```
P/E = price per share ÷ earnings per share
```

Worked examples

Example 1 Cost, dividends, and yield

Using Figure 21.1, find the purchase cost (no fee), the year's dividend income, and the dividend yield at the $40 purchase price.

SOLUTION

1. Cost: 50 × $40 = $2,000.
2. Annual dividend per share: 4 × $0.30 = $1.20.
3. Dividend income: 50 × $1.20 = $60.
4. Yield: $1.20 ÷ $40 = 0.03 = 3%.

> **Answer:** Cost $2,000; dividends $60; yield 3%

Example 2 Total return

Find the price return and total return for one share held for the year.

SOLUTION

1. Price change: $44 − $40 = $4.

2. Price return: $4 ÷ $40 = 10%.
3. Add dividends: $4 + $1.20 = $5.20 per share.
4. Total return: $5.20 ÷ $40 = 0.13 = 13%.
5. For 50 shares: 50 × $5.20 = $260 total gain.

> **Answer:** Price return 10%; total return 13% ($260 on the position)

Example 3 When the price falls

Another investor buys 20 shares at $25, receives $0.50 per share in dividends, and the price drops to $21. Find the total return.

SOLUTION

1. Cost: 20 × $25 = $500.
2. Price change for the position: 20 × ($21 − $25) = −$80.
3. Dividends: 20 × $0.50 = $10.
4. Total return: (−80 + 10) ÷ 500 = −70 ÷ 500 = −14%.

> **Answer:** −14%; the dividends reduced, but did not erase, the loss

Practice exercises 21.1

1. Find the cost of 30 shares at $60 with a $0 fee.
2. A stock priced at $50 pays $0.50 per quarter. Find its dividend yield.
3. You buy at $15, receive $0.40 in dividends, and the price rises to $18. Find total return to the nearest hundredth of a percent.

4. A share costs $48 and EPS is $3. Find the P/E ratio and explain it.
5. Using Figure 21.1's EPS of $2.50, find the P/E at the $40 and $44 prices.
6. A $1,000 position loses 20% in price and pays $15 in dividends. What is it worth including dividends?

RECAP QUIZ 21.1

1. A $25 stock pays $1 per year in dividends. The yield is:

- ○ a. 1%
- ○ b. 2.5%
- ○ c. 4%
- ○ d. 25%

2. Total return includes:

- ○ a. Only price change
- ○ b. Only dividends
- ○ c. Price change and dividends
- ○ d. Only fees

3. Price $30, EPS $2. P/E is:

- ○ a. 15
- ○ b. 28
- ○ c. 32
- ○ d. 60

SECTION 21.1 RECAP

- Yield = annual dividend ÷ price; P/E = price ÷ EPS.
- Total return = (price change + dividends) ÷ starting price.
- Dividends and price gains are never guaranteed.

Answer key with explanations: Section 21.1

PRACTICE EXERCISES

1. 30 × $60 = $1,800.
2. Annual dividend = 4 × $0.50 = $2.00; yield = 2 ÷ 50 = 4%.
3. (18 − 15 + 0.40) ÷ 15 = 3.40 ÷ 15 ≈ 22.67%.
4. 48 ÷ 3 = 16. Investors are paying $16 for each $1 of yearly earnings.
5. 40 ÷ 2.50 = 16; 44 ÷ 2.50 = 17.6.
6. Price value 1,000 × 0.80 = $800; plus $15 = $815, a total return of −18.5%.

RECAP QUIZ

1. **4%**: 1 ÷ 25 = 0.04 = 4%.
2. **Price change and dividends**: Total return adds income to the price change.
3. **15**: 30 ÷ 2 = 15.

21.2

Bonds: Coupon Payments, Par, and Current Yield

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate annual and semiannual coupon payments.
2. Convert bond price quotes to dollars.
3. Find current yield and explain why bond prices move opposite to interest rates.

REAL-LIFE SITUATION

A city bond quote

A hypothetical city issues bonds with a $1,000 face value and a 5% coupon, paid twice a year. Months later, interest rates for new bonds have risen, and the bond is quoted at 96.

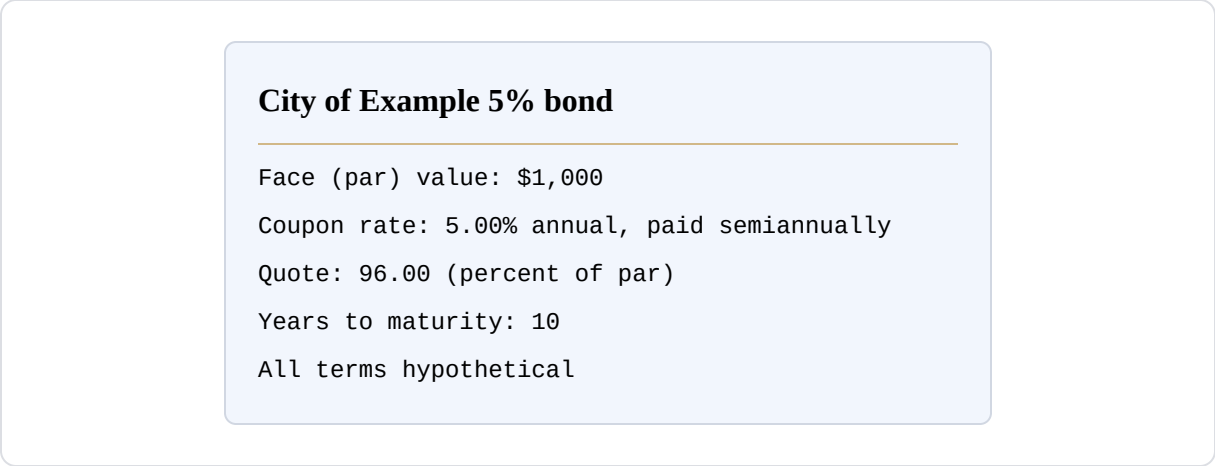


Figure 21.2 Hypothetical bond quote

The concept

A bond is a loan from an investor to a government or company. The issuer promises to pay interest, called the coupon, and to repay the face value, called par, at maturity.

The coupon is figured on par, not on the price you pay. A 5% coupon on a $1,000 bond pays $50 a year, or $25 every six months if paid semiannually.

Bond prices are quoted as a percent of par. A quote of 96 means 96% of $1,000, or $960. A bond below par trades at a discount; above par, at a premium.

When new bonds pay higher rates, older bonds with lower coupons become less attractive, so their prices fall. When rates drop, older bonds' prices rise. Current yield, the annual coupon divided by the price, reflects this. Bonds can also default, so higher yields usually come with more risk.

Read a bond quote

1. Par and quote
   `$1,000 par; quote 96.00 → $960 price`
2. Annual coupon
   `5% × $1,000 = $50; $25 semiannually`
3. Current yield
   `$50 ÷ $960 ≈ 5.21%; not yield to maturity`

Figure 21.2.V Read a bond quote. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

DIVIDEND
A share of a company's profit paid to its stockholders.

YIELD
The income an investment pays, shown as a percent of its price.

CAPITAL GAIN
Profit made by selling an investment for more than it cost.

KEY TERMS

Bond
A loan to an issuer that pays interest and returns the face value at maturity.

Par (face) value
The amount repaid at maturity, often $1,000.

Coupon rate
The yearly interest rate paid on par value.

Discount / premium
Price below par / price above par.

Current yield
Annual coupon ÷ current price.

FORMULAS & RULES

Annual coupon

```
Coupon = par × coupon rate
```

Price from quote

```
Price = quote ÷ 100 × par
```

Current yield

```
Current yield = annual coupon ÷ price × 100%
```

Worked examples

Example 1 Coupons and current yield at a discount

For the bond in Figure 21.2, find the annual coupon, each semiannual payment, the price, and the current yield.

SOLUTION

1. Annual coupon: $1,000 × 0.05 = $50.
2. Semiannual payment: $50 ÷ 2 = $25.
3. Price: 96 ÷ 100 × $1,000 = $960.

4 Current yield: $50 ÷ $960 ≈ 0.05208 ≈ 5.21%.

Answer: $50 per year ($25 twice); price $960; current yield ≈ 5.21%

Example 2 A premium bond

Rates later fall and the same bond is quoted at 104. Find the price and current yield.

SOLUTION

1 Price: 104 ÷ 100 × $1,000 = $1,040.

2 The coupon does not change: $50.

3 Current yield: $50 ÷ $1,040 ≈ 0.04808 ≈ 4.81%.

Answer: Price $1,040; current yield ≈ 4.81%, below the 5% coupon

Practice exercises 21.2

1. A $1,000 bond has a 4% coupon paid semiannually. Find each payment.
2. That 4% bond is priced at $950. Find its current yield.
3. What do 10 bonds quoted at 98 cost (par $1,000, no fees)?
4. A 6% bond with $1,000 par value is priced at $1,020. Find the current yield.
5. How much coupon income does one 5% $1,000 bond pay over 3 years?
6. Interest rates on new bonds rise. What usually happens to the price of an existing bond, and why?

RECAP QUIZ 21.2

1. When market interest rates rise, existing bond prices usually:

- ○ a. Rise
- ○ b. Fall
- ○ c. Stay at par
- ○ d. Double

2. A quote of 102 on a $1,000 bond means a price of:

- ○ a. $102
- ○ b. $1,002
- ○ c. $1,020
- ○ d. $1,200

3. Current yield equals:

- ○ a. Coupon ÷ par
- ○ b. Annual coupon ÷ price
- ○ c. Price ÷ par
- ○ d. Par ÷ price

SECTION 21.2 RECAP

- Coupons are based on par; prices are quoted as a percent of par.
- Current yield = annual coupon ÷ price. Prices and rates move in opposite directions.

Answer key with explanations: Section 21.2

PRACTICE EXERCISES

1. Annual 1,000 × 0.04 = $40; semiannual $40 ÷ 2 = $20.
2. 40 ÷ 950 ≈ 4.21%.
3. Each: 0.98 × 1,000 = $980; 10 × 980 = $9,800.
4. Annual coupon 1,000 × 0.06 = $60; 60 ÷ 1,020 ≈ 5.88%.
5. $50 × 3 = $150.

6. It falls, because its fixed coupon is less attractive than new bonds' higher payments.

RECAP QUIZ

1. **Fall**: Older, lower coupons become less attractive.
2. **$1,020**: 1.02 × 1,000 = $1,020.
3. **Annual coupon ÷ price**: It compares income to what you pay now.

21.3

Mutual Funds and ETFs: NAV, Expenses, and Diversification

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate net asset value (NAV) per share and shares purchased.
2. Find the yearly cost of an expense ratio and a sales load.
3. Compute a weighted portfolio return to see the effect of diversification.

REAL-LIFE SITUATION

Comparing two index funds

A student has $10,000 saved from summer jobs and a small creator side income. Two hypothetical funds track the same broad market. One charges 0.50% a year; the other charges 0.05%.

Fund	Expense ratio	Yearly cost on $10,000
Fund A	0.50%	$50.00
Fund B	0.05%	$5.00

Figure 21.3 Two hypothetical funds holding the same stocks

The concept

A mutual fund or exchange-traded fund (ETF) pools many investors' money to buy many stocks or bonds at once. Owning one share gives a small slice of everything the fund holds.

Net asset value (NAV) per share is the fund's total assets minus its liabilities, divided by the number of shares. Mutual funds are bought at NAV once per day; ETFs trade all day at market prices close to NAV.

Funds charge a yearly expense ratio, taken out of the fund's assets. Some mutual funds also charge a sales load when you buy. Because costs are certain and returns are not, small differences in fees add up over many years.

Diversification means spreading money across many investments so one bad result hurts less. A portfolio's return is the weighted average of its parts. Diversification reduces risk, but it cannot eliminate losses when the whole market falls.

Mutual Funds and ETFs: NAV, Expenses, and Diversification: solution map

1. Identify the question

   A fund holds $52,000,000 in assets, owes $2,000,000, and has 2,500,000 shares. Find NAV, then the number of shares $1,500 buys.

2. Calculation 1

   Net assets: 52,000,000 − 2,000,000 = $50,000,000.

3. Calculation 2

   NAV: 50,000,000 ÷ 2,500,000 = $20.00.

4. Calculation 3

   Shares bought: $1,500 ÷ $20 = 75 shares.

5. Interpret and check

   NAV $20.00; 75 shares

Figure 21.3.V Mutual Funds and ETFs: NAV, Expenses, and Diversification: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

DIVERSIFICATION

Spreading money across different investments to reduce risk.

PORTFOLIO

The full collection of investments a person or fund owns.

RISK

The chance that an investment or decision will turn out worse than expected.

KEY TERMS

Mutual fund / ETF
A pooled investment that holds many securities.

NAV
Net asset value per share: (assets − liabilities) ÷ shares.

Expense ratio
The yearly fee as a percent of the money invested.

Sales load
A commission charged by some funds when shares are bought.

Diversification
Spreading money across many investments to reduce risk.

FORMULAS & RULES

NAV

$$\text{NAV} = (\text{total assets} - \text{liabilities}) \div \text{shares outstanding}$$

Expense cost

$$\text{Yearly cost} = \text{amount invested} \times \text{expense ratio}$$

Weighted return

$$\text{Portfolio return} = w_1 r_1 + w_2 r_2 + \dots$$

Weights are decimals that add to 1.

Worked examples

Example 1 Finding NAV and shares bought

A fund holds $52,000,000 in assets, owes $2,000,000, and has 2,500,000 shares. Find NAV, then the number of shares $1,500 buys.

SOLUTION

1. Net assets: 52,000,000 − 2,000,000 = $50,000,000.
2. NAV: 50,000,000 ÷ 2,500,000 = $20.00.
3. Shares bought: $1,500 ÷ $20 = 75 shares.

Answer: NAV $20.00; 75 shares

Example 2 The cost of fees

Using Figure 21.3, find each fund's first-year cost on $10,000 and the difference.

SOLUTION

1. Fund A: 10,000 × 0.0050 = $50.
2. Fund B: 10,000 × 0.0005 = $5.
3. Difference: $50 − $5 = $45 in the first year, before any growth.

Answer: Fund A costs $45 more per year for the same holdings

Example 3 A diversified portfolio

A portfolio is 60% in a stock fund that returned 8% and 40% in a bond fund that returned −2%. Find the portfolio return.

SOLUTION

1. Stock part: 0.60 × 8% = 4.8%.
2. Bond part: 0.40 × (−2%) = −0.8%.
3. Total: 4.8% − 0.8% = 4.0%.

Answer: 4.0%

Practice exercises 21.3

1. A fund has $30,000,000 in assets, $1,000,000 in liabilities, and 1,450,000 shares. Find the NAV.
2. Find the yearly expense on $5,000 at a 0.75% expense ratio.
3. How many shares does $1,000 buy at a NAV of $25?
4. A portfolio is 50% at +6%, 30% at +2%, and 20% at −5%. Find its return.
5. A fund charges a 5% front-end load. How much of a $2,000 purchase is actually invested?
6. Why can't diversification guarantee you won't lose money?

RECAP QUIZ 21.3

1. A 0.10% expense ratio on $20,000 costs per year:

- ○ a. $2
- ○ b. $20
- ○ c. $200
- ○ d. $2,000

2. NAV is found by:

- ○ a. Assets ÷ shares
- ○ b. (Assets − liabilities) ÷ shares
- ○ c. Shares ÷ assets
- ○ d. Price × shares

3. Half a portfolio returns 10% and half returns 0%. The portfolio return is:

- ○ a. 0%
- ○ b. 5%
- ○ c. 10%
- ○ d. 20%

SECTION 21.3 RECAP

- NAV = (assets − liabilities) ÷ shares. Fees are certain; returns are not.
- Portfolio return is a weighted average. Diversification lowers risk but cannot remove it.

Answer key with explanations: Section 21.3

PRACTICE EXERCISES

1. (30,000,000 − 1,000,000) ÷ 1,450,000 = 29,000,000 ÷ 1,450,000 = $20.00.
2. 5,000 × 0.0075 = $37.50.
3. 1,000 ÷ 25 = 40 shares.
4. 0.5(6) + 0.3(2) + 0.2(−5) = 3 + 0.6 − 1 = 2.6%.
5. Load 2,000 × 0.05 = $100; invested 2,000 − 100 = $1,900.
6. When most investments fall together, as in a market-wide drop, a spread-out portfolio still loses value; diversification only reduces the damage from any single holding.

RECAP QUIZ

1. **$20**: 20,000 × 0.001 = $20.

2. **(Assets − liabilities) ÷ shares**: Subtract liabilities before dividing by shares.
3. **5%**: 0.5 × 10 + 0.5 × 0 = 5%.

21.4

Return on Investment (ROI) and Annualized Return

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate net gain and return on investment (ROI) for a purchase, project, or investment.
2. Include every cost and fee before computing ROI, and interpret negative ROI.
3. Convert a multi-year ROI to a simple and a compound annual rate so different time periods can be compared fairly.

REAL-LIFE SITUATION

Is the camera kit worth it?

A 17-year-old content creator buys a hypothetical $600 camera and lighting kit. Over the next year, better-looking videos bring in $900 of extra sponsorship pay, and she spends $60 on editing software for the new footage. Was the kit a good investment?

Item	Amount
Cost of kit (investment)	`$600`
Extra sponsorship pay	`$900`
Extra costs (software)	`$60`
Time period	`1 year`

Figure 21.4 Hypothetical one-year results of the camera kit

The concept

Return on investment (ROI) answers one question: for every dollar I put in, how much did I gain or lose? It works for stocks, a business purchase, a side hustle, or equipment.

First find the net gain: everything the investment brought in, minus every cost tied to it, including the original investment, fees, shipping, and extra expenses. Leaving out a cost makes ROI look better than it really was.

ROI is the net gain divided by the amount invested, written as a percent. A negative ROI means you lost money. ROI says nothing about risk or time on its own, so a large ROI earned over many years can be worse than a smaller one earned in one year.

To compare periods fairly, convert to a yearly rate. The simple annual rate divides ROI by the number of years. The compound annual rate, (1 + ROI)^(1/years) − 1, is the steady yearly growth that produces the same result and is the fairer comparison.

Return on Investment (ROI) and Annualized Return: solution map

1. Identify the question

   `Using Figure 21.4, find the net gain and ROI of the camera kit.`

2. Calculation 1

   `Net gain: $900 − $600 − $60 = $240.`

3. Calculation 2

   `ROI: $240 ÷ $600 = 0.40.`

4. Calculation 3

   `As a percent: 0.40 × 100% = 40%.`

5. Interpret and check

   `Net gain $240; ROI 40%`

Figure 21.4.V Return on Investment (ROI) and Annualized Return: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

ROI

Return on investment — the gain compared to what you put in, as a percent: (gain ÷ cost) × 100.

RETURN

The money or gain an investment produces, often shown as a percent.

NET PROFIT

The final profit after every cost is paid — sometimes called the bottom line.

KEY TERMS

Return on investment (ROI)
Net gain ÷ amount invested, as a percent.

Net gain
Total money returned minus all costs, including the original investment.

Annualized return
A multi-year return restated as a rate per year.

FORMULAS & RULES

Net gain

```
Net gain = total returned − amount invested − other costs
```

ROI

```
ROI = net gain ÷ amount invested × 100%
```

Simple annual ROI

```
Simple annual ROI = ROI ÷ years
```

Compound annual rate

```
Annual rate = (1 + ROI)^(1/years) − 1
```

Worked examples

Example 1 ROI on equipment

Using Figure 21.4, find the net gain and ROI of the camera kit.

SOLUTION

1. Net gain: $900 − $600 − $60 = $240.
2. ROI: $240 ÷ $600 = 0.40.
3. As a percent: 0.40 × 100% = 40%.

> **Answer:** Net gain $240; ROI 40%

Example 2 Comparing different time periods

Option A turns $2,000 into $2,300 in 1 year. Option B turns $5,000 into $5,600 over 2 years. Compare their ROI and their yearly rates.

SOLUTION

1. A: net gain $2,300 − $2,000 = $300; ROI = 300 ÷ 2,000 = 15% for one year.
2. B: net gain $5,600 − $5,000 = $600; ROI = 600 ÷ 5,000 = 12% over two years.
3. B simple annual: 12% ÷ 2 = 6% per year.
4. B compound annual: (1.12)^(1/2) − 1 = 1.0583 − 1 ≈ 5.83% per year.
5. B earned more dollars, but A grew faster each year.

> **Answer:** A: 15% per year. B: 12% total, about 5.83% per year compounded

Example 3 When fees make ROI negative

A reseller buys 4 pairs of sneakers at $180 each and sells them for $200 each. The selling app keeps a 10% fee and shipping costs $12 per pair. Find the ROI.

SOLUTION

1. Invested: 4 × $180 = $720.
2. Sales: 4 × $200 = $800. Fee: 0.10 × $800 = $80. Shipping: 4 × $12 = $48.
3. Net gain: $800 − $720 − $80 − $48 = −$48.
4. ROI: −48 ÷ 720 ≈ −0.0667 = −6.67%.

> **Answer:** About −6.67%; a $20 markup per pair became a loss after fees and shipping

Practice exercises 21.4

1. You spend $250 on supplies for a bake sale and take in $400. Find the ROI.
2. A $1,200 investment is worth $1,080 a year later. Find the ROI.
3. An $800 investment earns an ROI of 25%. Find the net gain and the ending value.
4. $3,000 grows to $3,630 over 2 years. Find the ROI, the simple annual ROI, and the compound annual rate.
5. Project A gains $90 on $300. Project B gains $200 on $1,000. Which has the higher ROI, and which earns more dollars?
6. Why can't you compare a 30% ROI earned over 3 years directly with a 15% ROI earned in 1 year?

RECAP QUIZ 21.4

1. You invest $200 and end with $250. The ROI is:

- ○ a. 20%
- ○ b. 25%
- ○ c. 50%
- ○ d. 125%

2. Which costs belong in the net gain calculation?

- ○ a. Only the purchase price
- ○ b. Only fees
- ○ c. The purchase price plus every fee and extra cost
- ○ d. None; use the selling price

3. A 20% ROI earned over 4 years has a simple annual ROI of:

- ○ a. 4%
- ○ b. 5%
- ○ c. 20%
- ○ d. 80%

SECTION 21.4 RECAP

- ROI = net gain ÷ amount invested; subtract every cost first.
- Negative ROI means a loss. ROI alone ignores time and risk.
- Compare different time periods with an annual rate: (1 + ROI) ^ (1/years) − 1.

Answer key with explanations: Section 21.4

PRACTICE EXERCISES

1. Net gain 400 − 250 = $150; ROI = 150 ÷ 250 = 60%.
2. Net gain 1,080 − 1,200 = −$120; ROI = −120 ÷ 1,200 = −10%.
3. Gain 0.25 × 800 = $200; ending value 800 + 200 = $1,000.
4. Gain $630; ROI 630 ÷ 3,000 = 21%; simple 21% ÷ 2 = 10.5% per year; compound (1.21) ^ (1/2) − 1 = 1.1 − 1 = 10% per year.

5. A: 90 ÷ 300 = 30%. B: 200 ÷ 1,000 = 20%. A has the higher ROI; B earns more dollars ($200 vs. $90).
6. ROI ignores time. 30% over 3 years is (1.30)^(1/3) − 1 ≈ 9.14% per year compounded, which is lower than 15% per year.

RECAP QUIZ

1. **25%**: Net gain $50; 50 ÷ 200 = 25%.
2. **The purchase price plus every fee and extra cost**: Leaving out any cost overstates ROI.
3. **5%**: 20% ÷ 4 = 5% per year.

21.5

Stock Market: Own It, Trade It, Understand It

LEARNING OBJECTIVES

After this section you will be able to:

1. Explain what a share represents, how companies raise money in the primary market, and how shares change hands in the secondary market.
2. Calculate market value, dollar and percent gains or losses, and separate realized from unrealized results.
3. Include the bid-ask spread and fees to find what really hit your account, and read a basic candlestick table and simple moving average without treating them as predictions.

REAL-LIFE SITUATION

What's the play? Before you risk your money, learn how the money moves

A 16-year-old opens a custodial brokerage account with a parent and watches a fictional company, Northside Robotics. The app shows a price, a chart, a bid, and an ask. Before buying anything, she writes down five days of the chart's numbers. Every company, price, fee, and return in this lesson is fictional. This is education, not investment advice.

Day	Open	High	Low	Close	Volume
Mon	$24.00	$25.10	$23.80	$24.90	1.2M
Tue	$24.90	$25.60	$24.50	$25.40	1.5M
Wed	$25.40	$25.50	$24.40	$24.70	2.1M
Thu	$24.70	$26.00	$24.60	$25.80	1.8M
Fri	$25.80	$26.40	$25.70	$26.20	1.4M

Figure 21.5 Chart reading lab: five fictional trading days of Northside Robotics (open, high, low, close, volume)

The concept

Think like an owner. A share is a small piece of ownership in a business. Companies sell new shares in the primary market (for example, an initial public offering) to raise money to grow. After that, investors buy and sell those shares to each other in the secondary market through brokers on exchanges, where market makers stand ready to buy and sell. When you buy on an app, your money usually goes to another investor, not to the company.

Market capitalization is price per share times shares outstanding: the market's price tag on the whole company. Volume is how many shares traded. Liquidity is how easily you can buy or sell without moving the price; heavily traded stocks usually have tighter spreads.

