WealthWiseGuys Math Quick Reference

Grades 7–12 · Also for adult learners · Draft handbook

Selected formulas from every lesson, key definitions, and one fully worked example per chapter. Match time periods and units; convert percentages to decimals before substitution. All prices and rates are hypothetical. Education, not personalized financial or tax advice.

1. Whole Numbers: How to Dissect and Solve Word Problems

Formulas & rules

1.1 · Rounding rule

```
Digit to the right ≥ 5 → round up; ≤ 4 → keep
```

All digits to the right of the rounding place become 0.

1.2 · Addition

```
Addend + Addend = Sum
```

1.2 · Subtraction

```
Minuend − Subtrahend = Difference
```

1.2 · Subtraction check

```
Difference + Subtrahend = Minuend
```

1.3 · Multiplication

```
Factor × Factor = Product
```

1.3 · Division check

```
Divisor × Quotient + Remainder = Dividend
```

Key definitions

Whole numbers

The numbers 0, 1, 2, 3, and so on, with no fractions or decimals.

Addends

The numbers being added.

Factors

Numbers being multiplied.

Worked example

Write 3,206,450 in words.

1. Split into periods: 3 | 206 | 450.
2. Millions period: three million.
3. Thousands period: two hundred six thousand.
4. Units period: four hundred fifty.

Answer: Three million, two hundred six thousand, four hundred fifty

2. Fractions

Formulas & rules

2.1 · Improper → mixed

$$a/b = q + r/b$$

q is the whole-number quotient and r is the remainder of a ÷ b.

2.1 · Mixed → improper

$$W\ n/d = (W \times d + n)/d$$

2.2 · Reduce

$$a/b = (a \div GCD)/(b \div GCD)$$

2.2 · Raise

$$a/b = (a \times k)/(b \times k)$$

2.2 · Like denominators

$$a/d \pm c/d = (a \pm c)/d$$

2.3 · Multiply

$$a/b \times c/d = (a \times c)/(b \times d)$$

2.3 · Divide

$$a/b \div c/d = a/b \times d/c$$

Key definitions

Numerator

The top number; how many parts are counted.

Equivalent fractions

Different fractions with the same value.

Reciprocal

A fraction flipped upside down; a number times its reciprocal equals 1.

Worked example

What fraction of the bag is cherry? Lemon?

1. Cherry: 5 of 12 → 5/12.
2. Lemon: 4 of 12 → 4/12, which simplifies to 1/3 (divide both by 4).

Answer: Cherry 5/12; lemon 4/12 = 1/3

3. Decimals

Formulas & rules

3.1 · Fraction → decimal

```
a/b = a ÷ b
```

3.1 · Decimal → fraction

```
0.65 = 65/100 = 13/20
```

3.2 · Sales tax

```
Tax = Subtotal × rate (as a decimal)
```

3.2 · Total

```
Total = Subtotal + Tax
```

3.2 · Decimal places in a product

```
places(a × b) = places(a) + places(b)
```

3.3 · Unit price

```
Unit price = Total price ÷ Number of units
```

3.3 · × 10^n

```
Move the decimal n places right
```

3.3 · ÷ 10^n

```
Move the decimal n places left
```

3.4 · Percent → decimal

```
p% = p ÷ 100
```

3.4 · Discount

```
Discount = Price × Rate
```

3.4 · Sale price

```
Sale price = Price × (1 − Rate)
```

3.4 · Cart total

```
Total = (Subtotal − Discount) × (1 + Tax rate)
```

Key definitions

Decimal point

The dot separating whole units from parts of a unit.

Subtotal

Total of items before tax.

Unit price
Price for one unit of measure.

Percent
A ratio out of 100.

Worked example

Round $3.479 to the nearest cent.

1. Cents are the hundredths place: 7.
2. The digit to the right is 9, so round up: 3.48.

Answer: $3.48

4. Banking

Formulas & rules

4.1 · Running balance

```
New balance = Previous balance + Deposits − Payments
```

4.1 · Net deposit

```
Cash + Coins + Checks − Cash received
```

4.2 · Adjusted bank balance

```
Statement balance + Deposits in transit − Outstanding checks
```

4.2 · Adjusted checkbook balance

```
Checkbook balance + Credits − Fees − NSF items
```

4.3 · Available balance

```
Available = Current − Pending debits − Holds
```

4.3 · Overdraft result

```
New balance = Available − Purchase − Fee
```

Key definitions

Checking account

A bank account for everyday deposits and payments.