Here's the catch. Every quote has two prices. The bid is the highest price buyers will pay right now; the ask is the lowest price sellers will accept. You usually buy at the ask and sell at the bid, so the bid-ask spread is a real cost

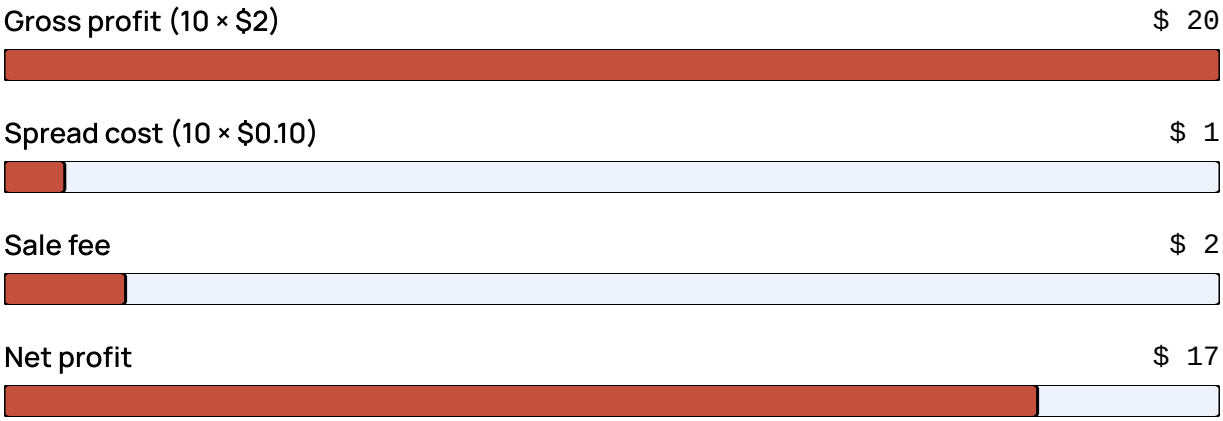
even when an app advertises zero commissions. A market order fills right away at the best available price, which can be worse than you expected when prices jump. A limit order sets the most you will pay (or the least you will accept); it protects your price but may never fill.

Realized vs. unrealized. A gain on a stock you still own is unrealized: it can shrink or disappear tomorrow. It becomes realized only when you sell. Total return adds dividends to the price change. Volatility measures how much a price swings; higher volatility means bigger possible gains and bigger possible losses.

Don't get fooled by the chart. Each candlestick shows a day's open, high, low, and close. A green candle closed above its open; a red candle closed below it. Support and resistance are price levels where buying or selling showed up before; they are observations, not guarantees. A simple moving average (SMA) is the average of recent closes; an exponential moving average (EMA) weights recent days more. Both lag price because they are built from the past, so a 20-day SMA reacts faster than a 50-day SMA, and neither predicts the future.

Investing, trading, and gambling are different. Investors buy ownership and usually hold for years, letting compounding and dividends work. Traders buy and sell over days or minutes, so costs repeat on every trade. Gambling is betting without an edge on outcomes you cannot control. An ETF or index fund holds many companies in one share, spreading risk; it can still fall in value.

Friction waterfall: what really hit your account



Bars share a zero baseline.

Figure 21.5.V Friction waterfall: what really hit your account. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

BID-ASK SPREAD

The gap between what buyers will pay (bid) and what sellers will accept (ask); a hidden cost on every trade.

LIMIT ORDER

An order to buy or sell only at a price you choose or better; it protects your price but may not fill.

LIQUIDITY

How quickly and easily something can be bought or sold for cash without changing its price much.

KEY TERMS

Primary market
Where a company sells new shares to raise money.

Secondary market
Where investors buy and sell existing shares with each other.

Market capitalization
Price per share × shares outstanding.

Bid / ask
The best price buyers will pay / the best price sellers will accept right now.

Market order / limit order
Fill now at the best available price / fill only at your price or better.

Unrealized gain
A paper gain on shares you still own; it becomes realized only when you sell.

Simple moving average (SMA)
The average of the last n closing prices.

Risk-to-reward ratio
Potential gain ÷ potential loss for a planned trade.

FORMULAS & RULES

Market value

```
Market value = shares × current price
```

Percent gain or loss

```
% change = (current value − cost) ÷ cost × 100%
```

Spread cost

```
Spread cost = spread per share × shares
```

Net trading profit

```
Net profit = sale proceeds − purchase cost − spread
cost − fees
```

Dividend yield

```
Yield = annual dividend per share ÷ price × 100%
```

Simple moving average

```
SMA = sum of last n closes ÷ n
```

Market capitalization

```
Market cap = price per share × shares outstanding
```

Worked examples

Example 1 You own a piece of the company

Run the numbers: a student buys 5 shares of a fictional company at $40 each. Three months later the price is $48. Find the original investment, current market value, dollar gain, and percent return. Then redo it if the price drops to $32 instead.

SOLUTION

1. Original investment: 5 × $40 = $200.
2. Market value at $48: 5 × $48 = $240.
3. Dollar gain: $240 − $200 = $40.
4. Percent return: $40 ÷ $200 = 0.20 = 20%.
5. Here's the catch: if the price drops to $32, value is 5 × $32 = $160, a change of $160 − $200 = −$40, or −20%.
6. What really hit? Nothing yet. Both results are unrealized until she sells, and the price can keep moving either way.

Answer: $200 invested; at $48 it is worth $240 (+$40, +20%); at $32 it is worth $160 (−$40, −20%). Both are unrealized.

Example 2 The trade looked better on the screen

A trader buys 10 shares at $25 and sells them at $27. The bid-ask spread costs a combined $0.10 per share on entry and exit, and there is a $2 fee on the sale. Find the gross profit, the net profit, and the net percent return.

SOLUTION

1. Purchase cost: 10 × $25 = $250. Sale proceeds: 10 × $27 = $270.
2. Gross profit: $270 − $250 = $20.
3. Here's the catch: spread cost 10 × $0.10 = $1, plus the $2 fee, is $3 of friction.
4. What really hit your account: $20 − $3 = $17.
5. Net return: $17 ÷ $250 = 0.068 = 6.8%, not the 8% the screen suggested.
6. Go or No-Go? Still a gain, but friction took 15% of the profit ($3 of $20). A zero-commission app still had a spread cost.

Answer: Gross $20 (8%); net $17 (6.8%) after $3 of spread and fees

Example 3 Investing versus trading

Two students each start with $500 for one year. Assumptions (hypothetical, not a forecast): the investor buys an index ETF once, pays no fee, and it gains 8% for the year. The trader makes 24 trades whose gains total $30 before costs, but pays $1.50 in spread and fees per trade. Compare the ending values.

SOLUTION

1. Investor: 500 × 1.08 = $540, a gain of $40 (8%).
2. Trader's costs: 24 × $1.50 = $36.

3. Trader's net: $30 − $36 = −$6, so the account ends at $494 (−1.2%).

4. Here's the catch: the trader picked more winners than losers, yet costs repeated on every trade.

5. This does not mean investing always wins. If the ETF had fallen 10% that year, the investor would end at $450. Results depend on costs, risk, timing, and consistency.

> **Answer:** Investor $540 (+8%); trader $494 (−1.2%) under these stated assumptions

Example 4 The dividend decision

A student owns 20 shares of a fictional company priced at $25 that pays an annual dividend of $0.50 per share. Find the dividend income and dividend yield. Then explain what happens to the share price when the dividend is paid.

SOLUTION

1. Dividend income: 20 × $0.50 = $10 per year.

2. Dividend yield: $0.50 ÷ $25 = 0.02 = 2%.

3. Here's the catch: the cash comes out of the company, so the price typically drops by about the dividend on the ex-dividend date: $25 − $0.50 = $24.50.

4. Value check: 20 × $24.50 = $490 in shares, plus $10 cash = $500, the same $500 as before.

5. Think like an owner: a dividend moves value from the company to you; wealth grows only if the business keeps earning. Dividends can be cut at any time.

> **Answer:** $10 per year; yield 2%; the payment by itself does not create extra wealth

Practice exercises 21.5

1. Go or No-Go? You buy 4 shares at $10.00 and sell at $10.50. The spread costs a combined $0.10 per share and the app charges a $2 fee on the sale. Find the net result and explain your decision.
2. Go or No-Go? A setup buys at $50 with a plan to exit at $46 if wrong and $51.50 if right. Find the risk, the reward, and the risk-to-reward ratio, and explain.
3. Long term: $1,000 grows at a hypothetical 6% per year for 10 years versus 3% per year. Find both ending values and the difference. What assumption matters most?
4. Stock or ETF? You put $1,000 into one company that falls 40%. Compare with $1,000 split equally across 50 companies in an ETF where that same company falls 40% and the rest are flat.
5. You buy 30 shares at $20, receive $0.80 per share in dividends over the year, and the price ends at $19. Find the total return in dollars and percent.
6. No trade is a decision: a fictional $8 stock jumps before the open and shows a bid of $7.80 and an ask of $8.20. What percent of the price is the spread, and why might waiting be reasonable?
7. Using Figure 21.5, find the 5-day simple moving average of the closes, and say whether Friday's close is above or below it.
8. Level up: a fictional company has 2,000,000 shares outstanding at $26.20. Find its market capitalization. Talk your math: explain why a $26 stock is not automatically 'cheaper' than a $260 stock.

RECAP QUIZ 21.5

1. You own shares that rose from $40 to $48 but you have not sold. Your gain is:

- ○ a. Realized
- ○ b. Unrealized

- ○ c. Guaranteed
- ○ d. Taxed already

2. A stock shows bid $19.95 and ask $20.05. If you buy now with a market order and sell immediately, you lose about:

- ○ a. $0.00 per share
- ○ b. $0.05 per share
- ○ c. $0.10 per share
- ○ d. $20 per share

3. Why does a 50-day moving average lag the price?

- ○ a. It is calculated from past closes
- ○ b. Brokers delay it
- ○ c. It predicts the future
- ○ d. It only uses volume

SECTION 21.5 RECAP

- A share is ownership. Market value = shares × price; % change = (value − cost) ÷ cost.
- Gains are unrealized until you sell. Spreads and fees come out of every trade, even 'zero-commission' ones.
- Charts and moving averages describe the past, not the future. Diversification lowers risk but never removes it. Choosing not to trade can be the smart move.

Answer key with explanations: Section 21.5

PRACTICE EXERCISES

1. Gross 4 × $0.50 = $2. Costs 4 × $0.10 = $0.40 plus $2 = $2.40. Net $2 − $2.40 = −$0.40. A gain on the screen became a loss after costs; No-Go at this size.
2. Risk $50 − $46 = $4. Reward $51.50 − $50 = $1.50. Reward ÷ risk = 1.5 ÷ 4 = 0.375. Even if it often works, one loss erases more than two wins; most would call it No-Go.

3. 1,000 × 1.06^10 ≈ $1,790.85; 1,000 × 1.03^10 ≈ $1,343.92; difference ≈ $446.93. The steady return rate is an assumption; real returns vary year to year and can be negative.
4. Single stock: 1,000 × 0.60 = $600 (−$400). ETF: each holding is $20; 20 × 0.40 = $8 loss, so $992. Diversification limited the damage, though a broad market drop would hit both.
5. Cost 30 × $20 = $600. Price change 30 × (−$1) = −$30. Dividends 30 × $0.80 = $24. Total −$30 + $24 = −$6; −6 ÷ 600 = −1%.
6. Spread $8.20 − $7.80 = $0.40; 0.40 ÷ 8 = 5%. The stock must rise 5% just to break even on a quick round trip, and a market order could fill worse. Waiting or a limit order protects your money.
7. Sum 24.90 + 25.40 + 24.70 + 25.80 + 26.20 = 127.00; 127 ÷ 5 = $25.40. Friday's $26.20 is above it. That describes the past week; it does not predict next week.
8. 2,000,000 × $26.20 = $52,400,000. Price per share alone says nothing about value; you must compare the whole company (market cap) to what it earns.

RECAP QUIZ

1. **Unrealized**: It becomes realized only when you sell; it can still change.
2. **$0.10 per share**: Buy at the ask, sell at the bid: $20.05 − $19.95 = $0.10.
3. **It is calculated from past closes**: It averages the last 50 closes, so it reacts after price moves.

CHAPTER 22

Business Statistics, Probability, and Data Interpretation

Box scores, odds tickers, creator dashboards, and stock apps all throw numbers at you. This chapter teaches you to summarize data, measure chance, see why the house wins, and catch a chart that is trying to fool you. All teams, players, prices, and odds are hypothetical.

22.1

Mean, Median, Mode, and Range

LEARNING OBJECTIVES

After this section you will be able to:

1. Find the mean, median, mode, and range of a data set.
2. Find the median when there is an even number of values.
3. Decide which average best describes data with an outlier.

REAL-LIFE SITUATION

A guard's last seven games

A high-school guard wants to tell a college recruiter what she "usually" scores. Her last seven games are listed in the stat sheet. Different averages give different answers, and she needs to know which one is fair.

Game	1	2	3	4	5	6	7
Points	18	24	12	30	24	15	31

Figure 22.1 Points scored, last seven games (hypothetical player)

The concept

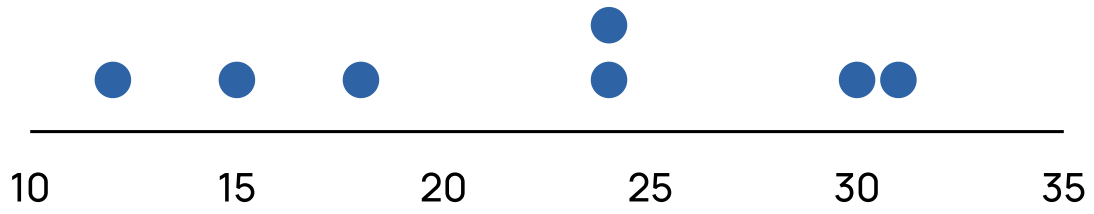
A statistic is a single number that summarizes a set of data. The three most common "averages" are called measures of center.

The mean is what most people call the average: add every value, then divide by how many values there are. The median is the middle value after you put the data in order. If there is an even number of values, the median is the mean of the two middle values. The mode is the value that appears most often; a data set can have one mode, several, or none.

The range measures spread, not center: it is the largest value minus the smallest. A large range means the results vary a lot from game to game.

An outlier is a value far away from the rest. Outliers pull the mean toward them but barely move the median, so the median is often the fairer "typical" value for incomes, prices, and earnings.

Distribution of seven game scores



1. Read the dot plot: repeated scores stack vertically.
2. Sort: 12, 15, 18, 24, 24, 30, 31. Median = 24; mode = 24.
3. Mean = 22 points; range = 19. Different summaries describe different features.

Original WealthWiseGuys diagram · Hypothetical teaching quantities.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

MEAN VS. MEDIAN

The mean is the arithmetic average; the median is the middle value. Outliers pull the mean more than the median.

OUTLIER

A value far away from the rest of the data, which can make an average misleading.

RISK

The chance that an investment or decision will turn out worse than expected.

KEY TERMS

Data set

A collection of numbers about the same thing, such as points per game.

Mean
The sum of the values divided by the number of values.

Median
The middle value of the ordered data.

Mode
The value that occurs most often.

Range
Largest value minus smallest value.

Outlier
A value much larger or smaller than the others.

FORMULAS & RULES

Mean

```
Mean = (sum of values) ÷ (number of values)
```

Range

```
Range = maximum − minimum
```

Median (even count)

```
Median = (middle value 1 + middle value 2) ÷ 2
```

Always sort the data first.

Worked examples

Example 1 The guard's stat sheet

Find the mean, median, mode, and range of 18, 24, 12, 30, 24, 15, 31.

SOLUTION

1. Sum: 18 + 24 + 12 + 30 + 24 + 15 + 31 = 154.
2. Mean: 154 ÷ 7 = 22 points.
3. Order the data: 12, 15, 18, 24, 24, 30, 31. The 4th of 7 values is the middle, so the median is 24.
4. Mode: 24 appears twice; every other value once. Mode = 24.
5. Range: 31 − 12 = 19 points.

Answer: Mean 22, median 24, mode 24, range 19

Example 2 A creator's payouts with an outlier

A video creator's monthly payouts were $40, $55, $35, $60, and $910 (one video went viral). Which average better describes a typical month?

SOLUTION

1. Mean: (40 + 55 + 35 + 60 + 910) ÷ 5 = 1,100 ÷ 5 = $220.
2. Order: 35, 40, 55, 60, 910. Median = $55.
3. Four of five months were $60 or less, so $220 overstates a normal month. The $910 outlier pulled the mean up.

Answer: The median, $55, is the better description of a typical month.

Example 3 Median of an even-sized set

Find the median of 3, 8, 10, 15.

SOLUTION

1. The data are already in order.
2. The two middle values are 8 and 10.
3. $(8 + 10) \div 2 = 9$.

Answer: 9

Practice exercises 22.1

1. Find the mean, median, mode, and range of 7, 9, 9, 10, 15.
2. Find the median of 21, 4, 17, 9, 30, 12.
3. Your first four test scores are 80, 92, 75, and 88. What score do you need on the fifth test for a mean of 85?
4. Hourly pay at a small shop is \$12, \$12, \$13, \$14, and \$49 (the owner). Which average describes a typical worker? Explain.
5. A bowler's mean over three games is 150. Two games were 140 and 165. What was the third?
6. List a data set of five numbers that has no mode.

RECAP QUIZ 22.1

1. What is the median of 2, 9, 4?

- ○ a. 4
- ○ b. 5
- ○ c. 9
- ○ d. 15

2. What is the mode of 5, 5, 6, 7, 7, 7?

- ○ a. 5
- ○ b. 6

○ c. 6.17

○ d. 7

3. Which measure is pulled most by one huge outlier?

○ a. Median

○ b. Mode

○ c. Mean

○ d. None of them

SECTION 22.1 RECAP

- Mean = sum ÷ count; median = middle of sorted data; mode = most frequent; range = max − min.
- When an outlier is present, report the median as the typical value.

Answer key with explanations: Section 22.1

PRACTICE EXERCISES

1. Sum 50, so mean 10; median 9; mode 9; range 15 − 7 = 8.
2. Order: 4, 9, 12, 17, 21, 30. Middle two are 12 and 17, so median = 14.5.
3. Needed total 5 × 85 = 425. Current total 335. 425 − 335 = 90.
4. Mean = 100 ÷ 5 = $20; median = $13. The median, because the owner's $49 is an outlier.
5. Total 3 × 150 = 450; 450 − 140 − 165 = 145.
6. Any five different numbers, for example 2, 4, 6, 8, 10.

RECAP QUIZ

1. **4**: Ordered: 2, 4, 9. The middle value is 4.
2. **7**: 7 appears three times, more than any other value.
3. **Mean**: Every value is added into the mean, so one extreme value moves it.

22.2

Probability: Measuring Chance

LEARNING OBJECTIVES

After this section you will be able to:

1. Find theoretical probability by counting equally likely outcomes.
2. Find experimental probability from game stats.
3. Use complements and the multiplication rule for independent events.

REAL-LIFE SITUATION

Free throws in the final minute

A forward has made 42 of his 60 free throws this season. His team is down by one with seconds left, and he is fouled for two shots. Commentators say he is "a 70% shooter." What does that number really tell us?

Attempts	Made	Missed	FT %
60	42	18	70.0%

Figure 22.2 Season free-throw stats (hypothetical player)

The concept

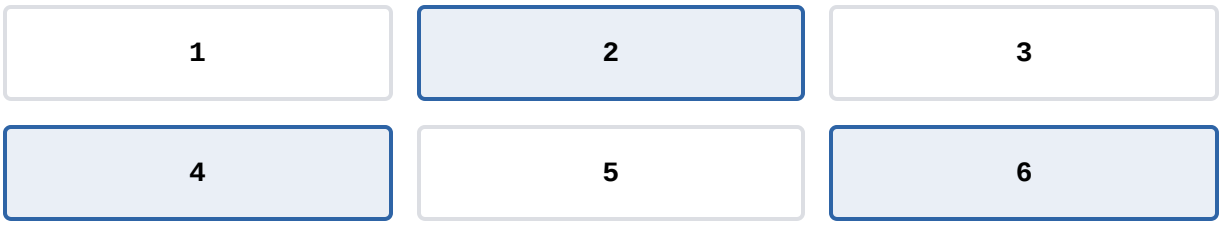
Probability measures how likely an event is, on a scale from 0 (impossible) to 1 (certain). It can be written as a fraction, a decimal, or a percent.

Theoretical probability comes from counting: if every outcome is equally likely, P(event) = favorable outcomes ÷ total outcomes. Experimental probability comes from what actually happened: successes ÷ trials. Sports percentages are experimental probabilities.

The complement of an event is everything that is not the event, and P(not A) = 1 − P(A). Two events are independent when one does not change the chance of the other. For independent events, P(A and B) = P(A) × P(B).

Probability describes the long run. A 70% shooter will still miss sometimes, and a streak of makes does not mean a miss is "due."

Probability: equally likely outcomes



1. An illustrative fair six-sided die has six equally likely outcomes.
2. Three outcomes are even: 2, 4, 6.
3. P(even) = 3 ÷ 6 = 1/2 = 50%. Equal likelihood is an assumption, not a rule for every real-world event.

Original WealthWiseGuys diagram · Hypothetical teaching quantities.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

PROBABILITY

The chance an event happens, written as a fraction, decimal, or percent between 0 and 1.

ODDS

Another way to write probability, comparing the chance of an event to the chance it does not happen.

EXPECTED VALUE

The average result you would expect if a random situation were repeated many times.

KEY TERMS

Outcome
One possible result of an experiment, like rolling a 4.

Event
A set of outcomes you are interested in, like rolling an even number.

Theoretical probability
Favorable outcomes ÷ total equally likely outcomes.

Experimental probability
Number of successes ÷ number of trials.

Complement
The event not happening.

Independent events
Events where one result does not affect the other.

FORMULAS & RULES

Probability

```
P(A) = favorable outcomes ÷ total outcomes
```

$0 \le P(A) \le 1$

Complement

```
P(not A) = 1 − P(A)
```

Independent events

```
P(A and B) = P(A) × P(B)
```

Worked examples

Example 1 Drawing from a bag

A bag holds 3 red, 5 blue, and 2 green tokens. One is drawn at random. Find P(blue) and P(not green).

SOLUTION

1. Total tokens: 3 + 5 + 2 = 10.
2. P(blue) = 5 ÷ 10 = 1/2 = 0.5.
3. P(green) = 2 ÷ 10 = 1/5, so P(not green) = 1 − 1/5 = 4/5 = 0.8.

> **Answer:** P(blue) = 1/2; P(not green) = 4/5

Example 2 Two free throws

Using 42 makes in 60 attempts, find the chance he makes both shots and the chance he misses both. Assume the shots are independent.

SOLUTION

1. Experimental probability of a make: 42 ÷ 60 = 0.7.
2. P(miss) = 1 − 0.7 = 0.3.
3. P(make both) = 0.7 × 0.7 = 0.49.
4. P(miss both) = 0.3 × 0.3 = 0.09.
5. Key point: the chance of at least one make is 1 − 0.09 = 0.91.

Answer: Make both 49%; miss both 9%

Example 3 Rolling two dice

Two fair dice are rolled. What is the probability that the sum is 7?

SOLUTION

1. Each die has 6 faces, so there are 6 × 6 = 36 equally likely outcomes.
2. Pairs that sum to 7: (1,6), (2,5), (3,4), (4,3), (5,2), (6,1). That is 6 outcomes.
3. P(sum 7) = 6 ÷ 36 = 1/6.

Answer: 1/6, about 16.7%

Practice exercises 22.2

1. A fair coin is flipped twice. What is P(heads both times)?
2. A spinner has 8 equal sections; 3 are gold. Find P(gold) as a fraction and a percent.
3. A batter has 27 hits in 90 at-bats. What is his batting average (experimental probability of a hit)?
4. The forecast gives a 35% chance of rain. What is the chance it does not rain?
5. One card is drawn from a standard 52-card deck. Find P(heart) and P(not a heart).
6. A player has won 5 coin flips in a row. What is the probability the next flip is heads? Explain.

RECAP QUIZ 22.2

1. Every probability is between:

- ○ a. 0 and 10
- ○ b. 0 and 1
- ○ c. −1 and 1
- ○ d. 1 and 100

2. If P(A) = 0.2, then P(not A) is:

- ○ a. 0.2
- ○ b. 0.02
- ○ c. 0.8
- ○ d. 1.2

3. Rolling one fair die, P(5) is:

- ○ a. 1/5
- ○ b. 1/6
- ○ c. 5/6
- ○ d. 1/2

SECTION 22.2 RECAP

- P(A) = favorable ÷ total; experimental P = successes ÷ trials.
- P(not A) = 1 − P(A). For independent events, multiply.
- Past results do not make an independent outcome "due."

Answer key with explanations: Section 22.2

PRACTICE EXERCISES

1. 1/2 × 1/2 = 1/4.
2. 3/8 = 37.5%.
3. 27 ÷ 90 = 0.300.
4. 1 − 0.35 = 0.65, or 65%.
5. 13/52 = 1/4; complement 3/4.
6. 1/2. Flips are independent; past results do not change the coin.

RECAP QUIZ

1. **0 and 1**: 0 means impossible and 1 means certain (0% to 100%).
2. **0.8**: 1 − 0.2 = 0.8.
3. **1/6**: One favorable face out of six.

22.3

Odds, Expected Value, and Risk Literacy

LEARNING OBJECTIVES

After this section you will be able to:

1. Convert American odds to implied probability.
2. Explain why the implied probabilities on an odds board add to more than 100%.
3. Use expected value to see the long-run cost of a bet or game of chance.

REAL-LIFE SITUATION

The odds on the broadcast

Sports broadcasts and apps now show odds on screen during games. You will see them whether or not you ever bet. This lesson teaches you to read them as a math and risk problem. In Pennsylvania you must be 21 to place a sports bet, and this lesson is not advice to gamble.

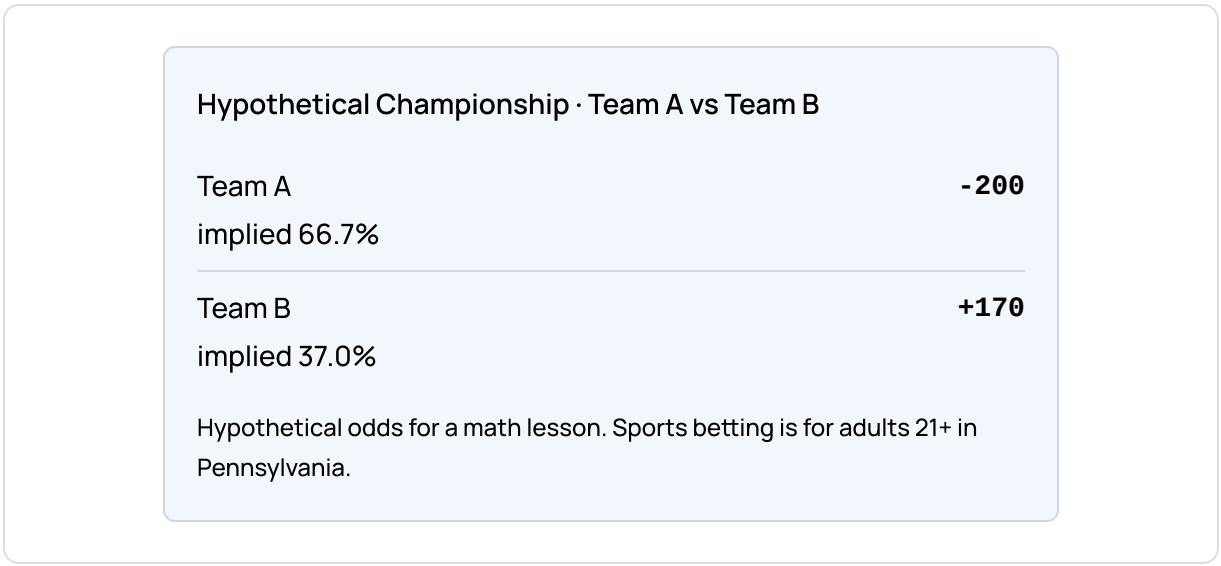


Figure 22.3 Odds board for a hypothetical championship game

The concept

American odds are written with a plus or minus sign. A plus number such as +170 shows the profit on a $100 winning bet. A minus number such as −200 shows how much you must risk to profit $100. The favorite has the minus sign.

Odds can be turned into an implied probability. For positive odds: 100 ÷ (odds + 100). For negative odds: |odds| ÷ (|odds| + 100). If the two sides of a fair game were priced honestly, the probabilities would add to exactly 100%.

On a real board they add to more than 100%. The amount above 100% is called the vig or overround. It measures the book's pricing spread — it is not a guaranteed profit margin, and it does not tell you any single bettor's expected loss; those depend on the true probabilities and how the money is spread.

Expected value (EV) is the average result per play if you repeated a choice many times: multiply each result by its probability and add. A negative EV means that, over time, you should expect to lose money. Nearly every casino game, lottery, and sportsbook line is built to have negative EV for the player.

Risk literacy means knowing the math before someone else uses it on you. If gambling stops being fun or feels out of control for you or someone you know, the free, confidential helpline is 1-800-GAMBLER.

Odds, Expected Value, and Risk Literacy: solution map

1. Identify the question

   `Find the implied probability for Team A at −200 and Team B at +170.`

2. Calculation 1

   `Team A (negative): 200 ÷ (200 + 100) = 200 ÷ 300 ≈ 0.6667, about 66.7%.`

3. Calculation 2

   `Team B (positive): 100 ÷ (170 + 100) = 100 ÷ 270 ≈ 0.3704, about 37.0%.`

4. Calculation 3

   `Total: 66.7% + 37.0% = 103.7%.`

5. Calculation 4

   `Key point: the extra is about 3.7 percentage points of overround. A fair board would total 100%.`

6. Interpret and check

   `Team A ≈ 66.7%, Team B ≈ 37.0%, total ≈ 103.7%`

Figure 22.3.V Odds, Expected Value, and Risk Literacy: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

SPECULATION

A very risky investment made in hopes of a quick, large gain.

INFLATION

A general rise in prices over time, which makes money buy less.

YIELD

The income an investment pays, shown as a percent of its price.

KEY TERMS

Odds
A way of stating the payout of a bet, which also implies a probability.

Favorite / underdog
The side expected to win (minus odds) / expected to lose (plus odds).

Implied probability
The chance of winning that the odds suggest.

Vig (overround)
How far implied probabilities exceed 100% — the book's pricing spread, not a guaranteed profit margin.

Expected value
Average gain or loss per play over many plays.

FORMULAS & RULES

Positive odds

```
Implied P = 100 ÷ (odds + 100)
```

Negative odds

```
Implied P = |odds| ÷ (|odds| + 100)
```

Expected value

```
EV = P(win) × gain − P(lose) × loss
```

Worked examples

Example 1 Reading the board

Find the implied probability for Team A at −200 and Team B at +170.

SOLUTION

1. Team A (negative): 200 ÷ (200 + 100) = 200 ÷ 300 ≈ 0.6667, about 66.7%.
2. Team B (positive): 100 ÷ (170 + 100) = 100 ÷ 270 ≈ 0.3704, about 37.0%.
3. Total: 66.7% + 37.0% = 103.7%.
4. Key point: the extra is about 3.7 percentage points of overround. A fair board would total 100%.

> **Answer:** Team A ≈ 66.7%, Team B ≈ 37.0%, total ≈ 103.7%

Example 2 The cost of a "50/50" bet

Both sides of an even game are listed at −110. You risk $110 to win $100. If the game really is a coin flip, what is the expected value of one bet?

SOLUTION

1. P(win) = 0.5, gain = $100. P(lose) = 0.5, loss = $110.
2. EV = 0.5 × 100 − 0.5 × 110 = 50 − 55 = −$5.

3 Over 100 such bets, the expected result is 100 × (−5) = −$500.

Answer: EV = −$5 per bet

Example 3 A raffle

A school raffle sells 500 tickets at $2 each for one $400 prize. What is the expected value of one ticket?

SOLUTION

1 P(win) = 1 ÷ 500. Expected prize = 400 × 1/500 = $0.80.

2 Subtract the ticket cost: 0.80 − 2.00 = −$1.20.

3 Key point: a fundraiser raffle is honest about this, since the extra money supports the school. Commercial gambling keeps it.

Answer: EV = −$1.20 per ticket

Practice exercises 22.3

1. Convert +300 to an implied probability.
2. Convert −150 to an implied probability.
3. At +250, what is the profit on a winning $20 bet?
4. At −150, what is the profit on a winning $30 bet?
5. A game costs $1 to play. You win $4 profit with probability 1/6; otherwise you lose the $1. Find the EV.
6. A board lists +100 and +100 on the two sides. What do the implied probabilities total, and is there a margin?

RECAP QUIZ 22.3

1. +100 odds imply a probability of:

- ○ a. 100%
- ○ b. 10%
- ○ c. 50%
- ○ d. 1%

2. Implied probabilities on a board total 104%. The extra 4 percentage points is:

- ○ a. A math error
- ○ b. The overround
- ○ c. Your expected profit
- ○ d. The tax rate

3. A negative expected value means:

- ○ a. You will lose the next bet
- ○ b. You will win eventually
- ○ c. You expect to lose money over many plays
- ○ d. The odds are illegal

SECTION 22.3 RECAP

- Plus odds = profit on $100; minus odds = risk needed to profit $100.
- Implied probabilities over 100% reveal the overround — the book's pricing spread, not a guaranteed profit.
- Negative EV means the game is designed for you to lose over time.

Answer key with explanations: Section 22.3

PRACTICE EXERCISES

1. 100 ÷ 400 = 0.25 = 25%.
2. 150 ÷ 250 = 0.6 = 60%.

3. 20 × 250 ÷ 100 = $50 profit.
4. 30 × 100 ÷ 150 = $20 profit.
5. (1/6)(4) − (5/6)(1) = 4/6 − 5/6 = −1/6 ≈ −$0.17 per play.
6. 50% + 50% = 100%. No margin; real boards almost never do this.

RECAP QUIZ

1. **50%**: 100 ÷ (100 + 100) = 0.5.
2. **The overround**: The overround builds an advantage into the quoted payouts; it does not guarantee profit on every event.
3. **You expect to lose money over many plays**: EV is a long-run average, not a prediction of one play.

22.4

Investing Returns and Misleading Charts

LEARNING OBJECTIVES

After this section you will be able to:

1. Compute percent change for a price or investment.
2. Show why a gain and an equal-percent loss do not cancel.
3. Spot a truncated axis and other misleading chart choices.

REAL-LIFE SITUATION

"Resale prices are exploding!"

A post shows the resale price of a hypothetical sneaker over three months with towering bars. The caption says prices are "exploding." The same

numbers drawn on a chart that starts at $0 tell a calmer story.

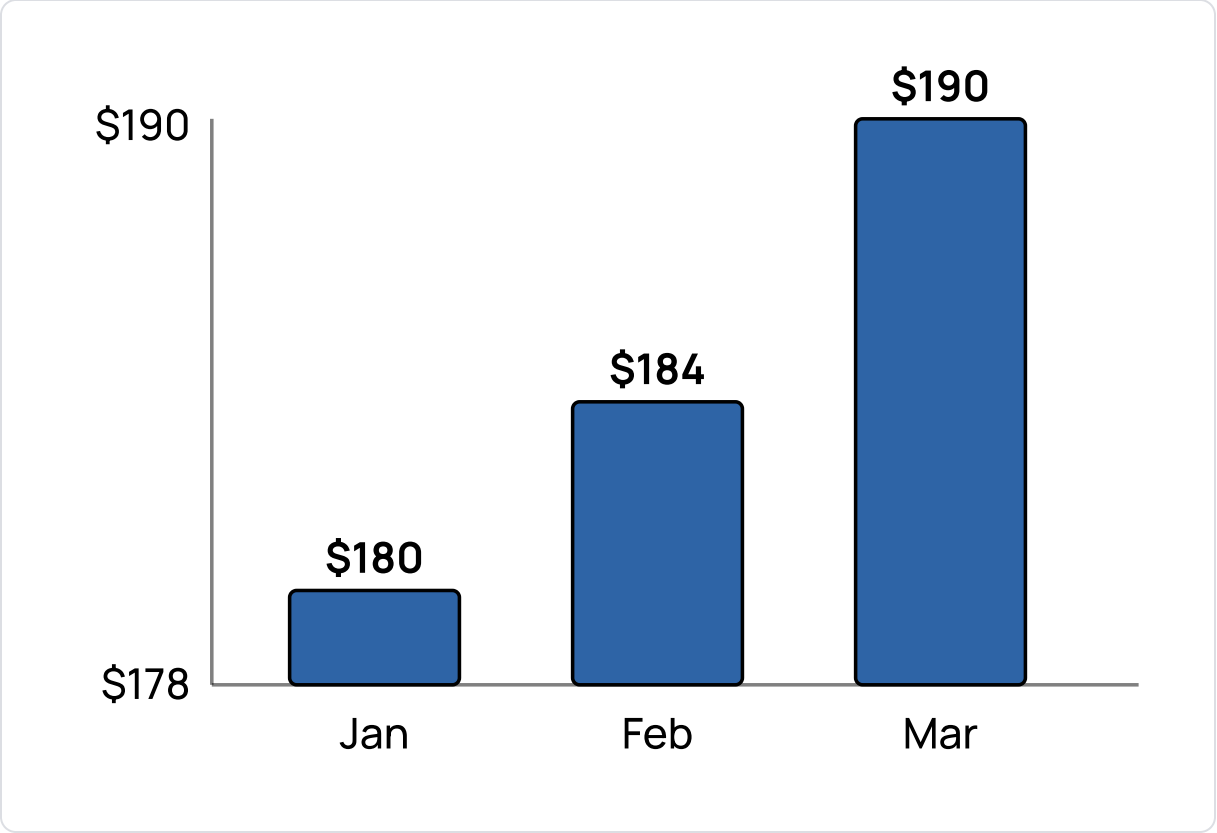


Figure 22.4a The post's chart: the axis starts at $178

The concept

Percent change compares the change to where you started: (new − old) ÷ old × 100%. It is how investment apps report gains and losses.

Percent gains and losses do not cancel. If an investment rises 50% and then falls 50%, the fall is 50% of a bigger number, so you end up below where you began. This is why steady growth beats wild swings.

Compound growth means each year's gain is earned on the new, larger balance: Balance = Start × $(1 + r)^n$. Historical averages are not promises; real returns vary and can be negative.

Charts can mislead without lying about a single number. Watch for a vertical axis that does not start at zero, uneven time intervals, missing

labels, cherry-picked start dates, and 3-D effects. Always read the axis before the bars.

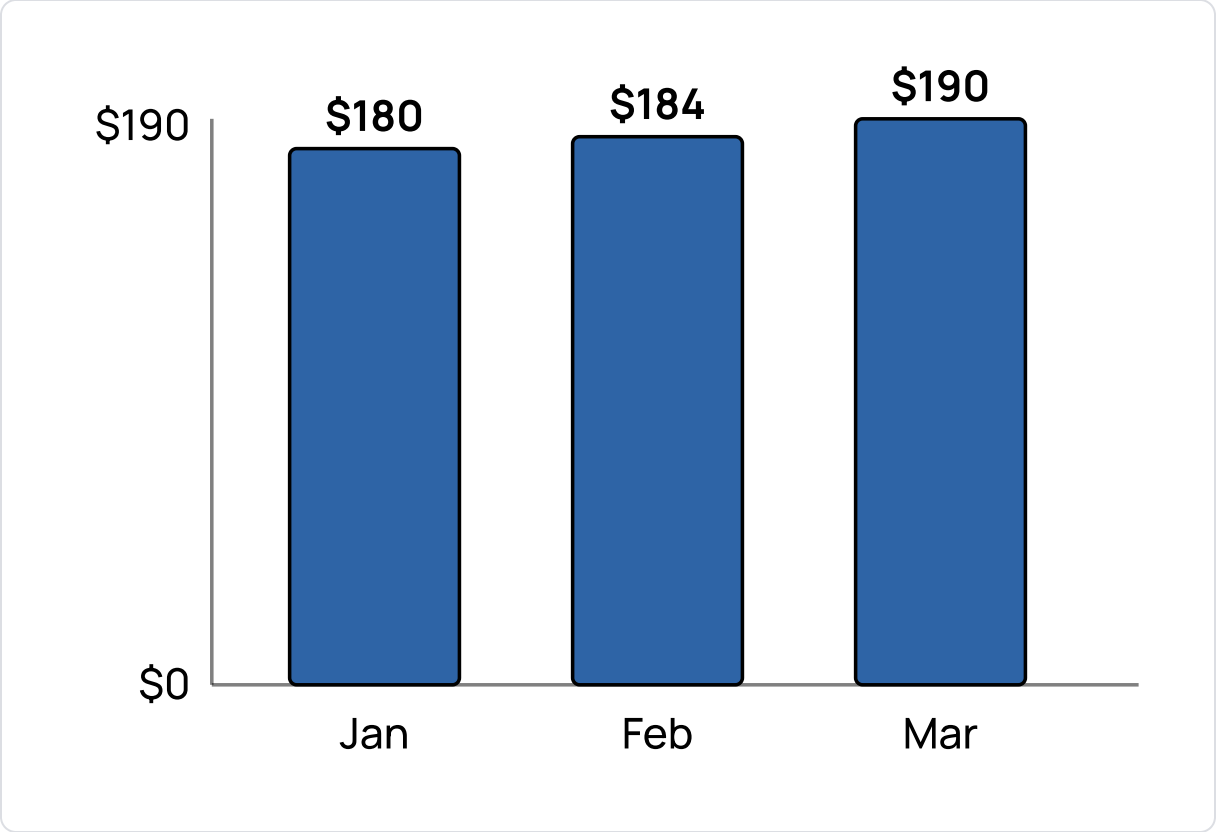


Figure 22.4b The same data with the axis starting at $0

Investing Returns and Misleading Charts: solution map

1. Identify the question

   `The sneaker went from $180 in January to $190 in March. Find the percent change.`

2. Calculation 1

   `Change: 190 − 180 = $10.`

3. Calculation 2

   `Percent change: 10 ÷ 180 ≈ 0.0556 = 5.56%.`

4. Calculation 3

   `Key point: in Figure 22.4a the March bar looks about 6 times taller than January, because the bars show only the part above $178 (12 vs. 2). The real increase is about 5.6%.`

5. Interpret and check

   `About a 5.6% increase`

Figure 22.4.V Investing Returns and Misleading Charts: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

RETURN

The money or gain an investment produces, often shown as a percent.

ROI

Return on investment — the gain compared to what you put in, as a percent: (gain ÷ cost) × 100.

CAPITAL GAIN

Profit made by selling an investment for more than it cost.

KEY TERMS

Percent change
The change in a value as a percent of its starting value.

Return
The gain or loss on an investment, usually shown as a percent.

Compound growth
Growth where each period's gain is added before the next gain is figured.

Truncated axis
An axis that starts above zero, which exaggerates differences.

FORMULAS & RULES

Percent change

$$\% \text{ change} = (\text{new} - \text{old}) \div \text{old} \times 100\%$$

Compound growth

$$A = P(1 + r)^n$$

P = start, r = rate per period as a decimal, n = periods.

Worked examples

Example 1 How big is the jump?

The sneaker went from $180 in January to $190 in March. Find the percent change.

SOLUTION

1. Change: 190 − 180 = $10.
2. Percent change: 10 ÷ 180 ≈ 0.0556 = 5.56%.
3. Key point: in Figure 22.4a the March bar looks about 6 times taller than January, because the bars show only the part above $178 (12 vs. 2). The real increase is about 5.6%.

> **Answer:** About a 5.6% increase

Example 2 Up 50%, down 50%

You invest $1,000. It gains 50% one year and loses 50% the next. What do you have?

SOLUTION

1. Year 1: 1,000 × 1.50 = $1,500.
2. Year 2: 1,500 × 0.50 = $750.
3. Overall: (750 − 1,000) ÷ 1,000 = −25%.

> **Answer:** $750, a 25% loss, even though the two percents "average" to 0%

Example 3 Compound growth (hypothetical)

A student puts $500 in a broad index fund. Suppose it grows 7% per year for 2 years. Find the balance. (7% is a hypothetical rate, not a prediction.)

SOLUTION

1. After year 1: $500 \times 1.07 = \$535.00$.
2. After year 2: $535 \times 1.07 = \$572.45$.
3. Check with the formula: $500 \times 1.07^2 = 500 \times 1.1449 = \$572.45$. ✓

> **Answer:** $572.45

Practice exercises 22.4

1. A stock goes from $40 to $46. Find the percent change.
2. A stock goes from $80 to $60. Find the percent change.
3. $200 rises 10%, then falls 10%. What is the final value?
4. A chart's axis starts at 95. Bars show 96 and 100. How many times taller does the second bar look, and what is the real percent increase?
5. $1,000 grows 5% per year for 3 years. Find the balance to the nearest cent.
6. Name two ways a chart can mislead besides a truncated axis.

RECAP QUIZ 22.4

1. The most common trick in a misleading bar chart is:

- ○ a. Using color
- ○ b. An axis that doesn't start at zero
- ○ c. Labeling the bars
- ○ d. Showing a title

2. $100 gains 20% and then loses 20%. The result is:

- ○ a. $100
- ○ b. $96
- ○ c. $104
- ○ d. $80

3. Percent change is found by dividing the change by the:

- ○ a. New value
- ○ b. Old value
- ○ c. Sum of both
- ○ d. Number 100

SECTION 22.4 RECAP

- % change = (new − old) ÷ old. Gains and equal-percent losses do not cancel.
- Compound growth: $A = P(1 + r)^n$. Past returns are not promises.
- Read the axis first. A chart that doesn't start at zero can make 5% look like 500%.

Answer key with explanations: Section 22.4

PRACTICE EXERCISES

1. 6 ÷ 40 = 0.15 = +15%.
2. −20 ÷ 80 = −0.25 = −25%.
3. 200 × 1.10 = 220; 220 × 0.90 = $198.
4. Visible heights 1 and 5, so it looks 5 times taller. Real increase 4 ÷ 96 ≈ 4.17%.
5. $1{,}000 \times 1.05^3 = 1{,}000 \times 1.157625 = \$1{,}157.63$.
6. Any two: uneven time intervals, cherry-picked start or end dates, missing labels or units, 3-D distortion, different scales on side-by-side charts.

RECAP QUIZ

1. **An axis that doesn't start at zero**: A truncated axis exaggerates small differences.
2. **$96**: 100 × 1.2 = 120; 120 × 0.8 = 96.

3. **Old value**: Percent change compares to the starting (old) value.

22.5

Interpreting Business Data: Weighted Averages and Percent of Total

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate sales revenue and shares of a weekly total.
2. Find a weighted average selling price using units as weights.
3. Distinguish unit share, revenue share, and profit.

REAL-LIFE SITUATION

Interpreting Business Data: Weighted Averages and Percent of Total in practice

A hypothetical Philadelphia food truck records one week of item sales below. Every item in a category sells at the listed price, with no discounts, refunds, or sales tax included. All amounts are invented; round final money to cents and percentages to two decimals.

Item	Units sold	Price per unit ($)	Revenue ($)
Rice bowl	120	8	960
Wrap	80	6	480
Lemon drink	200	3	600
Total	400	Varies	2040

Figure 22.5 Interpreting Business Data: Weighted Averages and Percent of Total (hypothetical data)

The concept

A business table can describe several different totals. Units sold count individual items, while revenue counts dollars received from selling them. Multiply units by price for each category before adding revenue. A row containing a total is a summary, not an additional category to add again.

A percent of total compares one category with the matching grand total. The rice bowl unit share uses 120 divided by 400 items; its revenue share uses $960 divided by $2,040. Both are legitimate but answer different questions. Always name the denominator and the time period.

A weighted average gives more influence to values represented more often. To find average revenue per item sold, weight each listed price by its unit count: add price times units, then divide by total units. The ordinary mean of the three menu prices treats each category as equally common, which is not how this week’s sales occurred.

Average selling price is not average spending per customer because one customer may buy several items. Revenue share also is not profit share; ingredient and operating costs are absent. Rounded percentage shares may not total exactly 100%, so keep unrounded values until the final display and explain small rounding differences.

Interpreting Business Data: Weighted Averages and Percent of Total: solution map

1. Identify the question
   `Use the table to find total revenue and average revenue per item sold.`
2. Calculation 1
   `Rice bowls: 120 × $8 = $960; wraps: 80 × $6 = $480.`
3. Calculation 2
   `Drinks: 200 × $3 = $600.`
4. Calculation 3
   `Total revenue = 960 + 480 + 600 = $2,040; total units = 120 + 80 + 200 = 400.`
5. Calculation 4
   `Weighted average price = 2,040 ÷ 400 = $5.10.`
6. Interpret and check
   `Total revenue is $2,040; the truck earns $5.10 per item on average, not per customer.`

Figure 22.5.V Interpreting Business Data: Weighted Averages and Percent of Total: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

REVENUE

The total money a business brings in from sales, before any costs are paid.

PROFIT MARGIN

Profit shown as a percent of revenue; a 10% margin means $10 of profit per $100 of sales.

PORTFOLIO

The full collection of investments a person or fund owns.

KEY TERMS

Weight
The count or importance attached to a value in an average.

Weighted average
Sum of value times weight, divided by total weight.

Revenue
Selling price multiplied by quantity sold, summed across sales.

Percent of total
A category amount divided by the matching overall amount, times 100%.

Sales mix
The distribution of sales among product categories.

FORMULAS & RULES

Item revenue

```
Revenue = units sold × price per unit
```

Weighted average price

```
Average price = sum(price × units) ÷ sum(units)
```

Percent of total

```
Share = category amount ÷ matching total × 100%
```

Worked examples

Example 1 Weekly average price

Use the table to find total revenue and average revenue per item sold.

SOLUTION

1. Rice bowls: 120 × $8 = $960; wraps: 80 × $6 = $480.
2. Drinks: 200 × $3 = $600.
3. Total revenue = 960 + 480 + 600 = $2,040; total units = 120 + 80 + 200 = 400.
4. Weighted average price = 2,040 ÷ 400 = $5.10.

> **Answer:** Total revenue is $2,040; the truck earns $5.10 per item on average, not per customer.

Example 2 Two different shares

Find rice bowls' unit share and revenue share.

SOLUTION

1. Identify unit denominator: 400 items.
2. Unit share = 120 ÷ 400 × 100% = 30.00%.
3. Identify revenue denominator: $2,040.