Bank statement

The bank's monthly record of your account activity.

Current balance

Money in the account from fully posted transactions.

Worked example

Find each running balance in Tyrell's register (Figure 4.2).

1. Start $820.00.
2. 10/1 deposit 612.50: 820.00 + 612.50 = 1,432.50.
3. 10/3 check #1041, 650.00: 1,432.50 − 650.00 = 782.50.
4. 10/5 debit SEPTA Key, 25.50: 782.50 − 25.50 = 757.00.
5. 10/8 check #1043, 45.00: 757.00 − 45.00 = 712.00.

Answer: Ending balance $712.00

5. Solving for the Unknown

Formulas & rules

5.1 · Addition equation

$$x + a = b \rightarrow x = b - a$$

5.1 · Subtraction equation

$$x - a = b \rightarrow x = b + a$$

5.1 · Multiplication equation

$$a \cdot x = b \rightarrow x = b \div a$$

5.1 · Division equation

$$x \div a = b \rightarrow x = b \times a$$

5.2 · Two-step

$$ax + b = c \rightarrow x = (c - b) \div a$$

5.2 · Distribute

$$a(x + b) = ax + ab$$

5.3 · Flat fee plus rate

$$\text{Total} = \text{Fixed fee} + \text{Rate} \times \text{Units}$$

5.3 · Parts of a total

$$\text{Part}_1 + \text{Part}_2 = \text{Total}$$

Key definitions

Variable

A letter standing for an unknown number.

Coefficient

The number multiplying a variable, like 3 in 3x.

Translate

Rewrite a sentence as an algebraic expression or equation.

Worked example

x + 7 = 19

1. Subtract 7 from both sides: x = 19 − 7.
2. x = 12.
3. Check: 12 + 7 = 19. ✓

Answer: x = 12 pieces

6. Percentages and Their Applications

Formulas & rules

6.1 · Find a part

```
Part = base × decimal rate
```

6.1 · Find a rate

```
Decimal rate = part ÷ base
```

6.1 · Find a base

```
Base = part ÷ decimal rate
```

6.2 · Percent change

```
Percent change = (new − old) ÷ old × 100%
```

6.2 · Increase multiplier

```
New = old × (1 + decimal increase rate)
```

6.2 · Decrease multiplier

```
New = old × (1 − decimal decrease rate)
```

6.2 · Rate difference

```
Percentage-point change = new percent − old percent
```

6.3 · Discount amount

```
Discount = price × decimal discount rate
```

6.3 · Sale price

```
Sale price = original price − discount
```

6.3 · Tax and total

```
Total = taxable subtotal + rounded tax
```

6.3 · Tip

```
Tip = stated tip base × decimal tip rate
```

Key definitions

Percent

A ratio expressed per hundred.

Percent change

Difference divided by the original amount, expressed as a percent.

Discount

A reduction from a stated price.

Worked example

What percent of the 80 club members chose the weekend event if 28 selected it?

1. Identify the part as 28 members and the base as 80 members.
2. Divide: 28 ÷ 80 = 0.35.
3. Convert: 0.35 × 100 = 35%.

Answer: 35% of surveyed members chose the weekend event.

7. Trade Discounts, Cash Discounts, and Purchasing

Formulas & rules

7.1 · Single discount

```
Net price = list price × (1 − decimal discount rate)
```

7.1 · Series factor

```
Net factor = (1 − d1) × (1 − d2)
```

7.1 · Equivalent discount

```
Equivalent decimal discount = 1 − net factor
```

7.2 · Cash discount

```
Cash discount = eligible balance × decimal cash discount rate
```

7.2 · Early payment

```
Payment = eligible balance − cash discount + excluded charges
```

7.2 · Full payment

```
Full payment = eligible balance + excluded charges
```

7.3 · Adjusted merchandise

```
Adjusted merchandise = invoice merchandise − returns − allowances
```

7.3 · Delivered cost

```
Delivered cost = adjusted merchandise − cash discount + freight + stated tax
```

7.3 · Unit cost

```
Cost per usable unit = delivered cost ÷ usable units
```

Key definitions

List price

Supplier reference price before trade discounts.