4 Revenue share = 960 ÷ 2,040 × 100% ≈ 47.06%.

> **Answer:** Rice bowls are 30.00% of units but 47.06% of revenue.

Practice exercises 22.5

1. Find wraps' unit share and revenue share.
2. Find drinks' revenue share.
3. Find the simple mean of the three prices and explain why it is not this week's average selling price.
4. Another week sells 10 bowls at $8 and 30 drinks at $3. Find average price.
5. A reader calls $2,040 the weekly profit. Explain what is missing.
6. What do this week's displayed revenue shares sum to?

RECAP QUIZ 22.5

1. Correct denominator for drinks' unit share?

- ○ a. $2,040
- ○ b. 400 items
- ○ c. 200 items
- ○ d. $600

2. Average selling price for this week?

- ○ a. $5.67
- ○ b. $6.00
- ○ c. $5.10
- ○ d. $17.00

3. Does the table reveal profit?

- ○ a. Yes, $2,040
- ○ b. Yes, $400

○ c. Yes, $5.10

○ d. No, costs are missing

SECTION 22.5 RECAP

- Weight prices by units sold, not by the number of menu categories.
- Name the denominator: unit share and revenue share answer different questions.
- Revenue and average selling price do not reveal profit or spending per customer.

Answer key with explanations: Section 22.5

PRACTICE EXERCISES

1. Unit share = 80 ÷ 400 × 100% = 20.00%; revenue share = 480 ÷ 2,040 × 100% ≈ 23.53%.
2. Drink revenue = 200 × 3 = $600; 600 ÷ 2,040 × 100% ≈ 29.41%.
3. (8 + 6 + 3) ÷ 3 = 17 ÷ 3 ≈ $5.67. It weights categories equally, unlike the actual 120, 80, and 200 unit counts; the weighted result is $5.10.
4. Revenue = 10 × 8 + 30 × 3 = $170; units = 40; weighted average = 170 ÷ 40 = $4.25.
5. The $2,040 equals sales revenue, 960 + 480 + 600. Profit requires subtracting food, labor, and other costs; those costs are not supplied.
6. 47.06% + 23.53% + 29.41% = 100.00%. Other data sets may differ slightly from 100% after rounding, even when the exact shares total 100%.

RECAP QUIZ

1. **400 items**: The category has 200 units out of 400 total units.
2. **$5.10**: Total revenue $2,040 ÷ 400 units = $5.10.
3. **No, costs are missing**: Revenue minus costs gives profit; the table has no cost data.

CHAPTER 23

Linear Equations, Graphs, and Business Relationships

A delivery fee, an hourly paycheck, and a food stand's costs can all become equations. Learn to solve those equations, graph their relationships, and explain what their answers mean. All businesses, prices, and earnings in this chapter are hypothetical; round money to the nearest cent when needed.

23.1

Writing and Solving Linear Equations

LEARNING OBJECTIVES

After this section you will be able to:

1. Translate fixed charges and per-unit charges into equations.
2. Solve linear equations using inverse operations.
3. Check solutions and interpret their units.

REAL-LIFE SITUATION

A neighborhood equipment rental

A hypothetical Philadelphia neighborhood group rents equipment for a cleanup. The charge is $18 plus $14 per hour. An equation lets the group

work backward from its bill to the rental time.

Hours h	Charge ($)
0	18
2	46
5	88

Figure 23.1 Hypothetical rental charges

The concept

A variable stands for an unknown or changing quantity. Before writing an equation, define the variable and its units. If h is rental time in hours, 14h means 14 dollars per hour multiplied by h hours.

An equation states that two expressions have equal values. A fixed charge appears once, while a per-unit charge is multiplied by the number of units. The rental model is C = 18 + 14h; when the bill is $88, it becomes 88 = 18 + 14h.

Solve by undoing operations in reverse order. Subtract an added amount before dividing by a coefficient. Perform the same operation on both sides so that the equality remains true. When parentheses occur, distribute correctly or undo the outside operation first.

Finally, substitute the proposed answer into the original equation. A correct calculation must also fit the situation: negative rental time is not meaningful, and a business that bills only whole hours may require a whole-number interpretation.

Writing and Solving Linear Equations: solution map

1. Identify the question
   `The rental bill is $88. At $18 plus $14 per hour, how many hours were charged?`
2. Calculation 1
   `Define h as the number of rental hours.`
3. Calculation 2
   `Write 18 + 14h = 88.`
4. Calculation 3
   `Subtract 18: 14h = 70.`
5. Calculation 4
   `Divide by 14: h = 5.`
6. Calculation 5
   `Check: 18 + 14 × 5 = 18 + 70 = 88.`
7. Interpret and check
   `The bill covers 5 hours.`

Figure 23.1.V Writing and Solving Linear Equations: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

VARIABLE COST

A cost that rises or falls with the number of items made or sold.

FIXED COST

A cost that stays the same no matter how many items are sold, like rent.

REVENUE

The total money a business brings in from sales, before any costs are paid.

KEY TERMS

Variable
A symbol representing an unknown or changing quantity.

Coefficient
A number multiplying a variable.

Constant
A fixed number in an expression.

Linear equation
An equation that can be written ax + b = c, with a nonzero.

Inverse operations
Operations that undo each other, such as addition and subtraction.

FORMULAS & RULES

Fixed-plus-variable charge

$$C = F + r \times x$$

F is the fixed charge and r is the charge per unit.

Solve a linear equation

$$x = (c - b) \div a$$

For ax + b = c, with a ≠ 0.

Distributive property

```
a × (x + b) = ax + ab
```

Worked examples

Example 1 Work backward from a rental bill

The rental bill is $88. At $18 plus $14 per hour, how many hours were charged?

SOLUTION

1. Define h as the number of rental hours.
2. Write 18 + 14h = 88.
3. Subtract 18: 14h = 70.
4. Divide by 14: h = 5.
5. Check: 18 + 14 × 5 = 18 + 70 = 88.

> **Answer:** The bill covers 5 hours.

Example 2 Undo grouped operations

Three identical supply kits cost $90 altogether. Each kit costs 4x + 6 dollars. Find x.

SOLUTION

1. Write 3(4x + 6) = 90.
2. Divide both sides by 3: 4x + 6 = 30.

3. Subtract 6: $4x = 24$.
4. Divide by 4: $x = 6$.
5. Check: $3(4 \times 6 + 6) = 3 \times 30 = 90$.

> **Answer:** $x = 6$; each kit costs $30.00.

Practice exercises 23.1

1. Evaluate $C = 12h + 15$ for $h = 6$.
2. Solve $10x + 15 = 95$.
3. Evaluate $3(x + 4)$ for $x = 7$.
4. Solve $4(5x + 3) = 72$.
5. A student says $2x + 6 = 22$ gives $x = 14$. Explain and correct the error.

RECAP QUIZ 23.1

1. Solve $8x + 10 = 50$.

- ○ a. 5
- ○ b. 7.5
- ○ c. 40
- ○ d. 480

2. Which expression equals $4(x + 2)$?

- ○ a. $4x + 2$
- ○ b. $x + 8$
- ○ c. $4x + 8$
- ○ d. $6x$

3. A service charges $9 plus $7 per hour. What is the charge for 5 hours?

- ○ a. $35
- ○ b. $44

○ c. $16

○ d. $80

SECTION 23.1 RECAP

- Define the variable, write the equation, and undo operations on both sides.
- Check the result in the original equation and explain its units.

Answer key with explanations: Section 23.1

PRACTICE EXERCISES

1. Substitute 6: C = 12 × 6 + 15 = 72 + 15 = $87.00.
2. Subtract 15 to get 10x = 80; divide by 10 to get x = 8. Check: 80 + 15 = 95.
3. First add inside parentheses: 7 + 4 = 11. Then 3 × 11 = 33.
4. Divide by 4: 5x + 3 = 18. Subtract 3: 5x = 15. Divide by 5: x = 3.
5. Subtracting 6 gives 2x = 16, not x = 16; dividing by 2 gives x = 8. Check: 2 × 8 + 6 = 22.

RECAP QUIZ

1. **5**: 8x = 40, so x = 40 ÷ 8 = 5.
2. **4x + 8**: Distribute 4 to both terms: 4x + 4 × 2 = 4x + 8.
3. **$44**: 9 + 7 × 5 = 9 + 35 = $44.00.

23.2

Slope, Intercepts, and Graphing Lines

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate slope from two ordered pairs.
2. Interpret slope and the vertical intercept in context.
3. Plot a linear relationship with labeled axes and an appropriate domain.

REAL-LIFE SITUATION

Preparing a delivery-cost graph

A hypothetical local delivery service charges $30 plus $5 per mile. Its table supplies points for a graph: miles belong on the horizontal axis, and total dollars belong on the vertical axis.

Miles x	Total cost C ($)	Ordered pair
0	30	(0, 30)
2	40	(2, 40)
6	60	(6, 60)

Figure 23.2 Plotting data for C = 5x + 30

The concept

A coordinate graph pairs each input with an output. The horizontal axis usually shows the input x, and the vertical axis shows the output y. Label both axes with quantities and units, and choose evenly spaced numerical scales.

Slope measures the change in output divided by the change in input. Subtract coordinates in the same order in the numerator and denominator. For a delivery model, slope has units of dollars per mile, not simply dollars.

In y = mx + b, m is the slope and b is the vertical intercept, the output when x = 0. Positive slope means the output rises as x increases; negative slope means it falls. A horizontal line has slope zero. A vertical line has undefined slope because its change in x is zero.

To graph a line, plot the intercept and at least one additional point, or calculate a small table. Draw the line through the points, but restrict it to meaningful inputs. Distance may vary continuously, whereas counts of tickets or shirts use separate whole-number points.

Slope, Intercepts, and Graphing Lines: solution map

1. Identify the question
   `Use (2, 40) and (6, 60) to find the delivery equation and predict the cost for 8 miles.`
2. Calculation 1
   `Slope: m = (60 − 40) ÷ (6 − 2) = 20 ÷ 4 = 5.`
3. Calculation 2
   `Use (2, 40): b = 40 − 5 × 2 = 30.`
4. Calculation 3
   `Write C = 5x + 30.`
5. Calculation 4
   `Plot (0, 30), (2, 40), and (6, 60) on axes labeled miles and dollars; draw the line for x ≥ 0.`
6. Calculation 5
   `At 8 miles: C = 5 × 8 + 30 = 70.`
7. Interpret and check
   `Slope $5 per mile, intercept $30.00, and an 8-mile charge of $70.00.`

Figure 23.2.V Slope, Intercepts, and Graphing Lines: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

PROFIT

What is left when expenses are subtracted from revenue.

BREAK-EVEN POINT

The sales level where revenue exactly equals total cost, so there is no profit and no loss.

MONTHLY PAYMENT

The amount paid each month toward a loan or installment plan.

KEY TERMS

Ordered pair
A point written (x, y), with horizontal coordinate first.

Slope
Change in vertical coordinate divided by change in horizontal coordinate.

Vertical intercept
Where a graph crosses the vertical axis; x = 0.

Domain
The allowed input values of a relationship.

Linear relationship
A relationship with a constant rate of change whose graph lies on a straight line.

FORMULAS & RULES

Slope

$$m = (y_2 - y_1) \div (x_2 - x_1)$$

The two x-values must differ.

Slope-intercept form

$$y = mx + b$$

Find the intercept

```
b = y − mx
```

Worked examples

Example 1 Graph a delivery charge

Use (2, 40) and (6, 60) to find the delivery equation and predict the cost for 8 miles.

SOLUTION

1. Slope: m = (60 − 40) ÷ (6 − 2) = 20 ÷ 4 = 5.
2. Use (2, 40): b = 40 − 5 × 2 = 30.
3. Write C = 5x + 30.
4. Plot (0, 30), (2, 40), and (6, 60) on axes labeled miles and dollars; draw the line for x ≥ 0.
5. At 8 miles: C = 5 × 8 + 30 = 70.

> **Answer:** Slope $5 per mile, intercept $30.00, and an 8-mile charge of $70.00.

Example 2 A decreasing balance

A prepaid balance is $26 at 0 sessions and $14 after 4 sessions. Find the equation and the balance after 6 sessions.

SOLUTION

1. Slope: m = (14 − 26) ÷ (4 − 0) = −12 ÷ 4 = −3.

2. The starting balance is the intercept, b = 26.
3. Write B = −3s + 26.
4. Substitute s = 6: B = −18 + 26 = 8.
5. Plot only whole-number session counts while the balance remains nonnegative.

> **Answer:** B = −3s + 26; after 6 sessions, $8.00 remains.

Practice exercises 23.2

1. Find the line through (0, 10) and (6, 34).
2. Using y = 4x + 10, find the point with x = 3.
3. Find the slope through (0, 17) and (4, 5), and interpret its sign.
4. Find the slope through (2, 12) and (7, 12).
5. A shirt-sales graph uses x for shirts sold. Should you connect every point as a continuous business model?

RECAP QUIZ 23.2

1. For y = 2x + 7, what is y when x = 5?

- ○ a. 12
- ○ b. 14
- ○ c. 17
- ○ d. 35

2. A line passes through (0, 7) and (6, 31). Its slope is:

- ○ a. 4
- ○ b. 6
- ○ c. 7
- ○ d. 24

3. What does b represent in C = mx + b?

- ○ a. The cost per unit
- ○ b. The cost at x = 0
- ○ c. The largest possible cost
- ○ d. The horizontal coordinate

SECTION 23.2 RECAP

- Slope is a rate with units; the intercept is the output at zero input.
- Label axes and graph only inputs that make sense in the situation.

Answer key with explanations: Section 23.2

PRACTICE EXERCISES

1. m = (34 − 10) ÷ 6 = 4. The intercept is 10, so y = 4x + 10.
2. y = 4 × 3 + 10 = 22, so the point is (3, 22).
3. m = (5 − 17) ÷ 4 = −3. The output decreases by 3 units for every 1-unit increase in input.
4. m = (12 − 12) ÷ (7 − 2) = 0 ÷ 5 = 0. The graph is horizontal.
5. No. Sales counts are 0, 1, 2, and so on; a value such as 2.5 shirts is not an allowed input. Plot whole-number points even if they lie on a line.

RECAP QUIZ

1. **17**: 2 × 5 + 7 = 17.
2. **4**: (31 − 7) ÷ (6 − 0) = 24 ÷ 6 = 4.
3. **The cost at x = 0**: Setting x = 0 gives C = b, the vertical intercept.

23.3

Revenue, Cost, Profit, and Break-Even

LEARNING OBJECTIVES

After this section you will be able to:

1. Build linear revenue, cost, and profit models.
2. Find and interpret a break-even quantity.
3. Distinguish a model's prediction from guaranteed sales.

REAL-LIFE SITUATION

A school-market stand

A hypothetical student stand sells handmade items for $12 each. It pays $80 to set up and $8 per item sold. The table compares money coming in with modeled costs.

Items sold	Revenue ($)	Cost ($)	Profit ($)
0	0	80	-80
20	240	240	0
40	480	400	80

Figure 23.3 Hypothetical stand results

The concept

Revenue is money earned from sales before expenses. If every item sells for p dollars, revenue from q items is pq. This model assumes a constant selling price and that q items actually sell.

Total cost combines fixed cost, which does not change over the modeled production range, with variable cost per item. A simple model is C = F + vq.

If items are produced but not sold, production quantity and sales quantity must be modeled separately.

Profit is revenue minus total cost. The difference $p - v$ is the contribution margin per item: the amount available to cover fixed costs and then contribute to profit. It is not the same as final profit per item.

Break-even occurs where revenue equals cost and profit is zero. On a graph, it is the intersection of the revenue and cost lines. When $p > v$, quantities above break-even produce a modeled profit; if the calculated break-even quantity is not whole, round upward to find the first whole-unit quantity with nonnegative profit.

These equations do not guarantee customers or account automatically for waste, refunds, taxes, or capacity limits. State which costs are included and test whether the assumed selling price and per-item cost remain reasonable.

Revenue, Cost, Profit, and Break-Even: solution map

1. Identify the question

   `For the stand in Figure 23.3, find break-even sales and the revenue at that point.`

2. Calculation 1

   `Write R = 12q and C = 80 + 8q.`

3. Calculation 2

   `Set revenue equal to cost: 12q = 80 + 8q.`

4. Calculation 3

   `Subtract 8q: 4q = 80.`

5. Calculation 4

   `Divide by 4: q = 20.`

6. Calculation 5

   `Check: R = 12 × 20 = 240 and C = 80 + 8 × 20 = 240.`

7. Interpret and check

   `Break-even is 20 items, with revenue and cost both $240.00.`

Figure 23.3.V Revenue, Cost, Profit, and Break-Even: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

LOAN
Money borrowed that must be repaid, usually with interest.

DEPOSIT
Money placed into a bank account.

BUDGET
A plan that shows expected income and how it will be spent.

KEY TERMS

Revenue
Sales income before subtracting expenses.

Fixed cost
A cost unchanged by quantity within the modeled range.

Variable cost
A cost that changes with the number of units.

Profit
Revenue minus total cost; a negative result is a loss.

Contribution margin
Selling price minus variable cost per unit.

Break-even
The point at which revenue equals cost.

FORMULAS & RULES

Revenue

$$R = p \times q$$

Total cost

$$C = F + v \times q$$

Profit

$$P = R - C = (p - v) \times q - F$$

Break-even quantity

$$q = F \div (p - v)$$

Requires p > v for a positive break-even quantity when F > 0.

Worked examples

Example 1 Locate break-even

For the stand in Figure 23.3, find break-even sales and the revenue at that point.

SOLUTION

1. Write R = 12q and C = 80 + 8q.
2. Set revenue equal to cost: 12q = 80 + 8q.
3. Subtract 8q: 4q = 80.
4. Divide by 4: q = 20.
5. Check: R = 12 × 20 = 240 and C = 80 + 8 × 20 = 240.

> **Answer:** Break-even is 20 items, with revenue and cost both $240.00.

Example 2 Calculate profit at a sales target

What profit does the stand model predict if it sells 35 items?

SOLUTION

1. Revenue: $R = 12 \times 35 = 420$.
2. Cost: $C = 80 + 8 \times 35 = 80 + 280 = 360$.
3. Profit: $P = 420 - 360 = 60$.
4. Check with contribution margin: $(12 - 8) \times 35 - 80 = 140 - 80 = 60$.

> **Answer:** The modeled profit is $60.00, assuming all 35 items sell and the listed costs apply.

Practice exercises 23.3

1. Find the stand's profit at 40 items.
2. A stand has $R = 9q$ and $C = 30 + 6q$. Find break-even.
3. For $R = 9q$ and $C = 30 + 6q$, find profit at 25 items.
4. Fixed cost is $54, price is $15, and variable cost is $9 per item. Find break-even.
5. Find profit at 5 items if $R = 10q$ and $C = 70 + 6q$.

RECAP QUIZ 23.3

1. For $R = 13q$ and $C = 48 + 7q$, profit at $q = 12$ is:

- ○ a. $24
- ○ b. $72
- ○ c. $108
- ○ d. $156

2. For $R = 13q$ and $C = 48 + 7q$, break-even is:

- ○ a. 6 items
- ○ b. 8 items
- ○ c. 12 items
- ○ d. 48 items

3. If price equals variable cost and fixed cost is positive, what happens?

- ○ a. Every sale covers fixed cost
- ○ b. Profit is always zero
- ○ c. There is no break-even quantity in this model
- ○ d. Revenue is zero

SECTION 23.3 RECAP

- Revenue is not profit: subtract both fixed and variable costs.
- Break-even solves R = C; a sales forecast is not a promise.

Answer key with explanations: Section 23.3

PRACTICE EXERCISES

1. P = (12 − 8) × 40 − 80 = 160 − 80 = $80.00.
2. 9q = 30 + 6q gives 3q = 30, so q = 10. Both revenue and cost equal $90.00.
3. R = 225 and C = 30 + 150 = 180. Profit = 225 − 180 = $45.00.
4. Contribution margin = 15 − 9 = 6. Break-even = 54 ÷ 6 = 9 items.
5. Revenue = 50; cost = 70 + 30 = 100. Profit = 50 − 100 = −$50.00, a $50.00 loss.

RECAP QUIZ

1. **$24**: P = (13 − 7) × 12 − 48 = 72 − 48 = $24.00.
2. **8 items**: q = 48 ÷ (13 − 7) = 48 ÷ 6 = 8.
3. **There is no break-even quantity in this model**: The contribution margin is zero, so P = −F regardless of quantity.

CHAPTER 24

Inequalities, Systems of Equations, and Decision-Making

Budgets create limits, competing plans create comparison points, and limited resources create trade-offs. Inequalities and systems help you describe those situations without guessing. All people, businesses, prices, and earnings are hypothetical; round money to the nearest cent when needed.

24.1

Inequalities and Budget Limits

LEARNING OBJECTIVES

After this section you will be able to:

1. Translate budget and minimum requirements into inequalities.
2. Solve inequalities, reversing the sign when required.
3. Interpret solution sets and whole-number restrictions.

REAL-LIFE SITUATION

Planning a student workshop

A hypothetical workshop has a $20 setup charge and an $8 supply cost per participant. The organizers have at most $100 to spend. The budget permits

a range of participant counts rather than just one number.

Participants q	Cost ($)	Within $100?
0	20	Yes
5	60	Yes
10	100	Yes

Figure 24.1 Hypothetical workshop budget

The concept

An inequality compares quantities that need not be equal. The symbols < and > mean strictly less than and strictly greater than. The symbols ≤ and ≥ include equality. Translate 'at most' as ≤ and 'at least' as ≥.

Solve much as you solve an equation: add or subtract the same quantity on both sides. Multiplying or dividing both sides by a positive number preserves the inequality direction. Multiplying or dividing by a negative number reverses it because the order of numbers reverses.

A solution set contains every value making the original statement true. On a number line, use an open endpoint for < or > and a closed endpoint for ≤ or ≥. Shade toward smaller or larger values as appropriate.

Context can narrow the algebraic solution set. A budget may give q ≤ 13.75, but participant counts must be nonnegative whole numbers. The largest feasible count is therefore 13, and checking the next integer confirms why rounding to 14 would violate the limit.

Inequalities and Budget Limits: solution map

1. Identify the question

   How many participants can attend if setup costs $20, supplies cost $8 each, and the budget is $100?

2. Calculation 1

   Define q as a nonnegative whole-number participant count.

3. Calculation 2

   Write $20 + 8q \le 100$.

4. Calculation 3

   Subtract 20: $8q \le 80$.

5. Calculation 4

   Divide by positive 8: $q \le 10$.

6. Calculation 5

   Check the endpoint: $20 + 8 \times 10 = 100$.

7. Interpret and check

   The feasible counts are 0 through 10 participants.

Figure 24.1.V Inequalities and Budget Limits: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

BUDGET

A plan that shows expected income and how it will be spent.

LOAN TERM

How long a loan lasts before it must be fully repaid.

TOTAL REPAYMENT COST

The full amount paid over a loan's life — principal plus all interest and fees.

KEY TERMS

Inequality
A statement comparing quantities using $<$, $>$, $\leq$, or $\geq$.

Solution set
All values that make a statement true.

Boundary
A value separating included and excluded parts of a solution set.

Constraint
A limit or requirement that a plan must satisfy.

Feasible value
A value satisfying every relevant constraint.

FORMULAS & RULES

Budget constraint

$$F + v \times q \leq B$$

B is the maximum budget.

Positive per-unit cost

$$q \leq (B - F) \div v$$

For v > 0; also apply nonnegative and whole-number restrictions.

Negative multiplication rule

```
If a < b and c < 0, then ac > bc
```

Worked examples

Example 1 Maximum workshop attendance

How many participants can attend if setup costs $20, supplies cost $8 each, and the budget is $100?

SOLUTION

1. Define q as a nonnegative whole-number participant count.
2. Write 20 + 8q ≤ 100.
3. Subtract 20: 8q ≤ 80.
4. Divide by positive 8: q ≤ 10.
5. Check the endpoint: 20 + 8 × 10 = 100.

> **Answer:** The feasible counts are 0 through 10 participants.

Example 2 Reverse the inequality

Solve −3x ≤ −12 and describe the number-line graph.

SOLUTION

1. Divide both sides by −3.

2. Reverse ≤ to ≥ because the divisor is negative.
3. Obtain $x \geq 4$.
4. Check $x = 4$: $-12 \leq -12$ is true; check $x = 5$: $-15 \leq -12$ is true.
5. Use a closed dot at 4 and shade to the right.

Answer: $x \geq 4$.

Practice exercises 24.1

1. Solve $15 + 9q \leq 60$ for nonnegative whole-number q.
2. A project costs $35 plus $12 per kit. With a $200 budget, what is the maximum whole-number kit count?
3. Solve $25 + 15h \geq 100$.
4. Solve $-4x + 10 > 22$.
5. Solve $14 + 6q < 50$ for nonnegative whole-number q.

RECAP QUIZ 24.1

1. Solve $10 + 5x \leq 40$.

- ○ a. $x \leq 6$
- ○ b. $x \geq 6$
- ○ c. $x < 8$
- ○ d. $x \leq 10$

2. When must an inequality sign reverse?

- ○ a. Whenever you subtract
- ○ b. When dividing by a negative number
- ○ c. When adding a positive number
- ○ d. Whenever there is a variable

3. Which phrase means $x \geq 10$?

- ○ a. Fewer than 10
- ○ b. At most 10
- ○ c. At least 10
- ○ d. Exactly 10

SECTION 24.1 RECAP

- Translate the boundary carefully: 'at most' includes equality; 'less than' does not.
- Reverse the sign after negative multiplication or division, then apply context restrictions.

Answer key with explanations: Section 24.1

PRACTICE EXERCISES

1. $9q \leq 45$, so $q \leq 5$. The feasible whole-number counts are 0 through 5.
2. $35 + 12q \leq 200$ gives $q \leq 165 \div 12 = 13.75$. Use at most 13 kits: cost \$191.00; 14 would cost \$203.00.
3. Subtract 25: $15h \geq 75$. Divide by 15: $h \geq 5$.
4. Subtract 10: $-4x > 12$. Divide by -4 and reverse the sign: $x < -3$. Use an open dot at -3 and shade left.
5. $6q < 36$ gives $q < 6$. Since equality is excluded, the allowed counts are 0 through 5.

RECAP QUIZ

1. **$x \leq 6$**: $5x \leq 30$, so $x \leq 6$.
2. **When dividing by a negative number**: Multiplying or dividing by a negative reverses numerical order.
3. **At least 10**: At least includes 10 and every larger allowed value.

24.2

Solving Systems and Comparing Plans

LEARNING OBJECTIVES

After this section you will be able to:

1. Solve two linear equations by substitution or elimination.
2. Interpret an intersection as a shared solution.
3. Recognize systems with no solution or infinitely many solutions.

REAL-LIFE SITUATION

Two hypothetical data plans

Plan A costs $12 plus $3 per data unit. Plan B costs $24 plus $1 per data unit. Comparing both equations identifies when the plans cost the same and which is cheaper on either side.

Data units g	Plan A ($)	Plan B ($)
0	12	24
4	24	28
6	30	30

Figure 24.2 Hypothetical plan charges

The concept

A system asks for values satisfying two or more equations at once. With two variables, a solution is an ordered pair. Graphically, it lies on both lines, so an intersection represents a shared solution.

Substitution replaces a variable with an equal expression from another equation. When both plans are written as a total cost, setting their cost expressions equal is substitution: the same output must equal both expressions.

Elimination adds or subtracts equations to remove a variable. First multiply an equation if necessary to create equal or opposite coefficients. After finding one variable, substitute back to find the other, and check both original equations.

Two distinct nonparallel lines meet once. Parallel distinct lines have no common solution, while two equations describing the same line have infinitely many solutions. An algebraic result such as $0 = 5$ signals inconsistency; a true identity such as $0 = 0$ signals dependence when the original equations describe the same line.

An intersection is not automatically the best choice. Compare prices before and after it and consider restrictions such as capacity, included features, and allowed inputs. A mathematical solution outside the permitted domain may not be usable.

Solving Systems and Comparing Plans: solution map

1. Identify the question

   `At what usage do Plans A and B cost the same?`

2. Calculation 1

   `Set costs equal: 12 + 3g = 24 + g.`

3. Calculation 2

   `Subtract g and 12: 2g = 12.`

4. Calculation 3

   `Divide by 2: g = 6.`

5. Calculation 4

   `Substitute: A = 12 + 3 × 6 = 30 and B = 24 + 6 = 30.`

6. Calculation 5

   `Because A starts lower but rises faster, A is cheaper below 6 units and B is cheaper above 6 units.`

7. Interpret and check

   `The plans meet at (6, 30): 6 data units and $30.00.`

Figure 24.2.V Solving Systems and Comparing Plans: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

DOWN PAYMENT
Money paid upfront when financing a purchase, which reduces the amount borrowed.

MONTHLY PAYMENT
The amount paid each month toward a loan or installment plan.

INTEREST
The cost of borrowing money, or the earnings on money saved or invested.

KEY TERMS

System of equations
Equations required to hold at the same time.

Substitution
Replacing a quantity with an expression equal to it.

Elimination
Combining equations to remove a variable.

Intersection
A point shared by graphs.

Inconsistent system
A system with no solution.

Dependent equations
Equations that describe the same relationship.

FORMULAS & RULES

Equal-cost comparison

$$F_1 + r_1x = F_2 + r_2x$$

Comparison input

$$x = (F_2 - F_1) \div (r_1 - r_2)$$

Requires different rates.

Two-category total

$$x + y = n$$

Worked examples

Example 1 Compare two plans

At what usage do Plans A and B cost the same?

SOLUTION

1. Set costs equal: 12 + 3g = 24 + g.
2. Subtract g and 12: 2g = 12.
3. Divide by 2: g = 6.
4. Substitute: A = 12 + 3 × 6 = 30 and B = 24 + 6 = 30.
5. Because A starts lower but rises faster, A is cheaper below 6 units and B is cheaper above 6 units.

> **Answer:** The plans meet at (6, 30): 6 data units and $30.00.

Example 2 Eliminate a ticket category

A hypothetical event sells 10 tickets: student tickets cost $2 and adult tickets cost $4. Revenue is $28. How many of each sold?

SOLUTION

1. Let s be student tickets and a be adult tickets: s + a = 10 and 2s + 4a = 28.
2. Multiply the count equation by 2: 2s + 2a = 20.
3. Subtract it from the revenue equation: 2a = 8.
4. Divide by 2: a = 4; then s = 10 − 4 = 6.
5. Check: 6 + 4 = 10 and 2 × 6 + 4 × 4 = 28.

> **Answer:** 6 student tickets and 4 adult tickets.

Practice exercises 24.2

1. Find the equal-cost input for A = 8 + 4x and B = 20 + 2x.
2. Solve x + y = 10 and 4x + 7y = 70.
3. Do y = 3x + 4 and y = 3x + 9 intersect?
4. How many solutions do y = 2x + 1 and 2y = 4x + 2 have?
5. Find the intersection of y = 5x + 1 and y = 3x + 5.

RECAP QUIZ 24.2

1. Which point satisfies y = 3x + 4?

- ○ a. (2, 6)
- ○ b. (2, 10)
- ○ c. (4, 10)
- ○ d. (0, 3)

2. A system's solution must satisfy:

- ○ a. Only the first equation
- ○ b. Only the equation with larger numbers

- ○ c. Every equation in the system
- ○ d. Neither equation

3. Different lines with the same slope have:

- ○ a. No intersection
- ○ b. Exactly two intersections
- ○ c. Infinitely many intersections
- ○ d. An intersection at zero

SECTION 24.2 RECAP

- Solve by substitution or elimination, then check both equations.
- An intersection can be a comparison threshold rather than a universally best choice.

Answer key with explanations: Section 24.2

PRACTICE EXERCISES

1. $8 + 4x = 20 + 2x$ gives $2x = 12$, so $x = 6$. Both costs are $32.00.
2. Multiply the first equation by 4: $4x + 4y = 40$. Subtract to get $3y = 30$, so $y = 10$ and $x = 0$. Check: $0 + 70 = 70$.
3. Setting them equal gives $3x + 4 = 3x + 9$, or $4 = 9$. This is false, so the parallel lines have no solution.
4. Divide the second equation by 2 to obtain $y = 2x + 1$. Both describe the same line, so there are infinitely many shared points.
5. $5x + 1 = 3x + 5$ gives $2x = 4$, so $x = 2$. Then $y = 11$; both equations give 11.

RECAP QUIZ

1. **(2, 10)**: At $x = 2$, $y = 3 \times 2 + 4 = 10$.
2. **Every equation in the system**: A shared solution makes all original equations true.
3. **No intersection**: Different intercepts with equal slopes produce parallel lines.

24.3

Feasible Regions and Resource Decisions

LEARNING OBJECTIVES

After this section you will be able to:

1. Represent multiple resource limits with inequalities.
2. Test proposed plans against every constraint.
3. Compare an objective at feasible corner points.

REAL-LIFE SITUATION

A student print-shop decision

A hypothetical student shop makes x poster packs and y card packs. It can prepare at most 8 packs total and has 12 machine-time units; poster packs use 2 units and card packs use 1. Modeled contribution is $15 per poster pack and $10 per card pack.

Poster packs x	Card packs y	Total packs	Machine units	Contribution ($)
0	0	0	0	0
0	6	6	6	60
4	4	8	12	100
6	0	6	12	90

Figure 24.3 Candidate plans and modeled contribution

The concept

A practical plan may have several limits at once. Here $x + y \le 8$ limits total packs, while $2x + y \le 12$ limits machine time. Also $x \ge 0$ and $y \ge 0$, and pack

counts must be whole numbers.

To graph an inequality in two variables, first draw its boundary line. Use a solid line when equality is allowed and a dashed line for a strict inequality. Test a point not on the line to determine which side to shade. The overlap of all permitted regions is the feasible region.

The objective is the quantity you want to maximize or minimize. Here contribution is Z = 15x + 10y. Contribution is not necessarily final profit: any fixed expenses not included in these amounts would still need to be subtracted.

For a linear objective over a nonempty bounded polygonal feasible region, a maximum occurs at a corner; sometimes an entire edge ties. Find corners by intersecting boundary lines and axes, discard infeasible intersections, and evaluate the objective at the remaining corners.

Whole-number restrictions require care. A fractional best corner may be unusable, and rounding it can violate constraints or miss the best integer plan. In this example the best continuous corner already has whole-number coordinates, so it also solves the whole-pack problem.

Feasible Regions and Resource Decisions: solution map

1. Identify the question

   Find the intersection of x + y = 8 and 2x + y = 12, and check feasibility.

2. Calculation 1

   Subtract the first equation from the second: x = 4.

3. Calculation 2

   Substitute into x + y = 8: y = 4.

4. Calculation 3

   Check total packs: 4 + 4 = 8.

5. Calculation 4

   Check machine time: 2 × 4 + 4 = 12.

6. Calculation 5

   Both counts are nonnegative whole numbers, so (4, 4) is feasible.

7. Interpret and check

   The boundaries intersect at 4 poster packs and 4 card packs.

Figure 24.3.V Feasible Regions and Resource Decisions: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

APR

Annual percentage rate — the yearly cost of borrowing, including interest and some fees, shown as a percent.

COLLATERAL

Property a borrower promises to a lender if the loan is not repaid, such as a car.

PROFIT MARGIN

Profit shown as a percent of revenue; a 10% margin means $10 of profit per $100 of sales.

KEY TERMS

Feasible region
All points satisfying every constraint.

Boundary line
The equality line associated with a linear inequality.

Objective function
A formula for the quantity to maximize or minimize.

Corner point
A vertex where boundary segments of a feasible region meet.

Optimal solution
A feasible solution with the best objective value.

FORMULAS & RULES

Pack limit

$$x + y \le 8$$

Machine-time limit

$$2x + y \le 12$$

Contribution objective

```
Z = 15x + 10y
```

Worked examples

Example 1 Find the shared boundary

Find the intersection of $x + y = 8$ and $2x + y = 12$, and check feasibility.

SOLUTION

1. Subtract the first equation from the second: $x = 4$.
2. Substitute into $x + y = 8$: $y = 4$.
3. Check total packs: $4 + 4 = 8$.
4. Check machine time: $2 \times 4 + 4 = 12$.
5. Both counts are nonnegative whole numbers, so $(4, 4)$ is feasible.

> **Answer:** The boundaries intersect at 4 poster packs and 4 card packs.

Example 2 Compare all corners

Find the highest contribution under $x + y \le 8$, $2x + y \le 12$, $x \ge 0$, and $y \ge 0$.

SOLUTION

1. The axis corners are $(0, 0)$, $(0, 8)$, and $(6, 0)$; the boundary intersection is $(4, 4)$.
2. At $(0, 0)$, $Z = 0$.

3. At (0, 8), $Z = 15 \times 0 + 10 \times 8 = 80$.
4. At (6, 0), $Z = 15 \times 6 = 90$.
5. At (4, 4), $Z = 60 + 40 = 100$, the greatest corner value.

> **Answer:** Make 4 poster packs and 4 card packs for a maximum modeled contribution of $100.00.

Practice exercises 24.3

1. Is (3, 5) feasible for the shop?
2. Is (5, 3) feasible?
3. Compare contribution at (3, 5) and (4, 4).
4. For $y = 0$, what is the largest feasible x?
5. A student says (0, 6) is a corner because it is listed in Figure 24.3. Correct the claim.

RECAP QUIZ 24.3

1. Which plan violates machine time?

- a. (4, 4)
- b. (3, 5)
- c. (5, 3)
- d. (6, 0)

2. At (5, 2), how many machine-time units are used?

- a. 7
- b. 10
- c. 12
- d. 14

3. What does 'feasible' mean?

- ○ a. Has the highest revenue
- ○ b. Satisfies every constraint
- ○ c. Uses every resource
- ○ d. Has whole-number revenue

SECTION 24.3 RECAP

- Test every constraint; satisfying one limit is not enough.
- Compare feasible corners for a linear objective, and handle integer restrictions separately.

Answer key with explanations: Section 24.3

PRACTICE EXERCISES

1. Total packs: $3 + 5 = 8 \le 8$. Machine time: $2 \times 3 + 5 = 11 \le 12$. Both counts are nonnegative integers, so yes.
2. Total packs equal 8, but machine time is $2 \times 5 + 3 = 13 > 12$. It is not feasible.
3. At (3, 5), $Z = 45 + 50 = \$95.00$. At (4, 4), $Z = 60 + 40 = \$100.00$, which is $5.00 higher.
4. The constraints give $x \le 8$ and $2x \le 12$, so $x \le 6$. The tighter limit permits 6 poster packs.
5. The point is feasible, but it is not a corner. On the y-axis, the limits give $y \le 8$ and $y \le 12$, so the upper corner is (0, 8). A candidate table need not list only corners.

RECAP QUIZ

1. **(5, 3)**: At (5, 3), $2x + y = 13$, exceeding 12.
2. **12**: $2 \times 5 + 2 = 12$; total packs are 7, so both limits are satisfied.
3. **Satisfies every constraint**: A feasible plan obeys all limits; it need not be optimal or use all resources.

CHAPTER 25

Functions, Quadratic Equations, and Maximum Profit

A function organizes a relationship so that each allowed input has one output. Quadratic functions add curves, allowing you to model changing revenue and locate a maximum under stated assumptions. All businesses, prices, and earnings are hypothetical; money is rounded to the nearest cent when needed, and modeled profit is not guaranteed.

25.1

Functions, Notation, Domain, and Range

LEARNING OBJECTIVES

After this section you will be able to:

1. Determine whether a relationship is a function.
2. Evaluate functions from formulas and tables.
3. Identify meaningful domains and corresponding ranges.

REAL-LIFE SITUATION

A creator workshop fee

A hypothetical workshop charges a $25 booking fee plus $12 per participant. If n participants attend, f(n) gives the total charge. Each allowed participant

count produces one charge.

Participants n	Charge f(n) ($)
0	25
2	49
5	85

Figure 25.1 Values of the hypothetical function f(n) = 25 + 12n

The concept

A function assigns exactly one output to each allowed input. Several inputs may have the same output, but one input cannot have two different outputs in the same function. A table with the same input paired with conflicting outputs does not describe a function.

Function notation names the rule and its input. Read f(4) as 'f of four,' not f multiplied by four. To evaluate it, replace every occurrence of the input variable with 4 and follow the order of operations.

The domain is the collection of permitted inputs; the range is the collection of outputs actually produced. A formula alone does not settle a real-world domain. A workshop with room for 20 participants uses n = 0, 1, 2, ..., 20, not negative values or fractional people.

A graph represents a function when no vertical line intersects it more than once. Each horizontal location then has at most one output. Tables and formulas also help distinguish linear functions, which have constant first differences for equally spaced inputs, from nonlinear functions, whose changes may vary.

Functions, Notation, Domain, and Range: solution map

1. Identify the question

   For f(n) = 25 + 12n, find f(4), then find the participant count producing a charge of $85.

2. Calculation 1

   Substitute 4: f(4) = 25 + 12 × 4.

3. Calculation 2

   Calculate: f(4) = 25 + 48 = 73.

4. Calculation 3

   For an $85 charge, write 25 + 12n = 85.

5. Calculation 4

   Subtract 25 and divide by 12: n = 60 ÷ 12 = 5.

6. Calculation 5

   Check: f(5) = 25 + 60 = 85.

7. Interpret and check

   Four participants cost $73.00; an $85.00 charge corresponds to 5 participants.

Figure 25.1.V Functions, Notation, Domain, and Range: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

PROFIT

What is left when expenses are subtracted from revenue.

PROFIT MARGIN

Profit shown as a percent of revenue; a 10% margin means $10 of profit per $100 of sales.

REVENUE

The total money a business brings in from sales, before any costs are paid.

KEY TERMS

Function
A rule assigning exactly one output to each allowed input.

Function notation
Notation such as f(x), naming a function and its input.

Domain
The allowed input values.

Range
The outputs produced by the allowed inputs.

Vertical-line test
A graph is a function if every vertical line meets it at most once.

FORMULAS & RULES

Workshop charge

$$f(n) = 25 + 12n$$

Evaluation pattern

$$\text{If } f(x) = ax + b, \text{ then } f(k) = a \times k + b$$

A nonlinear rule

$$g(x) = 2x^2 + 5$$

Square the input before multiplying by 2.

Worked examples

Example 1 Evaluate and reverse a function

For $f(n) = 25 + 12n$, find $f(4)$, then find the participant count producing a charge of $85.

SOLUTION

1. Substitute 4: $f(4) = 25 + 12 \times 4$.
2. Calculate: $f(4) = 25 + 48 = 73$.
3. For an $85 charge, write $25 + 12n = 85$.
4. Subtract 25 and divide by 12: $n = 60 \div 12 = 5$.
5. Check: $f(5) = 25 + 60 = 85$.

> **Answer:** Four participants cost $73.00; an $85.00 charge corresponds to 5 participants.

Example 2 Evaluate a nonlinear function

Find $g(3)$ for $g(x) = 2x^2 + 5$.

SOLUTION

1. Replace x with 3: $g(3) = 2 \times 3^2 + 5$.
2. Square first: $3^2 = 9$.

3. Multiply: $2 \times 9 = 18$.

4. Add: $18 + 5 = 23$.

Answer: $g(3) = 23$.

Practice exercises 25.1

1. Find $f(0)$ and $f(3)$ for $f(n) = 25 + 12n$.
2. Solve $f(n) = 61$ for the workshop function.
3. Find $g(4)$ for $g(x) = 2x^2 + 5$.
4. Does $\{(1, 8), (2, 8), (3, 10)\}$ represent a function?
5. For $g(x) = 2x^2 + 5$ with domain $\{0, 2\}$, find the range.

RECAP QUIZ 25.1

1. For $f(n) = 25 + 12n$, $f(2)$ is:

- ○ a. $37
- ○ b. $49
- ○ c. $74
- ○ d. $50

2. For $h(x) = 3x + 1$, $h(2)$ is:

- ○ a. 6
- ○ b. 5
- ○ c. 7
- ○ d. 9

3. Which table fails to represent a function?

- ○ a. (1, 4), (2, 4)
- ○ b. (0, 5), (1, 6)

○ c. (2, 3), (2, 7)

○ d. (−1, 2), (1, 2)

SECTION 25.1 RECAP

- A function gives one output per allowed input; f(x) is notation, not multiplication.
- Use the situation to restrict the domain, then calculate the resulting range.

Answer key with explanations: Section 25.1

PRACTICE EXERCISES

1. f(0) = 25 + 0 = $25.00; f(3) = 25 + 36 = $61.00. The zero-input charge is the booking fee.
2. 25 + 12n = 61 gives 12n = 36, so n = 3 participants.
3. g(4) = 2 × 16 + 5 = 32 + 5 = 37.
4. Yes. Each input appears with exactly one output. Inputs 1 and 2 may share the output 8.
5. g(0) = 5 and g(2) = 2 × 4 + 5 = 13, so the range is {5, 13}.

RECAP QUIZ

1. **$49**: 25 + 12 × 2 = 25 + 24 = $49.00.
2. **7**: 3 × 2 + 1 = 7.
3. **(2, 3), (2, 7)**: Input 2 is assigned two different outputs, 3 and 7.

25.2

Quadratic Equations and Break-Even Roots

LEARNING OBJECTIVES

After this section you will be able to:

1. Recognize quadratic expressions and equations.
2. Solve quadratics by factoring or the quadratic formula.
3. Interpret roots and reject solutions outside the domain.

REAL-LIFE SITUATION

A curved profit model

A hypothetical small batch has modeled profit $P(q) = -q^2 + 10q - 16$ dollars. Unlike a constant-margin model, profit eventually falls as quantity rises. The zero-profit quantities separate modeled losses from gains.

Quantity q	Profit P(q) ($)
0	-16
2	0
5	9
8	0

Figure 25.2 Values of a hypothetical quadratic profit model

The concept

A quadratic has a nonzero squared term and can be written $ax^2 + bx + c$. Its graph is a parabola. Positive a produces an upward-opening curve, while negative a produces a downward-opening curve.

A root is an input making the function zero. To solve a quadratic equation, first move all terms to one side. When an expression factors, the zero-product property says at least one factor must equal zero.

Not every quadratic factors conveniently. The quadratic formula works for any quadratic equation with $a \neq 0$. Substitute signed coefficients carefully, especially when b or c is negative, and keep exact values until the final rounding step.

The discriminant $b^2 - 4ac$ tells how many real roots exist: positive gives two distinct roots, zero gives one repeated root, and negative gives no real roots. A model with no real zero need not be meaningless; it simply does not cross the horizontal axis.

Interpret each root in context. A negative quantity may be algebraically valid but unusable. For a downward-opening profit model with two valid roots, profit is positive between them and negative outside them, within the model's permitted domain.

Quadratic Equations and Break-Even Roots: solution map

1. Identify the question

   Find the zero-profit quantities for $P(q) = -q^2 + 10q - 16$.

2. Calculation 1

   Set profit equal to zero: $-q^2 + 10q - 16 = 0$.

3. Calculation 2

   Multiply by -1: $q^2 - 10q + 16 = 0$.

4. Calculation 3

   Find numbers multiplying to 16 and adding to -10: -2 and -8.

5. Calculation 4

   Factor: $(q - 2)(q - 8) = 0$.

6. Calculation 5

   Set each factor to zero: $q = 2$ or $q = 8$. Substitution gives zero profit at both.

7. Interpret and check

   The model breaks even at 2 and 8 units; its profit is positive for $2 < q < 8$.

Figure 25.2.V Quadratic Equations and Break-Even Roots: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

FIXED COST

A cost that stays the same no matter how many items are sold, like rent.

VARIABLE COST

A cost that rises or falls with the number of items made or sold.

NET PROFIT

The final profit after every cost is paid — sometimes called the bottom line.

KEY TERMS

Quadratic
An expression or function whose highest variable power is 2.

Parabola
The curved graph of a quadratic function.

Root
An input at which a function equals zero.

Factor
An expression multiplied by another to form a product.

Discriminant
The value $b^2 - 4ac$ used to classify a quadratic's real roots.

FORMULAS & RULES

Standard quadratic equation

$$ax^2 + bx + c = 0$$

$a \neq 0$.

Quadratic formula

```
x = (−b ± √(b² − 4ac)) ÷ (2a)
```

Zero-product property

```
If A × B = 0, then A = 0 or B = 0
```

Discriminant

```
D = b² − 4ac
```

Worked examples

Example 1 Factor to find break-even

Find the zero-profit quantities for $P(q) = -q^2 + 10q - 16$.

SOLUTION

1. Set profit equal to zero: $-q^2 + 10q - 16 = 0$.
2. Multiply by −1: $q^2 - 10q + 16 = 0$.
3. Find numbers multiplying to 16 and adding to −10: −2 and −8.
4. Factor: $(q - 2)(q - 8) = 0$.
5. Set each factor to zero: $q = 2$ or $q = 8$. Substitution gives zero profit at both.

> **Answer:** The model breaks even at 2 and 8 units; its profit is positive for $2 < q < 8$.

Example 2 Use the quadratic formula

Solve $q^2 - 10q + 16 = 0$ using the quadratic formula.

SOLUTION

1. Identify $a = 1$, $b = -10$, and $c = 16$.
2. Compute $D = (-10)^2 - 4 \times 1 \times 16 = 100 - 64 = 36$.
3. Substitute: $q = (10 \pm \sqrt{36}) \div 2 = (10 \pm 6) \div 2$.
4. The smaller root is $(10 - 6) \div 2 = 2$.
5. The larger root is $(10 + 6) \div 2 = 8$.

> **Answer:** $q = 2$ or $q = 8$, matching the factored solution.

Practice exercises 25.2

1. Solve $x^2 - 7x + 12 = 0$.
2. Solve $x^2 - 16 = 0$. If x is a length, which root is usable?
3. Solve $2x^2 - 10x + 12 = 0$.
4. How many real roots does $x^2 + 4x + 5 = 0$ have?
5. Verify that $x = 3$ solves $x^2 - 5x + 6 = 0$.

RECAP QUIZ 25.2

1. The solutions of $x^2 - 9 = 0$ are:

- ○ a. Only 3
- ○ b. Only −3
- ○ c. −3 and 3
- ○ d. −9 and 9

2. A positive discriminant means:

- ○ a. No real roots
- ○ b. Two distinct real roots

○ c. One repeated real root

○ d. Every number is a root

3. Which equation is quadratic?

○ a. $4x + 2 = 0$

○ b. $x^2 - 5x + 6 = 0$

○ c. $7 = 7$

○ d. $x^3 + 1 = 0$

SECTION 25.2 RECAP

- Set the expression equal to zero before using factors or the quadratic formula.
- Check roots algebraically and against the situation's domain.