Cash discount

Reduction for meeting an early-payment condition.

Return

Goods sent back for a credit.

Worked example

A $600 supply order receives a 20% trade discount. Find its net merchandise price.

1. Convert 20% to 0.20.
2. Discount: 600 × 0.20 = $120.00.
3. Net price: 600 − 120 = $480.00.

Answer: Net merchandise price: $480.00.

8. Markups, Markdowns, Pricing, and Break-Even Analysis

Formulas & rules

8.1 · Markup dollars

```
Markup = selling price − cost
```

8.1 · Price from cost markup

```
Selling price = cost × (1 + decimal markup rate)
```

8.1 · Margin

```
Decimal margin = markup ÷ selling price
```

8.1 · Price from margin

```
Selling price = cost ÷ (1 − decimal margin)
```

8.2 · Markdown

```
Markdown dollars = regular price × decimal markdown rate
```

8.2 · Reduced price

```
Sale price = regular price − markdown dollars
```

8.2 · Remaining markup

```
Remaining markup = sale price − cost
```

8.2 · Recover regular price

```
Regular price = sale price ÷ (1 − decimal markdown rate)
```

8.3 · Contribution

```
Contribution per unit = price − variable unit cost
```

8.3 · Break-even

```
Break-even units = fixed cost ÷ contribution per unit
```

8.3 · Profit

```
Operating profit = units × contribution per unit − fixed cost
```

8.3 · Target volume

```
Target units = (fixed cost + target profit) ÷ contribution per unit
```

Key definitions

Cost

Amount assigned to acquiring or producing the item.

Markdown
Reduction from a regular selling price.

Fixed cost
Cost constant over the modeled activity range.

Worked example

An item costs $40. Set price using a 25% markup on cost.

1. Markup: 40 × 0.25 = $10.00.
2. Price: 40 + 10 = $50.00.
3. Check markup rate: 10 ÷ 40 = 0.25 = 25%.

Answer: Price: $50.00.

9. Wages, Salaries, Commissions, and Payroll

Formulas & rules

9.1 · Decimal time

```
Decimal hours = whole hours + minutes ÷ 60
```

9.1 · Regular wages

```
Regular wages = regular hours × base hourly rate
```

9.1 · Overtime rate

```
Overtime rate = base rate × stated multiplier
```

9.1 · Gross wages

```
Gross wages = regular wages + overtime wages
```

9.2 · Periodic salary

```
Gross pay per period = annual salary ÷ number of periods
```

9.2 · Straight commission

```
Commission = eligible sales × decimal commission rate
```

9.2 · Base plus commission

```
Gross pay = base pay + commission
```

9.2 · Tiered commission

```
Commission = sum of (sales in each tier × tier rate)
```

9.3 · Percentage deduction

```
Deduction = stated base × decimal deduction rate
```

9.3 · Total deductions

```
Total deductions = sum of rounded employee deductions
```

9.3 · Net pay

```
Net pay = gross pay − total employee deductions
```

9.3 · Employer cost

```
Employer cost = gross pay + employer-only costs
```

Key definitions

Hourly wage

Pay per paid hour of work.

Salary

Stated compensation for a period rather than pay directly per hour.

Net pay
Gross earnings less employee deductions.

Worked example

A student works 12 hours 30 minutes at $15 per hour, with no unpaid breaks or overtime. Find gross pay.

1. Convert minutes: 30 ÷ 60 = 0.50 hour.
2. Paid time: 12 + 0.50 = 12.50 hours.
3. Gross pay: 12.50 × 15 = $187.50.

Answer: Gross pay: $187.50.

10. Simple Interest and Borrowing

Formulas & rules

10.1 · Simple interest

```
I = P × r × t
```

10.1 · Maturity value

```
M = P + I
```

10.1 · Month conversion

```
t = months ÷ 12
```

10.1 · Actual/365 time

```
t = actual days ÷ 365
```

10.2 · Find principal

```
P = I ÷ (r × t)
```

10.2 · Find rate

```
r = I ÷ (P × t)
```

10.2 · Find time

```
t = I ÷ (P × r)
```

10.3 · Net proceeds

```
Proceeds = face amount − withheld fees
```

10.3 · Single-payment cost

```
Cost = maturity payment + separate required fees − proceeds
```

10.3 · Classroom annualization

```
Annualized indicator = cost ÷ proceeds ÷ t
```

Key definitions

Principal

The original sum borrowed or invested.

Inverse operation

An operation that undoes another, such as division undoing multiplication.

Origination fee

A charge for arranging a loan.

Worked example

Find interest and repayment on $600 at 8% simple annual interest for 9 months.

1. Convert 8% to r = 0.08.
2. Convert time: t = 9 ÷ 12 = 0.75 year.
3. I = 600 × 0.08 × 0.75 = $36.00.
4. M = 600 + 36 = $636.00.

Answer: Interest is $36.00 and maturity value is $636.00.

11. Promissory Notes and Financial Obligations

Formulas & rules

11.1 · Day-based interest

```
I = P × r × (days ÷ year basis)
```

11.1 · Maturity value

```
M = P + I
```

11.1 · Days remaining

```
Remaining days = stated term − days already counted
```

11.2 · Bank discount

```
D = M × d × t
```

11.2 · Proceeds

```
Proceeds = M − D
```

11.2 · Proceeds-based indicator

```
Annualized indicator = D ÷ proceeds ÷ t
```

11.3 · Accrued interest

```
I = outstanding principal × r × t
```

11.3 · Principal reduction

```
Reduction = payment − accrued interest
```

11.3 · New balance

```
New principal = old principal − reduction
```

11.3 · Payoff

```
Payoff = outstanding principal + accrued interest + unpaid separate fees
```

Key definitions

Promissory note

A written promise to pay under stated terms.

Face amount

The amount written on a note; for a non-interest-bearing discount note, the maturity obligation.

Accrued interest

Interest earned or owed since the last relevant date.

Worked example

Find the maturity date and value of the figure's April 10, 60-day, $2,000 note at 9%, actual/360.

1. April 11–30 contributes 30 − 10 = 20 days.
2. May contributes 31 days; 60 − 20 − 31 = 9 days remain in June.
3. The maturity date is June 9; exclude April 10 and include June 9.
4. I = 2,000 × 0.09 × 60 ÷ 360 = $30.00.
5. M = 2,000 + 30 = $2,030.00.

Answer: Payment of $2,030.00 is due June 9.

12. Compound Interest, Exponents, and Present Value

Formulas & rules

12.1 · Exponents and Compound Growth

$$FV = P \times (1 + i)^n$$

12.1 · Related relationship

$$Interest = FV - P$$

12.2 · Compounding Frequency and Effective Rates

$$FV = P \times (1 + r \div m)^n$$

12.2 · Related relationship

$$Effective\ annual\ rate = (1 + r \div m)^m - 1$$

12.3 · Present Value and Financial Equivalence

$$PV = FV \div (1 + i)^n$$

12.3 · Related relationship

$$FV = PV \times (1 + i)^n$$

Key definitions

Exponent

The number of repeated factors.

Nominal annual rate

An annual quoted rate before within-year compounding.

Present value

The equivalent amount at the start of a timeline.

Worked example

Find future value of $1,000.00 at 5% annually compounded for 2 years.

1. Use P = $1,000.00, i = 0.05, and n = 2.
2. Growth factor = $1.05^2 = 1.1025$.
3. FV = 1000 × 1.1025 = $1,102.50.
4. Interest = $1,102.50 − $1,000.00 = $102.50.

Answer: $1,102.50

13. Annuities, Sinking Funds, and Long-Term Savings

Formulas & rules

13.1 · Ordinary Annuities and Deposit Timing

$$FV = R \times ((1 + i)^n - 1) \div i$$

13.1 · Related relationship

$$\text{Total contributions} = R \times n$$

13.2 · Annuities Due and Present Value

$$FV_{due} = R \times ((1 + i)^n - 1) \div i \times (1 + i)$$

13.2 · Related relationship

$$PV_{ordinary} = R \times (1 - (1 + i)^{-n}) \div i$$

13.3 · Sinking Funds and Savings Goals

$$R = \text{target } FV \times i \div ((1 + i)^n - 1)$$

13.3 · Related relationship

$$\text{Shortfall} = \text{target} - \text{available balance}$$

Key definitions

Annuity
Equal payments at equally spaced intervals.

Annuity
Equal payments at equally spaced intervals.

Annuity
Equal payments at equally spaced intervals.