Answer key with explanations: Section 25.2

PRACTICE EXERCISES

1. Factor as $(x - 3)(x - 4) = 0$ because $-3 + (-4) = -7$ and $(-3)(-4) = 12$. Thus $x = 3$ or 4.
2. $(x - 4)(x + 4) = 0$ gives $x = 4$ or -4. A length must be nonnegative, so only 4 is usable.
3. Divide by 2: $x^2 - 5x + 6 = 0$. Factor $(x - 2)(x - 3) = 0$, giving $x = 2$ or 3.
4. $D = 4^2 - 4 \times 1 \times 5 = 16 - 20 = -4$. The negative discriminant means no real roots.
5. Substitute 3: $3^2 - 5 \times 3 + 6 = 9 - 15 + 6 = 0$, so it is a root.

RECAP QUIZ

1. **-3 and 3**: $(x - 3)(x + 3) = 0$ gives both roots.
2. **Two distinct real roots**: A positive square root produces two different values in the quadratic formula.
3. **$x^2 - 5x + 6 = 0$**: Its highest variable power is 2 and its squared-term coefficient is nonzero.

25.3

The Vertex and Maximum Modeled Profit

LEARNING OBJECTIVES

After this section you will be able to:

1. Build a quadratic profit function from price and cost assumptions.
2. Find and evaluate a parabola's vertex.
3. Apply capacity and whole-number restrictions to an optimization result.

REAL-LIFE SITUATION

Pricing a hypothetical community event

A hypothetical Philadelphia student event models ticket price as $p(q) = 30 - 0.20q$ dollars when q tickets sell. Its cost is $200 plus $10 per attendee. The assumed price-demand relationship creates a quadratic profit model, not a guaranteed sales forecast.

Tickets q	Modeled profit ($)
0	-200
10	-20
30	220
50	300
70	220

Figure 25.3 Hypothetical event profit $P(q) = -0.20q^2 + 20q - 200$

The concept

When the selling price depends on quantity, revenue need not be linear. Multiplying $p(q) = 30 - 0.20q$ by q gives $R(q) = 30q - 0.20q^2$. Subtracting $C(q) = 200 + 10q$ produces $P(q) = -0.20q^2 + 20q - 200$.

A parabola's vertex is its turning point. For $f(x) = ax^2 + bx + c$, the vertex input is $-b \div (2a)$. When a is negative, the parabola opens downward and the vertex gives its unrestricted maximum. When a is positive, the vertex is a minimum.

Calculate the output by substituting the vertex input into the original function. In a business problem, also recover the corresponding price, revenue, and cost so the final recommendation is understandable and can be checked.

Optimization must respect the domain. Here nonnegative ticket price requires $q \le 150$, and actual attendance uses whole numbers. Capacity may impose a smaller upper bound. For a continuous quadratic on a closed interval, compare an allowed vertex with the interval endpoints; for whole-number quantities, compare the allowed integers closest to the vertex and any relevant boundaries.

The result is only as dependable as the assumptions. Actual demand, refunds, staffing costs, and capacity may differ. Maximum modeled profit is neither investment advice nor a promise that a business or event will earn that amount.

The Vertex and Maximum Modeled Profit: solution map

1. Identify the question

   For the event, find the quantity, price, and profit at the unrestricted vertex. Assume capacity is at least 50.

2. Calculation 1

   Build profit: $P(q) = q(30 - 0.20q) - (200 + 10q) = -0.20q^2 + 20q - 200$.

3. Calculation 2

   With $a = -0.20$ and $b = 20$, $q_v = -20 \div (2 \times -0.20) = 50$.

4. Calculation 3

   Price: $p(50) = 30 - 0.20 \times 50 = \$20.00$.

5. Calculation 4

   Revenue: $R = 50 \times 20 = \$1{,}000.00$.

6. Calculation 5

   Cost: $C = 200 + 10 \times 50 = \$700.00$; profit $= 1{,}000 - 700 = \$300.00$.

7. Calculation 6

   Since $a < 0$ and 50 is an allowed whole-number quantity, the vertex gives the modeled maximum.

8. Interpret and check

   The model's maximum is $300.00 at 50 tickets priced at $20.00 each.

Figure 25.3.V The Vertex and Maximum Modeled Profit: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

COST
The amount paid to make or buy an item before it is sold.

SELLING PRICE
The price a customer pays for an item or service.

BREAK-EVEN POINT
The sales level where revenue exactly equals total cost, so there is no profit and no loss.

KEY TERMS

Vertex
The turning point of a parabola.

Axis of symmetry
The vertical line through the vertex, dividing a parabola into mirror halves.

Maximum
The greatest output over the allowed domain.

Minimum
The smallest output over the allowed domain.

Price-demand model
An assumed relationship between selling price and quantity sold.

FORMULAS & RULES

Revenue with changing price

$$R(q) = q \times p(q)$$

Profit

$$P(q) = R(q) - C(q)$$

Vertex input

$$x_v = -b \div (2a)$$

For $ax^2 + bx + c$, $a \neq 0$.

Vertex output

$$y_v = a \times x_v^2 + b \times x_v + c$$

Worked examples

Example 1 Find the modeled profit maximum

For the event, find the quantity, price, and profit at the unrestricted vertex. Assume capacity is at least 50.

SOLUTION

1. Build profit: $P(q) = q(30 - 0.20q) - (200 + 10q) = -0.20q^2 + 20q - 200$.
2. With $a = -0.20$ and $b = 20$, $q_v = -20 \div (2 \times -0.20) = 50$.
3. Price: $p(50) = 30 - 0.20 \times 50 = \$20.00$.
4. Revenue: $R = 50 \times 20 = \$1{,}000.00$.
5. Cost: $C = 200 + 10 \times 50 = \$700.00$; profit $= 1{,}000 - 700 = \$300.00$.
6. Since $a < 0$ and 50 is an allowed whole-number quantity, the vertex gives the modeled maximum.

Answer: The model's maximum is $300.00 at 50 tickets priced at $20.00 each.

Example 2 Check symmetry and a capacity limit

Compare profit at 40 and 60 tickets. If capacity is only 40, what quantity maximizes modeled profit?

SOLUTION

1. At 40: $P(40) = -0.20 \times 1{,}600 + 800 - 200 = \$280.00$.
2. At 60: $P(60) = -0.20 \times 3{,}600 + 1{,}200 - 200 = \$280.00$.
3. These quantities are equally far from the vertex $q = 50$, so their outputs match.
4. With capacity 40, the allowed domain is whole-number q from 0 through 40.
5. The quadratic increases up to $q = 50$, so its greatest allowed value occurs at $q = 40$.

Answer: Both quantities produce $280.00 in the model; with capacity 40, choose 40 tickets for the maximum feasible modeled profit.

Practice exercises 25.3

1. Find the vertex and maximum of $P(q) = -q^2 + 12q - 20$ for $q \geq 0$.
2. Find the maximum of $P(q) = -2q^2 + 24q - 40$ for $q \geq 0$.
3. Does $f(x) = x^2 + 8x + 3$ have an unrestricted maximum or minimum?
4. For the event model, calculate $P(41)$. Why is it unusable if capacity is 40?
5. For $P(q) = -q^2 + 7q$ and whole-number $q \geq 0$, find the maximizing quantities.

RECAP QUIZ 25.3

1. For $P(q) = -q^2 + 10q$, the vertex quantity is:

- a. −5
- b. 5
- c. 10
- d. 20

2. What is the maximum value of $P(q) = -q^2 + 10q$ for $q \geq 0$?

- a. 5
- b. 10
- c. 25
- d. 50

3. A vertex quantity exceeds capacity. What should you do?

- a. Ignore capacity
- b. Always round the vertex upward
- c. Optimize over the allowed domain
- d. Assume the model guarantees sales

SECTION 25.3 RECAP

- For a downward-opening quadratic, an allowed vertex gives the maximum.
- Check price, capacity, whole-number quantities, and modeling assumptions before interpreting the result.

Answer key with explanations: Section 25.3

PRACTICE EXERCISES

1. $q_v = -12 \div (-2) = 6$. $P(6) = -36 + 72 - 20 = 16$. Since $a < 0$, the maximum is 16 at $q = 6$.
2. $q_v = -24 \div (-4) = 6$. $P(6) = -72 + 144 - 40 = 32$, the maximum.

3. Since $a = 1 > 0$, it has a minimum. The vertex input is $-8 \div 2 = -4$; $f(-4) = 16 - 32 + 3 = -13$. There is no unrestricted maximum.
4. $P(41) = -0.20 \times 1{,}681 + 820 - 200 = -336.20 + 620 = \$283.80$. It exceeds $P(40)$, but 41 attendees violate the capacity constraint.
5. The continuous vertex is $q = -7 \div (-2) = 3.5$. Check adjacent integers: $P(3) = -9 + 21 = 12$ and $P(4) = -16 + 28 = 12$. Both 3 and 4 maximize the whole-number model.

RECAP QUIZ

1. **5**: $q_v = -10 \div (2 \times -1) = 5$.
2. **25**: At $q = 5$, $P = -25 + 50 = 25$; the downward-opening parabola has its maximum there.
3. **Optimize over the allowed domain**: An infeasible vertex cannot be chosen; evaluate the objective within the actual constraints.

CHAPTER 26

Exponential Growth, Logarithms, and Financial Modeling

Repeated percent changes power many financial models, but assumptions matter as much as formulas. Use hypothetical business and consumer situations to calculate growth, reverse it with logarithms, and compare borrowing costs without treating forecasts as promises.

26.1

Exponential Growth and Decay

LEARNING OBJECTIVES

After this section you will be able to:

1. Distinguish repeated addition from repeated multiplication.
2. Calculate growth and decay with a constant percent rate.
3. Interpret model predictions and their limits.

REAL-LIFE SITUATION

Exponential Growth and Decay in practice

A hypothetical Philadelphia repair club tracks a $500 equipment reserve and a $1,000 machine. Rates are modeling assumptions, not promised

returns. Round final money amounts to cents; keep intermediate powers unrounded.

Period	Reserve at 10% growth	Machine at 20% decay
0	500	1000
1	550	800
2	605	640

Figure 26.1 Exponential Growth and Decay (hypothetical data)

The concept

Linear change adds the same amount each period. Exponential change multiplies by the same factor. Adding $50 to $500 produces $550 and then $600; growing by 10% produces $550 and then $605 because the second increase uses a larger base.

A percent rate must become a decimal before entering the model. A 10% increase gives a factor of 1.10, while a 20% decrease gives 0.80. The exponent counts complete periods; match the rate period to the time unit.

For decay, the remaining amount is multiplied again, not reduced by the original dollar loss each time. Under a 20% annual decline, a $1,000 machine loses $200 in year one but $160 in year two. This model estimates resale value; it is not an accounting or tax rule.

A constant-rate model is a conditional forecast. Real sales, equipment values, and account rates change. State the starting value, period, and assumption, and do not round every intermediate result unless the actual account credits rounded amounts each period.

Exponential Growth and Decay: solution map

1. Identify the question
   `Model $500 growing by 10% per year for 3 years.`
2. Calculation 1
   `Convert 10% to 0.10.`
3. Calculation 2
   `Factor = 1 + 0.10 = 1.10.`
4. Calculation 3
   `Compute 1.10³ = 1.331.`
5. Calculation 4
   `Multiply $500 × 1.331 = $665.50.`
6. Interpret and check
   `The modeled balance is $665.50.`

Figure 26.1.V Exponential Growth and Decay: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

COMPOUND INTEREST

Interest earned on both the original amount and the interest already earned.

FUTURE VALUE

What an amount of money today will be worth at a later date with interest.

PRESENT VALUE

What a future amount of money is worth today at a given interest rate.

KEY TERMS

Exponential model
A model with a constant multiplicative change per period.

Growth factor
One plus the decimal growth rate.

Decay factor
One minus the decimal decline rate.

Initial value
The quantity at period zero.

Exponent
The number of repeated factor multiplications.

FORMULAS & RULES

Growth

$$A = P(1 + r)^n$$

Decay

$$A = P(1 - d)^n$$

Linear comparison

```
A = P + kn
```

Worked examples

Example 1 A growing reserve

Model $500 growing by 10% per year for 3 years.

SOLUTION

1. Convert 10% to 0.10.
2. Factor = 1 + 0.10 = 1.10.
3. Compute $1.10^{3} = 1.331$.
4. Multiply $500 × 1.331 = $665.50.

> **Answer:** The modeled balance is $665.50.

Example 2 Equipment resale

Estimate a $1,000 machine after 3 years of 20% annual decay.

SOLUTION

1. Convert 20% to 0.20.
2. Remaining factor = 1 − 0.20 = 0.80.
3. Compute $0.80^{3} = 0.512$.
4. Multiply $1,000 × 0.512 = $512.00.

> **Answer:** Estimated value: $512.00, not a guaranteed sale price.

Practice exercises 26.1

1. Grow $200 by 5% for 2 years.
2. Reduce $800 by 25% each year for 2 years.
3. Compare adding $20 yearly to $200 with growing $200 by 10% yearly, after 2 years.
4. A subscriber count goes 100, 120, 144. Identify the constant factor and rate.
5. A model predicts 400 orders after two years. Is that a guarantee?

RECAP QUIZ 26.1

1. Which is a 15% decline factor?

- ○ a. 1.15
- ○ b. 0.15
- ○ c. 0.85
- ○ d. −0.15

2. What is $100 after two 10% increases?

- ○ a. $120
- ○ b. $121
- ○ c. $110
- ○ d. $200

3. Which sequence is exponential?

- ○ a. 10, 20, 30
- ○ b. 8, 12, 16
- ○ c. 5, 10, 15
- ○ d. 3, 6, 12

SECTION 26.1 RECAP

- Use a multiplicative factor for a constant percent change.
- Match time units and state assumptions; round money at the end.

Answer key with explanations: Section 26.1

PRACTICE EXERCISES

1. $200 \times 1.05^2 = 220.50$, so the result is $220.50.
2. Factor = 0.75; $800 \times 0.75^2 = \$450.00$.
3. Linear: $200 + 2 \times 20 = \$240.00$. Exponential: $200 \times 1.10^2 = \$242.00$, $2.00 more.
4. $120 \div 100 = 1.2$ and $144 \div 120 = 1.2$; rate = $1.2 - 1 = 20\%$ per period.
5. No. Repeated multiplication assumes a stable rate; changing demand can change actual orders. Treat 400 as a conditional prediction.

RECAP QUIZ

1. **0.85**: $1 - 0.15 = 0.85$.
2. **$121**: $100 \times 1.10^2 = \$121.00$.
3. **3, 6, 12**: $6 \div 3 = 12 \div 6 = 2$, a constant factor.

26.2

Logarithms and Time to a Target

LEARNING OBJECTIVES

After this section you will be able to:

1. Interpret a logarithm as an exponent.

2. Solve for time in a positive exponential model.
3. Round a threshold time to a feasible whole period.

REAL-LIFE SITUATION

Logarithms and Time to a Target in practice

A hypothetical student business models a $400 reserve growing at 10% annually until it reaches $600. This is a math scenario, not investment advice; money is rounded to cents and times to two decimals unless a whole period is required.

Year	Modeled reserve
0	$400.00
4	$585.64
5	$644.20

Figure 26.2 Logarithms and Time to a Target (hypothetical data)

The concept

An exponent answers how much results after a known number of periods. A logarithm reverses that question: what exponent produces a specified amount? For example, log base 2 of 8 equals 3 because $2^3 = 8$.

To solve $A = P \times b^n$, first divide by P. Then take the same logarithm on both sides. The power rule brings the exponent down, giving $n = \log(A \div P) \div \log(b)$. Common logarithms and natural logarithms give the same ratio.

The logarithm inputs must be positive, and the base must be positive and different from 1. A zero growth rate gives $b = 1$, so division by $\log(1)$ would divide by zero. With no growth, a larger target is never reached by this model.

A decimal answer describes an interpolated model time. If crediting or checking happens only at year-end, round up for the first year at or above a

growing target and verify the surrounding years. Decay thresholds also require checking the stated inequality; never round a deadline down automatically.

Logarithms and Time to a Target: solution map

1. Identify the question
   `When does $400 at modeled 10% annual growth first reach $600 at a year-end?`
2. Calculation 1
   `Divide: 600 ÷ 400 = 1.5.`
3. Calculation 2
   `Solve n = ln(1.5) ÷ ln(1.10) ≈ 4.25 years.`
4. Calculation 3
   `Round up to year 5 for year-end checks.`
5. Calculation 4
   `Verify: year 4 is $585.64; year 5 is $644.20.`
6. Interpret and check
   `Year 5 is the first qualifying year-end.`

Figure 26.2.V Logarithms and Time to a Target: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

INFLATION

A general rise in prices over time, which makes money buy less.

INVESTMENT

Money put into something with the hope it will grow or earn income later.

ANNUITY

A series of equal payments made at regular intervals, such as monthly.

KEY TERMS

Logarithm
The exponent needed to obtain a positive number from a valid base.

Natural logarithm
A logarithm with base e, available as ln on calculators.

Threshold
A target level that must be reached or crossed.

Discrete period
A separate counting interval such as a full year.

Power rule
The rule $\log(b^n) = n \times \log(b)$ for positive b.

FORMULAS & RULES

Inverse relationship

```
log base b of x = n means bⁿ = x
```

Time to target

```
n = ln(A ÷ P) ÷ ln(b)
```

Doubling time

```
n = ln(2) ÷ ln(1 + r)
```

Worked examples

Example 1 Reserve target

When does $400 at modeled 10% annual growth first reach $600 at a year-end?

SOLUTION

1. Divide: 600 ÷ 400 = 1.5.
2. Solve n = ln(1.5) ÷ ln(1.10) ≈ 4.25 years.
3. Round up to year 5 for year-end checks.
4. Verify: year 4 is $585.64; year 5 is $644.20.

> **Answer:** Year 5 is the first qualifying year-end.

Example 2 A shrinking inventory

A supply of 1,600 units halves weekly. When will 200 units remain?

SOLUTION

1. Set $200 = 1{,}600 \times 0.5^{n}$.
2. Divide: 200 ÷ 1,600 = 0.125.
3. Compute n = ln(0.125) ÷ ln(0.5) = 3.
4. Check: $1{,}600 \times 0.5^{3} = 200$.

> **Answer:** After 3 weeks.

Practice exercises 26.2

1. Find log base 3 of 81.
2. A count grows from 50 to 400 by doubling each week. Find the time.
3. Find doubling time at 5% growth per year, to two decimals.
4. A $1,000 reserve grows 20% annually. First year-end at or above $1,700?
5. Can $300 at 0% growth reach $450 without deposits?

RECAP QUIZ 26.2

1. log base 10 of 1,000 equals what?

- ○ a. 2
- ○ b. 3
- ○ c. 10
- ○ d. 100

2. A target is crossed at model time 6.2 years. First qualifying year-end?

- ○ a. 6
- ○ b. 6.2
- ○ c. 7
- ○ d. 5

3. Which logarithm input is valid in real-number math?

- ○ a. 0
- ○ b. −4
- ○ c. −1
- ○ d. 0.5

SECTION 26.2 RECAP

- Logarithms solve for unknown exponents.

- Verify whole-period thresholds in the original model.

Answer key with explanations: Section 26.2

PRACTICE EXERCISES

1. $3^4 = 81$, so the logarithm is 4.
2. $400 \div 50 = 8 = 2^3$, so 3 weeks.
3. $\ln(2) \div \ln(1.05) \approx 14.21$ years in the interpolated model.
4. $\ln(1.7) \div \ln(1.2) \approx 2.91$. Year 2: \$1,440.00; year 3: \$1,728.00. Choose year 3.
5. No: $300 \times 1^n = 300$ in every period. The logarithm time formula has denominator $\ln(1) = 0$ and cannot be used.

RECAP QUIZ

1. **3**: $10^3 = 1{,}000$.
2. **7**: Year 6 is too early; use year 7 and verify.
3. **0.5**: 0.5 is positive; real logarithms require positive inputs.

26.3

Compounding, Loans, and Fair Comparisons

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate compound balances with different crediting periods.
2. Calculate a fixed-payment loan and total borrowing cost.
3. Compare alternatives using consistent assumptions.

REAL-LIFE SITUATION

Compounding, Loans, and Fair Comparisons in practice

A hypothetical Philadelphia student compares a savings model and a fixed-payment equipment loan. All quoted rates are invented, not offers. Ignore fees and taxes unless stated; use unrounded formula values and round money to cents at the end.

Model	Principal	Annual rate	Schedule
Savings	$1,000	12% nominal	Monthly compounding
Loan	$1,000	12% nominal	12 monthly payments

Figure 26.3 Compounding, Loans, and Fair Comparisons (hypothetical data)

The concept

A nominal annual rate does not by itself tell you how often interest is credited. Divide that rate by the number of periods per year, then multiply the years by the same number. Monthly compounding at a nominal 12% uses 1% per month, not 12% per month.

The effective annual rate describes the actual one-year growth factor minus 1 under the stated compounding rule. It allows a cleaner comparison of savings models with different crediting schedules, assuming no fees or withdrawals. It is not a forecast of variable investment returns.

A fixed-payment installment loan charges interest on the unpaid balance. Each payment covers that period's interest first, then reduces principal. For a positive periodic rate, the payment formula balances the present value of all end-of-period payments against the borrowed amount.

Total scheduled payments minus principal gives borrowing cost when there are no fees. A rounded scheduled payment may require an adjusted final payment, so formula totals are estimates until an actual repayment schedule is supplied. Compare total cash outflow, term, fees, and ownership consequences, not just the smallest monthly payment.

Compounding, Loans, and Fair Comparisons: solution map

1. Identify the question

   `Find $1,000 after one year at 12% nominal, compounded monthly.`

2. Calculation 1

   `Monthly rate = 0.12 ÷ 12 = 0.01.`

3. Calculation 2

   `Number of periods = 12 × 1 = 12.`

4. Calculation 3

   $A = 1{,}000 \times 1.01^{12} \approx \$1{,}126.83.$

5. Calculation 4

   $\text{EAR} = (1.01^{12} - 1) \times 100\% \approx 12.68\%.$

6. Interpret and check

   `Balance $1,126.83; effective annual rate about 12.68%.`

Figure 26.3.V Compounding, Loans, and Fair Comparisons: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

YIELD

The income an investment pays, shown as a percent of its price.

RETURN

The money or gain an investment produces, often shown as a percent.

RISK

The chance that an investment or decision will turn out worse than expected.

KEY TERMS

Nominal annual rate
An annual quoted rate before the effect of within-year compounding.

Effective annual rate
The equivalent one-year growth rate after compounding.

Principal
The original borrowed or deposited amount, or unpaid loan balance in context.

Amortization
Reduction of a loan balance through scheduled payments.

Periodic rate
The rate applicable to one payment or crediting period.

FORMULAS & RULES

Compound balance

```
A = P(1 + r ÷ m)^(mt)
```

Effective annual rate

```
EAR = (1 + r ÷ m)^m − 1
```

Loan payment

```
M = P × i ÷ [1 − (1 + i)^(−n)]
```

Borrowing cost without fees

```
Cost = total payments − P
```

Worked examples

Example 1 Monthly compounding

Find $1,000 after one year at 12% nominal, compounded monthly.

SOLUTION

1. Monthly rate = 0.12 ÷ 12 = 0.01.
2. Number of periods = 12 × 1 = 12.
3. $A = 1{,}000 \times 1.01^{12} \approx \$1{,}126.83$.
4. $EAR = (1.01^{12} - 1) \times 100\% \approx 12.68\%$.

> **Answer:** Balance $1,126.83; effective annual rate about 12.68%.

Example 2 Equipment loan

Estimate payments and interest on $1,000 at 12% nominal, paid monthly over one year, with no fees.

SOLUTION

1. i = 0.12 ÷ 12 = 0.01; n = 12.
2. M = 1,000 × 0.01 ÷ [1 − 1.01^(−12)] ≈ $88.85.
3. Using the unrounded formula payment, total ≈ $1,066.19.
4. Subtract principal: $1,066.19 − $1,000.00 = $66.19.

Answer: About $88.85 monthly and $66.19 interest; the final payment may be adjusted for rounding.

Practice exercises 26.3

1. Find $500 after 2 years at 4% compounded annually.
2. Find the effective annual rate for 8% nominal compounded semiannually.
3. A $600 zero-interest loan has 12 equal monthly payments and no fees. Find each payment.
4. A loan advances $900 and requires 10 payments of $96, with no other fees. Find borrowing cost.
5. On a loan balance of $1,000, the monthly rate is 1% and payment is $90. Find first-month interest and ending balance.

RECAP QUIZ 26.3

1. 12% nominal compounded monthly means what monthly rate?

- ○ a. 12%
- ○ b. 1%
- ○ c. 0.12%
- ○ d. 6%

2. A no-fee $500 loan is repaid with $560 total. Cost?

- ○ a. $1,060
- ○ b. $500
- ○ c. $60
- ○ d. $56

3. What should accompany a monthly-payment comparison?

- ○ a. Only the lender name
- ○ b. Only the payment date

○ c. Only the color of the form

○ d. Term, fees, and total cash paid

SECTION 26.3 RECAP

- Match the periodic rate to the number of compounding or payment periods.
- Use total costs and stated assumptions, not a monthly payment alone.