Worked example

Deposit $100.00 at each month-end for 3 months at an assumed 1% monthly rate. Find value just after deposit 3.

1. R = $100.00; i = 0.01; n = 3.
2. Factor = $1.01^2 + 1.01 + 1 = 3.0301$.
3. FV = 100 × 3.0301 = $303.01.
4. Contributions = 3 × 100 = $300.00; interest = $3.01.

Answer: $303.01

14. Installment Purchases, Credit Cards, and Consumer Debt

Formulas & rules

14.1 · Amount financed

```
Amount financed = cash price − down payment
```

14.1 · Payment total

```
Payment total = payment amount × number of payments
```

14.1 · Finance charge

```
Finance charge = payment total − amount financed
```

Assumes all borrowing charges are in the scheduled payments.

14.1 · Total purchase cost

```
Total purchase cost = down payment + payment total + separate required fees
```

14.2 · Weighted balance

```
Average daily balance = sum of (balance × days) ÷ cycle days
```

14.2 · Daily rate

```
Daily rate = APR as a decimal ÷ 365
```

The convention assumed in this section.

14.2 · Cycle interest

```
Interest = average daily balance × daily rate × cycle days
```

14.2 · Statement reconciliation

```
Ending balance = beginning balance + purchases + fees + interest − payments − credits
```

14.3 · Monthly interest

```
Interest = starting balance × monthly rate
```

14.3 · Principal reduction

```
Principal reduction = payment − interest
```

Assumes no other charges.

14.3 · Next balance

```
Next balance = starting balance + interest − payment
```

14.3 · Payment burden

```
Payment-to-income percent = monthly debt payments ÷ monthly income × 100
```

Key definitions

Cash price
The price of an item before borrowing charges.

Billing cycle
The period covered by one statement.

Principal
The unpaid borrowed amount, excluding newly calculated interest in this model.

Worked example

Use Figure 14.1 to find Offer A's amount financed, finance charge, and total purchase cost.

1. Amount financed: $900 − $180 = $720.
2. Scheduled payments: 12 × $65 = $780.
3. Finance charge: $780 − $720 = $60.
4. Total purchase cost: $180 + $780 = $960.

Answer: The student borrows $720, pays a $60 finance charge, and spends $960 altogether.

15. Homeownership, Mortgages, and Amortization

Formulas & rules

15.1 · Down payment

```
Down payment = purchase price × down payment rate
```

15.1 · Initial mortgage

```
Mortgage principal = purchase price − down payment
```

No financed costs or other loans in this model.

15.1 · Cash due

```
Cash due = down payment + closing costs + separate prepaids − prior deposits − credits
```

15.1 · Loan-to-value

```
LTV percent = loan balance ÷ property value × 100
```

15.2 · Monthly allowance

```
Monthly allowance = annual cost ÷ 12
```

15.2 · PITI

```
PITI = principal and interest + monthly taxes + monthly insurance
```

15.2 · Housing budget

```
Housing budget = PITI + maintenance + utilities + other applicable costs
```

15.3 · Fixed payment

$$M = P \times r \div [1 - (1 + r)^{-n}]$$

P is principal; r is the positive periodic rate; n is the number of payments. At zero interest, M = P ÷ n.

15.3 · Periodic interest

```
Interest = beginning balance × periodic rate
```

15.3 · Principal paid

```
Principal paid = regular payment − interest + extra principal
```

15.3 · Ending balance

```
Ending balance = beginning balance − principal paid
```

Key definitions

Mortgage principal

The amount borrowed and still unpaid on a home loan.

P&I

The principal-and-interest portion of a mortgage payment.

Amortization
Gradual loan repayment through interest and principal payments.

Worked example

Complete the purchase worksheet in Figure 15.1.

1. Down payment: $200,000 × 0.10 = $20,000.
2. Mortgage principal: $200,000 − $20,000 = $180,000.
3. Closing costs: $200,000 × 0.03 = $6,000.
4. Cash needed: $20,000 + $6,000 + $2,400 = $28,400.

Answer: The mortgage is $180,000, but the buyer needs $28,400 in cash under these assumptions.

16. Understanding Financial Statements

Formulas & rules

16.1 · Gross profit

```
Gross profit = revenue − cost of goods sold
```

16.1 · Operating income

```
Operating income = gross profit − operating expenses
```

16.1 · Net income

```
Net income = operating income + other revenues − other expenses
```

16.1 · Net margin

```
Net profit margin = net income ÷ revenue × 100
```

Revenue must be nonzero.