Answer key with explanations: Section 26.3

PRACTICE EXERCISES

1. $500 \times 1.04^2 = \$540.80$.
2. Periodic rate = $0.08 \div 2 = 0.04$; $1.04^2 - 1 = 0.0816 = 8.16\%$.
3. At zero interest use principal ÷ count, not the positive-rate formula: $600 \div 12 = \$50.00$.
4. Total = $10 \times 96 = \$960.00$; cost = $960 - 900 = \$60.00$.
5. Interest = $1{,}000 \times 0.01 = \$10.00$. Principal repaid = $90 - 10 = \$80.00$. Ending balance = $\$920.00$.

RECAP QUIZ

1. **1%**: $12\% \div 12 = 1\%$.
2. **$60**: $560 - 500 = \$60.00$.
3. **Term, fees, and total cash paid**: A lower payment can hide a longer term and higher total cost.

CHAPTER 27

Ratios, Geometry, Measurement, and Business Applications

A business buys space, materials, and quantities as well as products. Use ratios and measurement to turn hypothetical plans into workable estimates, with clear units and sensible rounding.

27.1

Ratios, Rates, and Scale Drawings

LEARNING OBJECTIVES

After this section you will be able to:

1. Simplify ratios while preserving order.
2. Use unit rates and proportions in business decisions.
3. Translate between drawing lengths and actual lengths.

REAL-LIFE SITUATION

Ratios, Rates, and Scale Drawings in practice

A hypothetical food truck near a Philadelphia recreation center plans a stand layout and a drink batch. All quantities and prices are invented. Round final money to cents and preserve units throughout.

Drawing length (cm)	Actual length (m)
1	2
3	6
4	8

Figure 27.1 Ratios, Rates, and Scale Drawings (hypothetical data)

The concept

A ratio compares two quantities in a stated order. A syrup-to-water ratio of 2:5 means two parts syrup for every five parts water, not two parts syrup out of five total. There are seven total parts. Labeling the quantities prevents a surprisingly common mixing error.

A rate compares quantities with different units, such as dollars per kilogram or meals per hour. Divide to find a unit rate, then compare like units. A lower package price can still have a higher cost per unit if the package is smaller.

A proportion states that two ratios are equal. Multiplying or dividing both quantities by the same positive factor preserves a mixture or scale. Cross products provide another method, but units should still tell you what the unknown represents.

A scale drawing replaces actual distances with proportional drawing distances. If 1 cm represents 2 m, convert each measured length using that relation. Area changes with the square of the length factor, so a drawing-area conversion cannot use the length multiplier just once.

Ratios, Rates, and Scale Drawings: solution map

1. Identify the question
   `Make 21 liters of drink using syrup:water = 2:5.`
2. Calculation 1
   `Total parts = 2 + 5 = 7.`
3. Calculation 2
   `Each part = 21 ÷ 7 = 3 liters.`
4. Calculation 3
   `Syrup = 2 × 3 = 6 liters.`
5. Calculation 4
   `Water = 5 × 3 = 15 liters; 6 + 15 = 21.`
6. Interpret and check
   `Use 6 liters syrup and 15 liters water.`

Figure 27.1.V Ratios, Rates, and Scale Drawings: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

COST

The amount paid to make or buy an item before it is sold.

MARKUP

The amount added to a cost to set a selling price; the markup rate uses cost as its base.

PROFIT MARGIN

Profit shown as a percent of revenue; a 10% margin means $10 of profit per $100 of sales.

KEY TERMS

Ratio
An ordered comparison of two quantities by division.

Unit rate
A rate with a denominator of one unit.

Proportion
An equation asserting that two ratios are equal.

Scale
A fixed relationship between drawing and actual measurements.

Part-to-whole ratio
A comparison of a component with the entire amount.

FORMULAS & RULES

Unit rate

```
Unit rate = total quantity ÷ number of units
```

Proportion

```
a ÷ b = c ÷ d means a × d = b × c
```

Scale conversion

```
Actual length = drawing length × actual units per
```

```
drawing unit
```

Worked examples

Example 1 A drink batch

Make 21 liters of drink using syrup:water = 2:5.

SOLUTION

1. Total parts = 2 + 5 = 7.
2. Each part = 21 ÷ 7 = 3 liters.
3. Syrup = 2 × 3 = 6 liters.
4. Water = 5 × 3 = 15 liters; 6 + 15 = 21.

> **Answer:** Use 6 liters syrup and 15 liters water.

Example 2 A booth plan

A rectangular booth is 3 cm by 4 cm on a drawing with 1 cm representing 2 m. Find actual dimensions and area.

SOLUTION

1. Actual short side = 3 × 2 = 6 m.
2. Actual long side = 4 × 2 = 8 m.
3. Actual area = 6 × 8 = 48 m^2.
4. Drawing area = 12 cm^2; area multiplier = 2^2 = 4 m^2 per cm^2.

Answer: The booth is 6 m by 8 m, with area 48 m².

Practice exercises 27.1

1. Simplify a 12:18 ratio of cups to lids.
2. Compare 3 kg for $12 with 5 kg for $18.
3. A recipe uses 4 cups flour for 10 servings. How much for 25 servings?
4. At 1 cm to 3 m, what actual length does 7 cm represent?
5. A 1:4 concentrate-to-water mixture totals 15 liters. Find concentrate.

RECAP QUIZ 27.1

1. Simplify 8:12.

- ○ a. 3:2
- ○ b. 2:3
- ○ c. 4:3
- ○ d. 1:4

2. Cost per notebook for 6 costing $15?

- ○ a. $9.00
- ○ b. $90.00
- ○ c. $2.50
- ○ d. $0.40

3. A length scale factor of 3 gives what area factor?

- ○ a. 3
- ○ b. 6
- ○ c. 1
- ○ d. 9

SECTION 27.1 RECAP

- Keep ratio order and units explicit.
- Scale lengths once and areas by the square of the length factor.

Answer key with explanations: Section 27.1

PRACTICE EXERCISES

1. Divide both terms by 6: 12:18 = 2:3; order remains cups:lids.
2. 12 ÷ 3 = $4.00/kg; 18 ÷ 5 = $3.60/kg. The 5 kg package costs $0.40 less per kg.
3. Scale factor = 25 ÷ 10 = 2.5; flour = 4 × 2.5 = 10 cups.
4. 7 × 3 = 21 m.
5. There are 1 + 4 = 5 parts. Each is 15 ÷ 5 = 3 liters, so concentrate is 3 liters, not 15 ÷ 4.

RECAP QUIZ

1. **2:3**: Divide both terms by 4.
2. **$2.50**: 15 ÷ 6 = $2.50 per notebook.
3. **9**: Both length and width multiply by 3, so area multiplies by $3^2 = 9$.

27.2

Area, Perimeter, and Material Costs

LEARNING OBJECTIVES

After this section you will be able to:

1. Distinguish boundary length from surface area.

2. Calculate rectangular, triangular, and circular measurements.
3. Estimate material purchases with waste and package rounding.

REAL-LIFE SITUATION

Area, Perimeter, and Material Costs in practice

A hypothetical student vendor needs floor covering and boundary tape for a 6 m by 4 m rectangular booth. Prices exclude tax; no current tax rules are implied. Round money to cents and round package counts upward.

Quantity	Amount
Length	6 m
Width	4 m
Area	24 m²
Perimeter	20 m

Figure 27.2 Area, Perimeter, and Material Costs (hypothetical data)

The concept

Perimeter measures distance around a boundary, so it uses length units. Area measures the surface enclosed, so it uses square units. A booth may need tape around its edge and mats across its floor; those are different purchasing problems.

Rectangle area is length times width, while its perimeter counts two lengths and two widths. A triangle occupies half the area of a rectangle with the same base and perpendicular height. The slanted side is not the height unless it is perpendicular to the base.

For circles, radius runs from center to edge and diameter is twice the radius. Circumference measures the curved boundary; area uses radius squared. Use the calculator value of π unless a problem specifies an approximation.

An estimate should allow for cuts, breakage, and packaging. Add a stated waste allowance to the required area, divide by coverage per package, then round up to whole packages. Waste assumptions are planning choices, not universal construction standards; also account for openings and overlap when relevant.

Area, Perimeter, and Material Costs: solution map

1. Identify the question
   `Cover a 6 m by 4 m booth with 10% waste. Each pack covers 5 m² and costs $18.`
2. Calculation 1
   `Area = 6 × 4 = 24 m².`
3. Calculation 2
   `Required material = 24 × 1.10 = 26.4 m².`
4. Calculation 3
   `Packs = 26.4 ÷ 5 = 5.28; buy 6.`
5. Calculation 4
   `Cost = 6 × $18 = $108.00.`
6. Interpret and check
   `Buy 6 packs for $108.00 before any tax.`

Figure 27.2.V Area, Perimeter, and Material Costs: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

INVENTORY
The stock of goods a business holds ready to sell.

OVERHEAD
Ongoing business costs such as rent, utilities, and insurance that do not change with each sale.

BUDGET
A plan that shows expected income and how it will be spent.

KEY TERMS

Perimeter
Total distance around a polygon.

Area
The amount of enclosed surface in square units.

Radius
Distance from the center of a circle to its edge.

Perpendicular height
A height meeting its base at a right angle.

Waste allowance
Extra material budgeted for unusable cuts or damage.

FORMULAS & RULES

Rectangle

```
A = l × w; P = 2(l + w)
```

Triangle

```
A = b × h ÷ 2
```

Circle

```
A = πr²; C = 2πr
```

Material planning

```
Packages = round up [area × (1 + waste rate) ÷ coverage per package]
```

Worked examples

Example 1 Covering the booth

Cover a 6 m by 4 m booth with 10% waste. Each pack covers 5 m² and costs $18.

SOLUTION

1. Area = 6 × 4 = 24 m².
2. Required material = 24 × 1.10 = 26.4 m².
3. Packs = 26.4 ÷ 5 = 5.28; buy 6.
4. Cost = 6 × $18 = $108.00.

> **Answer:** Buy 6 packs for $108.00 before any tax.

Example 2 A round sign

Find the area and edge length of a circular sign of radius 0.5 m.

SOLUTION

1. Diameter = $2 \times 0.5 = 1$ m.
2. Area = $\pi \times 0.5^2 = 0.25\pi \approx 0.7854$ m^2.
3. Circumference = $2\pi \times 0.5 = \pi \approx 3.1416$ m.
4. Round measurements to two decimals: 0.79 m^2 and 3.14 m.

Answer: Area about 0.79 m^2; edge about 3.14 m.

Practice exercises 27.2

1. Find area and perimeter of a 9 m by 3 m rectangle.
2. Find triangle area with base 8 cm and perpendicular height 5 cm.
3. How many 4 m^2 packs cover 30 m^2 with no waste allowance?
4. Tape costs $1.50 per meter. Tape all edges of a 6 m by 4 m booth.
5. A square side doubles from 2 m to 4 m. Compare areas.

RECAP QUIZ 27.2

1. Which units measure floor area?

- ○ a. Meters
- ○ b. Square meters
- ○ c. Liters
- ○ d. Dollars per meter

2. Area of a triangle with base 6 and height 4?

- ○ a. 24
- ○ b. 20
- ○ c. 12
- ○ d. 10

3. Need 10.1 packages; how many must be bought?

- ○ a. 10
- ○ b. 10.1
- ○ c. 9
- ○ d. 11

SECTION 27.2 RECAP

- Match the formula and units to the material being purchased.
- Apply waste before rounding package counts upward.

Answer key with explanations: Section 27.2

PRACTICE EXERCISES

1. Area = $9 \times 3 = 27$ m²; perimeter = $2(9 + 3) = 24$ m.
2. $8 \times 5 \div 2 = 20$ cm².
3. $30 \div 4 = 7.5$ packs; round up to 8 packs, covering 32 m².
4. Perimeter = $2(6 + 4) = 20$ m; cost = $20 \times 1.50 = \$30.00$.
5. Original area = $2^2 = 4$ m²; new area = $4^2 = 16$ m². Ratio = $16 \div 4 = 4$, not 2.

RECAP QUIZ

1. **Square meters**: Area counts square units.
2. **12**: $6 \times 4 \div 2 = 12$ square units.
3. **11**: Whole packages require rounding up to 11.

27.3

Volume, Conversions, and Capacity

LEARNING OBJECTIVES

After this section you will be able to:

1. Calculate box and cylinder volumes.
2. Convert units before combining measurements.
3. Use capacity and density to plan business quantities.

REAL-LIFE SITUATION

Volume, Conversions, and Capacity in practice

A hypothetical food truck uses a rectangular water tank with inside dimensions 50 cm by 40 cm by 30 cm. Capacity calculations ignore wall thickness because dimensions are internal. Money is rounded to cents; physical limits are rounded conservatively.

Measurement	Value
Length	50 cm
Width	40 cm
Height	30 cm
Capacity	60 L

Figure 27.3 Volume, Conversions, and Capacity (hypothetical data)

The concept

Volume measures three-dimensional space in cubic units. A rectangular container has volume length times width times height. A cylinder has a circular base area multiplied by its perpendicular height. Use internal dimensions to estimate usable capacity.

Convert all dimensions to compatible units before multiplying. Since a meter contains 100 centimeters, a cubic meter contains 100^3 cubic centimeters, not just 100. A liter equals 1,000 cm^3, and a milliliter equals 1 cm^3.

Capacity is the amount a container can hold. A usable-fill percentage may reserve headspace for expansion or safe handling. Once the usable amount is known, divide by serving size and round down to count complete servings; rounding up would promise product you do not have.

Volume and mass are not interchangeable. Density connects them through mass per unit volume. A delivery weight estimate must include both product mass and container mass, and real loading plans require verified equipment limits rather than a textbook estimate.

Volume, Conversions, and Capacity: solution map

1. Identify the question

   `Find tank capacity at 80% fill for internal dimensions 50 cm by 40 cm by 30 cm, and count 250 mL servings.`

2. Calculation 1

   `Volume = 50 × 40 × 30 = 60,000 cm³.`

3. Calculation 2

   `Capacity = 60,000 ÷ 1,000 = 60 L.`

4. Calculation 3

   `Usable volume = 60 × 0.80 = 48 L = 48,000 mL.`

5. Calculation 4

   `Servings = 48,000 ÷ 250 = 192.`

6. Interpret and check

   `Usable capacity 48 L, enough for 192 full servings.`

Figure 27.3.V Volume, Conversions, and Capacity: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

EXPENSE

Money a business spends to run, such as rent, supplies, or wages.

INVOICE

A document listing goods or services sold and the amount owed.

BREAK-EVEN POINT

The sales level where revenue exactly equals total cost, so there is no profit and no loss.

KEY TERMS

Volume
Space occupied or enclosed, expressed in cubic units.

Capacity
The amount a container can hold.

Liter
A volume unit equal to 1,000 cm^3.

Density
Mass per unit volume.

Tare mass
The mass of an empty container.

FORMULAS & RULES

Rectangular volume

$$V = l \times w \times h$$

Cylinder volume

$$V = \pi r^2 h$$

Mass from density

$$\text{Mass} = \text{density} \times \text{volume}$$

Serving count

```
Complete servings = round down (usable volume ÷ serving volume)
```

Worked examples

Example 1 Tank capacity

Find tank capacity at 80% fill for internal dimensions 50 cm by 40 cm by 30 cm, and count 250 mL servings.

SOLUTION

1. Volume = 50 × 40 × 30 = 60,000 cm³.
2. Capacity = 60,000 ÷ 1,000 = 60 L.
3. Usable volume = 60 × 0.80 = 48 L = 48,000 mL.
4. Servings = 48,000 ÷ 250 = 192.

Answer: Usable capacity 48 L, enough for 192 full servings.

Example 2 Cylindrical container

A cylindrical container has inside radius 10 cm and height 30 cm. Find liters to two decimals.

SOLUTION

1. Base area = π × 10² = 100π cm².
2. Volume = 100π × 30 = 3,000π cm³.

3 Liters = $3{,}000\pi \div 1{,}000 = 3\pi$ L.

4 Round $3\pi \approx 9.42478$ L to 9.42 L.

Answer: Capacity is about 9.42 L.

Practice exercises 27.3

1. Convert 2.5 L to milliliters.
2. Find liters in a box 20 cm by 30 cm by 40 cm.
3. How many complete 300 mL servings come from 5 L?
4. A liquid has density 1.2 kg/L. Find the loaded mass of 10 L in a 2 kg container.
5. A box is 0.5 m by 20 cm by 30 cm. Find liters.

RECAP QUIZ 27.3

1. 1 L equals how many cm^3?

- a. 100
- b. 1,000
- c. 10
- d. 1

2. Box volume uses which calculation?

- a. $l + w + h$
- b. $2(l + w)$
- c. $l \times w \times h$
- d. $l \times w$

3. 8 L available, each serving 0.3 L. Full servings?

- a. 27
- b. 26.67
- c. 24
- d. 26

SECTION 27.3 RECAP

- Convert dimensions first, then calculate volume.
- Round servings down and include container mass in load estimates.

Answer key with explanations: Section 27.3

PRACTICE EXERCISES

1. 2.5 × 1,000 = 2,500 mL.
2. 20 × 30 × 40 = 24,000 cm³; divide by 1,000 to get 24 L.
3. 5 L = 5,000 mL; 5,000 ÷ 300 ≈ 16.67. Round down to 16; 200 mL remains.
4. Product mass = 1.2 × 10 = 12 kg; add tare 2 kg to obtain 14 kg.
5. Convert 0.5 m = 50 cm. Volume = 50 × 20 × 30 = 30,000 cm³ = 30 L.

RECAP QUIZ

1. **1,000**: A liter contains 1,000 cubic centimeters.
2. **l × w × h**: Three dimensions multiply to cubic units.
3. **26**: 8 ÷ 0.3 ≈ 26.67, so only 26 complete servings.

CHAPTER 28

Mathematical Modeling, Entrepreneurship, and Economics

A useful business model turns assumptions into quantities you can check. Build hypothetical profit, pricing, and cash-flow models, then test their limits before using them to support a decision.

28.1

Building Revenue, Cost, and Profit Models

LEARNING OBJECTIVES

After this section you will be able to:

1. Identify variables, assumptions, and feasible inputs.
2. Build linear revenue and cost models.
3. Solve a break-even equation and interpret whole units.

REAL-LIFE SITUATION

Building Revenue, Cost, and Profit Models in practice

A hypothetical Philadelphia student sells printed event programs for $8 each. Monthly fixed costs are $120 and variable cost is $3 per program. Assume every produced program sells and ignore taxes; these are invented planning figures. Round money to cents.

Programs	Revenue ($)	Total cost ($)	Profit ($)
0	0	120	-120
24	192	192	0
40	320	240	80

Figure 28.1 Building Revenue, Cost, and Profit Models (hypothetical data)

The concept

A mathematical model simplifies a situation so you can ask useful questions. Define the input, output, units, and assumptions before writing equations. Here q is the number of programs sold in one month, so q must be a nonnegative whole number within production capacity.

Revenue is money from sales, not profit. If price is constant, revenue equals price times units sold. Costs include fixed expenses that do not change within the modeled range and variable expenses that change with output. Unsold inventory would require a more detailed model.

Profit equals revenue minus total cost. The contribution margin per unit is price minus variable cost; it is the amount each sale contributes toward fixed costs and then profit. If that margin is zero or negative, additional sales cannot cover a positive fixed cost in this simple model.

Break-even occurs where revenue equals cost. A fractional solution is a mathematical intersection, but a seller of whole items needs the next whole unit to avoid a loss. Test nearby quantities and do not extend a linear model past a range where costs or prices change.

Building Revenue, Cost, and Profit Models: solution map

1. Identify the question

   `Find break-even for price $8, variable cost $3, and fixed cost $120.`

↓

2. Calculation 1

   `Margin = 8 − 3 = $5 per program.`

↓

3. Calculation 2

   `Set profit to zero: 5q − 120 = 0.`

↓

4. Calculation 3

   `Solve q = 120 ÷ 5 = 24.`

↓

5. Calculation 4

   `Check R = 8 × 24 = $192 and C = 120 + 3 × 24 = $192.`

↓

6. Interpret and check

   `Break-even is 24 programs.`

Figure 28.1.V Building Revenue, Cost, and Profit Models: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

BUSINESS

An organization that makes or sells goods or services to earn money.

REVENUE

The total money a business brings in from sales, before any costs are paid.

EXPENSE

Money a business spends to run, such as rent, supplies, or wages.

KEY TERMS

Variable
A symbol representing a quantity that may change.

Fixed cost
A cost constant within the stated period and activity range.

Variable cost
A cost that changes with units produced or sold.

Contribution margin
Selling price minus variable cost per unit.

Break-even
The output level at which profit equals zero.

FORMULAS & RULES

Revenue

$$R = p \times q$$

Total cost

$$C = F + v \times q$$

Profit

$$\text{Profit} = (p - v) \times q - F$$

Break-even quantity

```
q = F ÷ (p − v), when p > v
```

Worked examples

Example 1 Program break-even

Find break-even for price $8, variable cost $3, and fixed cost $120.

SOLUTION

1. Margin = 8 − 3 = $5 per program.
2. Set profit to zero: 5q − 120 = 0.
3. Solve q = 120 ÷ 5 = 24.
4. Check R = 8 × 24 = $192 and C = 120 + 3 × 24 = $192.

> **Answer:** Break-even is 24 programs.

Example 2 A profit goal

How many programs must sell for at least $200 monthly profit?

SOLUTION

1. Use profit = 5q − 120.
2. Set 5q − 120 ≥ 200.
3. Solve q ≥ 320 ÷ 5 = 64.
4. Check 5 × 64 − 120 = $200.00.

Answer: Sell at least 64 programs if capacity and demand allow.

Practice exercises 28.1

1. Find profit at 40 programs using the situation.
2. With fixed cost $100 and margin $6, find the least whole quantity avoiding a loss.
3. Price is $5 and variable cost is $5 with $60 fixed cost. Is break-even possible?
4. For 30 units at $10 each, fixed cost $80 and variable cost $4, find revenue and profit.
5. A model predicts 80 sales but capacity is 60. Can the plan be used unchanged?

RECAP QUIZ 28.1

1. Revenue at $7 each for 20 units?

- ○ a. $27
- ○ b. $140
- ○ c. $13
- ○ d. $2.86

2. Price $9, unit cost $4: contribution margin?

- ○ a. $13
- ○ b. $36
- ○ c. $5
- ○ d. $4

3. Break-even equation yields 12.4 indivisible units. Minimum avoiding loss with positive margin?

- ○ a. 12
- ○ b. 12.4
- ○ c. 11
- ○ d. 13

SECTION 28.1 RECAP

- State assumptions and a feasible domain before calculating.
- Revenue is not profit; contribution margin must cover fixed costs.

Answer key with explanations: Section 28.1

PRACTICE EXERCISES

1. Revenue = 8 × 40 = $320; cost = 120 + 3 × 40 = $240; profit = $80.00.
2. 100 ÷ 6 ≈ 16.67. At 17 units, profit = 6 × 17 − 100 = $2.00; 16 units loses $4.00.
3. Margin = 5 − 5 = 0; profit is −$60 at every quantity, so no.
4. Revenue = 30 × 10 = $300; cost = 80 + 4 × 30 = $200; profit = $100.00.
5. No. Restrict the feasible domain to at most 60 units or model the added cost of capacity; 80 exceeds the stated limit by 20.

RECAP QUIZ

1. **$140**: 7 × 20 = $140.00.
2. **$5**: 9 − 4 = $5.00.
3. **13**: Round up to 13 units and verify.

28.2

Demand, Pricing, and Economic Trade-Offs

LEARNING OBJECTIVES

After this section you will be able to:

1. Use a simple price-demand relationship.

2. Compare revenue and profit at feasible prices.
3. Explain opportunity cost and limits of economic models.

REAL-LIFE SITUATION

Demand, Pricing, and Economic Trade-Offs in practice

A hypothetical snack stand estimates weekly demand $q = 100 - 10p$, where p is price in dollars. Unit cost is $2 and weekly fixed cost is $40. Treat the relationship as a local estimate for whole-dollar prices from $2 to $8, not a promise of customer behavior. Round money to cents.

Price ($)	Demand	Revenue ($)	Profit ($)
4	60	240	80
6	40	240	120
8	20	160	80

Figure 28.2 Demand, Pricing, and Economic Trade-Offs (hypothetical data)

The concept

Demand describes how many units buyers are willing and able to purchase under stated conditions. A downward-sloping demand model predicts fewer purchases at higher prices while other conditions remain unchanged. It does not establish that price is the only cause of a sales change.

Price times quantity gives revenue. Raising price can lower quantity enough that revenue falls; multiplying a higher price by the old quantity is not a valid forecast when demand responds. Substitute the proposed price into the demand equation first.

Profit subtracts both variable and fixed cost from revenue. The revenue-maximizing price can differ from the profit-maximizing price because serving more customers also costs money. A table of feasible options is often sufficient for a small business decision; a claim of a best price must specify the allowed choices.

Opportunity cost is the value of the next-best alternative given up. Choosing a higher-profit event may mean missing another event or study time. Model estimates should be tested with observations, and nonprice factors such as access, fairness, and workload also matter.

Demand, Pricing, and Economic Trade-Offs: solution map

1. Identify the question

   `Compare $4 and $6 prices using the situation.`

2. Calculation 1

   `At $4, q = 100 − 10 × 4 = 60.`

3. Calculation 2

   `Revenue = 4 × 60 = $240; profit = (4 − 2) × 60 − 40 = $80.`

4. Calculation 3

   `At $6, q = 100 − 10 × 6 = 40.`

5. Calculation 4

   `Revenue = 6 × 40 = $240; profit = (6 − 2) × 40 − 40 = $120.`

6. Interpret and check

   `Both yield $240 revenue, but $6 gives $40 more profit.`

Figure 28.2.V Demand, Pricing, and Economic Trade-Offs: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

PROFIT

What is left when expenses are subtracted from revenue.

NET PROFIT

The final profit after every cost is paid — sometimes called the bottom line.

BUDGET

A plan that shows expected income and how it will be spent.

KEY TERMS

Demand
Quantity buyers are willing and able to buy under stated conditions.

Feasible option
A choice satisfying the model constraints.

Revenue maximum
The largest sales revenue among allowed choices.

Opportunity cost
Value of the next-best alternative forgone.

Other things equal
An assumption that influences other than the one studied stay unchanged.

FORMULAS & RULES

Demand model

$$q = a - b \times p$$

Revenue with demand

$$R = p \times (a - b \times p)$$

Profit with demand

```
Profit = (p − v) × (a − b × p) − F
```

Worked examples

Example 1 Two price options

Compare $4 and $6 prices using the situation.

SOLUTION

1. At $4, q = 100 − 10 × 4 = 60.
2. Revenue = 4 × 60 = $240; profit = (4 − 2) × 60 − 40 = $80.
3. At $6, q = 100 − 10 × 6 = 40.
4. Revenue = 6 × 40 = $240; profit = (6 − 2) × 40 − 40 = $120.

> **Answer:** Both yield $240 revenue, but $6 gives $40 more profit.

Example 2 Best among three choices

Compare profits at prices $5, $6, and $7.

SOLUTION

1. At $5, demand = 50; profit = (5 − 2) × 50 − 40 = $110.
2. At $6, demand = 40; profit = (6 − 2) × 40 − 40 = $120.
3. At $7, demand = 30; profit = (7 − 2) × 30 − 40 = $110.

4 Compare $110, $120, $110.

> **Answer:** $6 is best among these three choices; this comparison alone does not prove a global optimum.

Practice exercises 28.2

1. Find demand at $3.
2. Find revenue and profit at $8.
3. At $5, revenue is $250. Is all $250 profit?
4. One event nets $90; another nets $70, and only one can be attended. What is the monetary opportunity cost of choosing the $90 event?
5. Can this demand model be trusted at $12?

RECAP QUIZ 28.2

1. Demand at $7 in this model?

- ○ a. 70
- ○ b. 30
- ○ c. 93
- ○ d. 107

2. What must be subtracted from revenue for profit?

- ○ a. Only price
- ○ b. Only sales count
- ○ c. Total costs
- ○ d. Nothing

3. Opportunity cost is what?

- ○ a. Every historical expense
- ○ b. Only an invoice

○ c. The chosen option's revenue

○ d. The next-best alternative forgone

SECTION 28.2 RECAP

- Update demand when testing a new price.
- Compare profit, not revenue alone, and acknowledge forgone alternatives.

Answer key with explanations: Section 28.2

PRACTICE EXERCISES

1. 100 − 10 × 3 = 70 units.
2. q = 100 − 80 = 20; revenue = 8 × 20 = $160; profit = 6 × 20 − 40 = $80.00.
3. No. Demand = 50; cost = 2 × 50 + 40 = $140; profit = 250 − 140 = $110.00.
4. The forgone next-best event nets $70, so monetary opportunity cost is $70. The net advantage is $90 − $70 = $20.
5. No. It is outside the $2–$8 modeled range and predicts 100 − 120 = −20 units, which is not a meaningful sales quantity.

RECAP QUIZ

1. **30**: 100 − 10 × 7 = 30.
2. **Total costs**: Profit = revenue − total costs.
3. **The next-best alternative forgone**: It measures the value sacrificed by the choice.

28.3

Cash Flow, Scenarios, and Model Validation

LEARNING OBJECTIVES

After this section you will be able to:

1. Separate cash timing from profitability.
2. Build a simple cash forecast with scenario analysis.
3. Compare forecasts with actual results and revise assumptions.

REAL-LIFE SITUATION

Cash Flow, Scenarios, and Model Validation in practice

A hypothetical teen design service starts a month with $300 cash. It collects $200 immediately and invoices another $400 due next month, while paying $450 in operating expenses now. No other items occur; taxes, borrowing, and depreciation are excluded. Round money to cents.

Item	This month cash ($)	This month earned revenue ($)
Opening cash	300	0
Immediate collections	200	200
Work invoiced for next month	0	400
Operating payments	-450	0
Ending cash	50	Not revenue

Figure 28.3 Cash Flow, Scenarios, and Model Validation (hypothetical data)

The concept

Profit and cash flow answer different questions. Profit compares earned revenue with the expenses associated with earning it under a stated accounting model. Cash flow follows when money enters or leaves. A profitable service can run short of cash while waiting for customers to pay.

A simple cash forecast starts with opening cash, adds expected collections, and subtracts cash payments. Do not count an unpaid invoice as a collection. Also avoid counting opening cash as current sales; it is a balance carried forward, not new revenue.

Scenario analysis changes assumptions to examine a range of outcomes. A low, base, and high collection case can reveal whether a cash buffer is adequate. These are conditional cases rather than probabilities unless there is evidence supporting assigned likelihoods.

Validation compares a forecast with observed results. Define an error direction, investigate why differences occurred, and revise inputs rather than hiding the discrepancy. Small samples, late payments, seasonality, and unusual events can make yesterday’s pattern a poor guide to next month.

Cash Flow, Scenarios, and Model Validation: solution map

1. Identify the question

   `Find profit and ending cash for the design service.`

2. Calculation 1

   `Earned revenue = 200 + 400 = $600.`

3. Calculation 2

   `Simplified profit = 600 − 450 = $150.`

4. Calculation 3

   `Cash collections are only $200; $400 remains receivable.`

5. Calculation 4

   `Ending cash = 300 + 200 − 450 = $50.`

6. Interpret and check

   `Profit is $150.00 while ending cash is only $50.00.`

Figure 28.3.V Cash Flow, Scenarios, and Model Validation: solution map. Hypothetical teaching data.

MONEY WORDS

Plain-language words you will use in this lesson — learn them before we reach the formulas.

CASH FLOW

The movement of money into and out of a business or account.

LOAN

Money borrowed that must be repaid, usually with interest.

EQUITY

What is left of an asset after debts are subtracted; in a home, value minus what you still owe.

KEY TERMS

Cash flow
Cash receipts and cash payments over a period.

Receivable
An amount owed by a customer but not yet collected.

Cash buffer
Cash retained to cover timing gaps or unexpected payments.

Scenario analysis
Comparison of outcomes under different assumptions.

Forecast error
Actual result minus forecast result under the convention used here.

FORMULAS & RULES

Ending cash

```
Ending cash = opening cash + collections − payments
```

Simplified profit

```
Profit = earned revenue − associated expenses
```

Forecast error

```
Error = actual − forecast
```

Percentage forecast error

```
Error percent = (actual − forecast) ÷ forecast × 100%,
for positive forecast
```

Worked examples

Example 1 Profit without much cash

Find profit and ending cash for the design service.

SOLUTION

1. Earned revenue = 200 + 400 = $600.
2. Simplified profit = 600 − 450 = $150.
3. Cash collections are only $200; $400 remains receivable.
4. Ending cash = 300 + 200 − 450 = $50.

> **Answer:** Profit is $150.00 while ending cash is only $50.00.

Example 2 Collections scenarios

With $300 opening cash and $450 payments, compare collections of $100, $200, and $350.

SOLUTION

1. Low case: 300 + 100 − 450 = −$50.
2. Base case: 300 + 200 − 450 = $50.
3. High case: 300 + 350 − 450 = $200.

4 The low case needs at least $50 of extra opening cash or changed timing to avoid a shortfall.

> **Answer:** Ending cash ranges from a $50 shortfall to $200; these are not guaranteed outcomes.

Practice exercises 28.3

1. Opening cash is $250, collections $500, and payments $620. Find ending cash.
2. A $300 invoice is unpaid at month-end. How much cash does it add now?
3. Forecast sales were 80 units; actual sales were 72. Find signed and percentage error.
4. Opening cash $100, collections $150, payments $320. Minimum extra cash needed to avoid a negative balance at period end?
5. A forecast worked for one week. Is it validated forever?

RECAP QUIZ 28.3

1. Which adds cash immediately?

- ○ a. Unpaid invoice
- ○ b. Customer cash payment
- ○ c. Forecast profit
- ○ d. An unaccepted quote

2. Cash $100 + collections $80 − payments $200 equals?

- ○ a. $380
- ○ b. $20
- ○ c. −$20
- ○ d. $180

3. Actual 110, forecast 100: signed percentage error?

- ○ a. −10%
- ○ b. 0%

○ c. 110%

○ d. 10%

SECTION 28.3 RECAP

- Track collections separately from earned revenue.
- Test adverse scenarios and compare forecasts with observations.

Answer key with explanations: Section 28.3

PRACTICE EXERCISES

1. 250 + 500 − 620 = $130.00.
2. It adds $0 now; the $300 is receivable until collected. Revenue timing is a separate question.
3. Error = 72 − 80 = −8 units; percentage error = −8 ÷ 80 × 100% = −10%.
4. Ending cash = 100 + 150 − 320 = −$70.00, so at least $70.00 extra is needed at the period-end level; timing within the period could require more.
5. No. One week provides limited evidence. Compare multiple future observations and update assumptions when demand or payment timing changes.

RECAP QUIZ

1. **Customer cash payment**: A received cash payment increases cash now.
2. **−$20**: 100 + 80 − 200 = −20, a shortfall.
3. **10%**: (110 − 100) ÷ 100 × 100% = 10%.

APPENDIX A

Money Dictionary

Every money word in the textbook, in plain language. Color-coded by category: business, banking, credit, financing, and investing.

Business — How companies earn, spend, and price.

business
An organization that makes or sells goods or services to earn money.

revenue
The total money a business brings in from sales, before any costs are paid.

expense
Money a business spends to run, such as rent, supplies, or wages.

profit
What is left when expenses are subtracted from revenue.

gross profit
Revenue minus the direct cost of the items sold, before other expenses.

net profit
The final profit after every cost is paid — sometimes called the bottom line.

profit margin
Profit shown as a percent of revenue; a 10% margin means $10 of profit per $100 of sales.

markup
The amount added to a cost to set a selling price; the markup rate uses cost as its base.

markdown
A reduction from the regular selling price, often to clear out inventory.

cost
The amount paid to make or buy an item before it is sold.

selling price
The price a customer pays for an item or service.

wholesale
The price or market for buying goods in bulk, usually from a supplier.

retail
The price or market for selling goods to the public, usually in small amounts.

invoice
A document listing goods or services sold and the amount owed.

vendor
A company or person that supplies goods or services to a business.

inventory
The stock of goods a business holds ready to sell.

overhead
Ongoing business costs such as rent, utilities, and insurance that do not change with each sale.

fixed cost
A cost that stays the same no matter how many items are sold, like rent.

variable cost
A cost that rises or falls with the number of items made or sold.

break-even point
The sales level where revenue exactly equals total cost, so there is no profit and no loss.

depreciation
The way a costly item, such as a machine, loses value and its cost is spread over its useful life.

asset
Something valuable that a person or business owns, like cash, equipment, or a building.

liability
Money or obligations a person or business owes to others.

equity
What is left of an asset after debts are subtracted; in a home, value minus what you still owe.

balance sheet
A snapshot listing what a business owns (assets) and owes (liabilities) on one date.

income statement
A report showing revenue, expenses, and profit over a period of time.

cash flow
The movement of money into and out of a business or account.

budget
A plan that shows expected income and how it will be spent.

sales tax
A percentage added by the government to the price of many purchases.

discount
An amount taken off a regular price.

commission
Pay based on a percentage of the sales a worker makes.

payroll
The system and records a business uses to pay its workers.

gross pay
Total earnings before any deductions are taken out.

net pay
Take-home pay after taxes and other deductions.

salary
A fixed amount of pay, usually stated per year, that does not depend on hours worked.

overtime
Extra pay, often 1.5 times the normal rate, for hours worked beyond a set limit.

deduction
Money removed from gross pay for taxes, benefits, or other reasons.

payment terms
The rules in an agreement about when a bill must be paid and any discounts for paying early.

purchase order
A formal request from a buyer listing items to buy at agreed prices.

insurance premium
The amount paid, often monthly, to keep an insurance policy active.

deductible
The amount you pay yourself before an insurance policy starts paying.

Banking — Accounts, statements, and everyday money movement.

checking account
A bank account for everyday spending, usually with a debit card and checks.

savings account
A bank account meant to hold money for later, often earning a small amount of interest.

deposit
Money placed into a bank account.

withdrawal
Money taken out of a bank account.

balance
The amount of money in an account at a given moment.

account statement
A bank's record of deposits, withdrawals, fees, and the balance for a period of time.

bank reconciliation
Comparing your own record of an account with the bank's statement and explaining any differences.

outstanding check
A check you wrote that the recipient has not cashed yet.

deposit in transit
Money you have recorded that the bank has not yet processed.

overdraft
Spending more than the balance in an account, which may trigger fees.

debit card
A card that takes money directly from your checking account when you pay.

mobile banking
Using a phone app to check balances, deposit checks, or move money.

pending transaction
A payment the bank has authorized but not yet completed, so it may not appear in the final balance.

FDIC insurance
A government program that protects money in insured bank accounts up to a set limit if the bank fails.

simple interest
Interest calculated only on the original principal, not on interest already earned.

interest rate
The percent used to calculate how much interest is earned or charged.

Credit — Borrowing, cards, and credit records.

credit
The ability to buy now and pay later, or money borrowed that must be repaid.

credit card
A card that lets you borrow up to a limit and repay over time, usually with interest.

APR
Annual percentage rate — the yearly cost of borrowing, including interest and some fees, shown as a percent.

minimum payment
The smallest amount due on a credit card bill each month; paying only this keeps you in debt longer.

credit limit
The most you can borrow on a credit card or credit line.

credit score
A number that estimates how reliably someone repays debts.

debt
Money that is owed and must be paid back.

installment plan
Paying for something in a set number of equal payments over time.

late fee
An extra charge when a payment arrives after its due date.

grace period
A short time after the due date before a fee or interest charge applies.

statement balance
The amount owed on a credit account as of the statement date.

balance transfer
Moving debt from one card to another, sometimes to get a lower interest rate.

default
Failing to repay a loan as agreed, which can harm credit and lead to collection.

cosigner
A person who agrees to repay a loan if the borrower cannot.

Financing — Loans, payments, and the cost of borrowing.

loan
Money borrowed that must be repaid, usually with interest.

principal
The original amount of money borrowed or invested, before interest.

interest
The cost of borrowing money, or the earnings on money saved or invested.

loan term
How long a loan lasts before it must be fully repaid.

total repayment cost
The full amount paid over a loan's life — principal plus all interest and fees.

finance charge
The total dollar cost of borrowing — all interest and fees added together, on top of the amount financed.

monthly payment
The amount paid each month toward a loan or installment plan.

down payment
Money paid upfront when financing a purchase, which reduces the amount borrowed.

amortization
Spreading a loan into scheduled payments so that early payments are mostly interest and later payments are mostly principal.

mortgage
A long-term loan used to buy a home, with the property as security.

collateral
Property a borrower promises to a lender if the loan is not repaid, such as a car.

refinance
Replacing an existing loan with a new one, often to get a lower rate or payment.

origination fee
A fee a lender charges to set up a loan.

closing costs
Fees paid when a home or property purchase is finalized, such as appraisal and title fees.

escrow
Money held by a third party, such as property tax collected along with a mortgage payment.

promissory note
A written promise to repay a specific amount by a certain date.

maturity date
The date a loan or note must be fully repaid.

bank discount
Interest deducted from a loan in advance, so the borrower receives less than the face amount.

sinking fund
Money set aside regularly so a future obligation can be paid without borrowing.

lease
A contract to use an item for a period of time by making payments, without owning it.

prepayment penalty
A fee charged for paying off a loan early.

Investing — Putting money to work and measuring results.

investment
Money put into something with the hope it will grow or earn income later.

return
The money or gain an investment produces, often shown as a percent.

ROI
Return on investment — the gain compared to what you put in, as a percent: (gain ÷ cost) × 100.

stock
A share of ownership in a company.

bond
A loan to a government or company that pays interest and repays the face amount at maturity.

mutual fund
A pool of money from many investors used to buy a mix of stocks, bonds, or both.

dividend
A share of a company's profit paid to its stockholders.

compound interest
Interest earned on both the original amount and the interest already earned.

future value
What an amount of money today will be worth at a later date with interest.

present value
What a future amount of money is worth today at a given interest rate.

diversification
Spreading money across different investments to reduce risk.

inflation
A general rise in prices over time, which makes money buy less.

yield
The income an investment pays, shown as a percent of its price.

capital gain
Profit made by selling an investment for more than it cost.

portfolio
The full collection of investments a person or fund owns.

annuity
A series of equal payments made at regular intervals, such as monthly.

risk
The chance that an investment or decision will turn out worse than expected.

speculation
A very risky investment made in hopes of a quick, large gain.

bid-ask spread
The gap between what buyers will pay (bid) and what sellers will accept (ask); a hidden cost on every trade.

limit order
An order to buy or sell only at a price you choose or better; it protects your price but may not fill.

liquidity
How quickly and easily something can be bought or sold for cash without changing its price much.

APPENDIX B

RBS

Read Before You Sign

How to understand a financing agreement before you put your name on it.

Financing agreements are written in a specific vocabulary. If you know what the key words mean, you can answer the three questions that matter: **How much am I borrowing? What does it cost in total? What happens if something goes wrong?** Everything on this page is a fictional teaching example — no real store, contract, or offer is described.

A 7-step checklist

1. Find the cash price, the down payment, and the amount financed (the principal) — that is what you are really borrowing.
2. Find the APR — the yearly cost of borrowing as a percent. Check whether it is nominal (before compounding) or effective, and how often interest compounds.
3. Find the monthly payment and the number of payments, then multiply them together.
4. Compare that total with the cash price: the difference is your total repayment cost.
5. Check the fees: late fee, origination fee, and any prepayment penalty.
6. Check what happens if you cannot pay: collateral, default, and credit reporting clauses.
7. Add up the worst case, decide before you sign, and never sign under time pressure.

An annotated fictional contract

Below is a made-up 12-month agreement for a phone. Read each numbered clause, then read the matching plain-language note underneath. Notice how every important number traces back to the amount financed: $960.

FICTIONAL TEACHING EXAMPLE. NOT A REAL OFFER, CONTRACT, OR LEGAL ADVICE. ALL PEOPLE, STORES, AND AMOUNTS ARE INVENTED.

Example Only — 12-Month Phone Finance Agreement

Sample Wireless Store (fictional) · Sample Customer (fictional) · October 22, 2026

1. Cash price: $960.00. Down payment: $0.00. Amount financed: $960.00. [1]
2. Nominal APR: 33.0% — a monthly rate of 2.75%, compounded monthly. [2]
3. Payment: $95.01 per month for 12 months; the final payment is $95.00 to clear the balance. [3]
4. Total of payments: $1,140.11. Finance charge: $180.11. [4]
5. Late fee: $25 for any payment more than 10 days late. [5]
6. Prepayment: the full balance may be paid off early with no penalty. [6]
7. Security interest: the store keeps a legal claim on the phone until it is paid off. [7]
8. Missed payments may be reported to credit bureaus after 30 days. [8]

PLAIN-LANGUAGE NOTES

[1] principal: The original amount borrowed — the principal — is $960.00. Because the down payment is $0, it equals the full cash price. Every other number in this contract is measured against it.

[2] APR: The yearly cost of borrowing as a percent: 12 × 2.75% = 33.0%. Because interest compounds monthly, the effective yearly cost is even higher — about 38.5%.

[3] monthly payment: Multiply to see what you will actually pay: $95.01 × 11 + $95.00 = $1,140.11.

[4] finance charge: The finance charge is every dollar of interest and fees added to the $960.00 you borrowed: $1,140.11 − $960.00 = $180.11.

[5] late fee: Missing a due date adds costs and can be reported to credit bureaus.

[6] prepayment penalty: Good news: some contracts charge a fee for paying early. This fictional one does not — always look for this clause.

[7] collateral: The phone acts as collateral — if payments stop, the store can take it back.

[8] credit score: Late payments can lower your credit score, making future loans cost more.

Figure RBS.1 Annotated fictional financing agreement

Cash today or financing over time?

The same $960 phone costs $960 if you pay cash, but $1,140.11 if you finance it over 12 months — a finance charge of $180.11. The contract's nominal APR is 33.0% (2.75% per month, compounded monthly); because interest compounds, the effective yearly cost is about 38.5%. Financing is not automatically wrong — it can be worth it when you keep your cash for something more urgent — but you should always know the total before you decide.

The same $960.00 phone, two paths

	Pay cash today	Finance for 12 months
Money paid today	$960.00 cash price	$0.00 down payment
Monthly cost	$0	$95.01 per month
Total paid	$960.00	$1,140.11
Finance charge $1,140.11 − $960.00	$0	$180.11
Nominal APR	—	33.0% (2.75% per month)
Effective yearly cost	—	About 38.5% — interest compounds monthly, so the real yearly cost is higher than the nominal APR.
What you gain	The phone is yours immediately; no debt.	You keep the $960 now and build a payment history — if every payment is on time.
What can go wrong	Money is tied up in the phone.	Late payments add fees and can lower your credit score.

Figure RBS.2 Cash today versus financing over 12 months (fictional)

Money Words you need here

PRINCIPAL

The original amount of money borrowed or invested, before interest.

APR

Annual percentage rate — the yearly cost of borrowing, including interest and some fees, shown as a percent.

LOAN TERM

How long a loan lasts before it must be fully repaid.

TOTAL REPAYMENT COST

The full amount paid over a loan's life — principal plus all interest and fees.

FINANCE CHARGE

The total dollar cost of borrowing — all interest and fees added together, on top of the amount financed.

MONTHLY PAYMENT

The amount paid each month toward a loan or installment plan.

COLLATERAL

Property a borrower promises to a lender if the loan is not repaid, such as a car.

LATE FEE

An extra charge when a payment arrives after its due date.

PREPAYMENT PENALTY

A fee charged for paying off a loan early.

DEFAULT

Failing to repay a loan as agreed, which can harm credit and lead to collection.

COSIGNER

A person who agrees to repay a loan if the borrower cannot.

CREDIT SCORE

A number that estimates how reliably someone repays debts.

DOWN PAYMENT

Money paid upfront when financing a purchase, which reduces the amount borrowed.

INSTALLMENT PLAN

Paying for something in a set number of equal payments over time.

INTEREST

The cost of borrowing money, or the earnings on money saved or invested.

RECAP QUIZ RBS

1. A contract says “Amount financed: $960.00.” Which Money Word describes the $960.00?

- ○ a. interest
- ○ b. principal
- ○ c. collateral
- ○ d. late fee

2. You pay about $1,140.11 over 12 months for a $960.00 phone. What was the cost of borrowing?

- ○ a. $960.00
- ○ b. $95.01
- ○ c. $1,140.11
- ○ d. $180.11

3. Which term means the store can take the phone back if payments stop?

- ○ a. collateral
- ○ b. grace period
- ○ c. dividend
- ○ d. escrow

4. Which term is the yearly cost of borrowing shown as a percent?

- ○ a. yield
- ○ b. ROI
- ○ c. APR
- ○ d. markdown

5. Which clause should you look for before signing if you might pay the balance off early?

- ○ a. prepayment penalty
- ○ b. grace period
- ○ c. security interest
- ○ d. balance transfer

6. Which number shows the full amount you will pay over the life of the loan?

- ○ a. statement balance
- ○ b. credit limit
- ○ c. total repayment cost
- ○ d. minimum payment

7. Paying only the minimum payment on a credit card usually:

- ○ a. pays the debt off quickly
- ○ b. keeps you in debt longer and adds interest
- ○ c. improves your credit score fastest
- ○ d. removes the late fee

8. Before financing any purchase, a smart first question is:

- ○ a. “What is the monthly payment?”
- ○ b. “What is the total repayment cost?”

○ c. “What color is it?”

○ d. “Can I pay the minimum?”

Answer key with explanations: Read Before You Sign

1. **principal**: The principal is the original amount borrowed — $960.00. Interest and fees are added on top.
2. **$180.11**: $1,140.11 − $960.00 = $180.11 — the finance charge, the total cost of borrowing beyond the cash price.
3. **collateral**: The phone is collateral — property promised to the lender if the loan is not repaid.
4. **APR**: APR is the annual percentage rate — the yearly cost of borrowing as a percent. This fictional contract’s nominal APR is 33.0% (2.75% per month, compounded monthly).
5. **prepayment penalty**: A prepayment penalty is a fee for paying off a loan early. This fictional contract has none — always check.
6. **total repayment cost**: Total repayment cost = principal + all interest and fees. It is the true price of financing.
7. **keeps you in debt longer and adds interest**: Minimum payments are small, so the balance lingers and interest keeps adding up. Pay more than the minimum when you can.
8. **“What is the total repayment cost?”**: The monthly payment can look small while the total cost is large. Compare the total repayment cost with the cash price.