16.2 · Accounting equation

```
Assets = liabilities + equity
```

16.2 · Residual equity

```
Equity = assets − liabilities
```

16.2 · Working capital

```
Working capital = current assets − current liabilities
```

16.2 · Current ratio

```
Current ratio = current assets ÷ current liabilities
```

Requires nonzero current liabilities.

16.3 · Simplified operating cash

```
Operating cash = net income + depreciation − receivables increase + operating payables increase
```

All other adjustments are zero in this model.

16.3 · Net cash change

```
Cash change = operating cash + investing cash + financing cash
```

16.3 · Ending cash

```
Ending cash = beginning cash + cash change
```

Key definitions

Revenue

Amounts earned from sales or services during the stated period.

Asset
A resource controlled by a business as a result of past events.

Operating cash flow
Cash generated or used by the main operating activities.

Worked example

Calculate the food cart's gross profit, operating income, and net income.

1. Gross profit: $5,000 − $1,800 = $3,200.
2. Operating income: $3,200 − $2,200 = $1,000.
3. Net income: $1,000 − $200 = $800.

Answer: Gross profit is $3,200, operating income is $1,000, and net income is $800.

17. Depreciation and Business Assets

Formulas & rules

17.1 · Recorded cost

```
Cost = purchase price + necessary delivery + necessary installation
```

17.1 · Straight-line expense

```
Annual depreciation = (cost − residual value) ÷ useful life
```

17.1 · Book value

```
Book value = cost − accumulated depreciation
```

17.1 · Partial-year expense

```
Partial-year depreciation = annual depreciation × months in service ÷ 12
```

The convention used in these examples.

17.2 · Per-unit allocation

```
Depreciation per unit = (cost − residual value) ÷ estimated total units
```

17.2 · Period expense

```
Period depreciation = actual units × depreciation per unit
```

Cap expense at the remaining depreciable amount.

17.2 · Remaining allocation

```
Remaining depreciable amount = cost − residual value − accumulated depreciation
```

17.3 · Double-declining rate

```
Double-declining rate = 2 ÷ useful life
```

17.3 · Ordinary period calculation

```
Depreciation = beginning book value × double-declining rate
```

Apply the residual floor and the stated final-year convention.

17.3 · Final-year adjustment

```
Final-year depreciation = beginning book value − residual value
```

Used in this chapter's no-switch schedule.

17.3 · Disposal result

```
Gain or loss = net sale proceeds − book value
```

Positive is a gain; negative is a loss.

Key definitions

Capitalized cost

An amount recorded as part of an asset rather than expensed immediately.

Units-of-production method
Depreciation based on actual activity relative to estimated lifetime activity.

Accelerated depreciation
A method that generally allocates more depreciation in earlier periods.

Worked example

Find the workshop machine's recorded cost and annual straight-line depreciation.

1. Recorded cost: $9,000 + $600 + $400 = $10,000.
2. Depreciable amount: $10,000 − $1,000 = $9,000.
3. Annual expense: $9,000 ÷ 5 = $1,800.

Answer: Record the machine at $10,000 and allocate $1,800 per full year.

18. Inventory, Overhead, and Operating Costs

Formulas & rules

18.1 · Goods available

```
Available cost = beginning inventory cost + net purchases
```

18.1 · Recorded sales cost

```
COGS = units sold × cost per unit (equal-cost goods)
```

18.1 · Shrinkage

```
Shrinkage cost = (expected units − counted units) × unit cost
```

18.1 · Turnover

```
Turnover = COGS ÷ [(beginning inventory + ending inventory) ÷ 2]
```

18.2 · Overhead rate

```
Rate = overhead pool ÷ total allocation-base units
```

18.2 · Allocated overhead

```
Job overhead = rate × job activity
```

18.2 · Full unit cost

```
Full unit cost = direct unit cost + (allocated overhead ÷ units made)
```

18.3 · Unit contribution

```
Contribution per unit = price − variable cost per unit
```

18.3 · Operating profit

```
Profit = units × contribution per unit − fixed costs
```

18.3 · Break-even units

```
Break-even units = fixed costs ÷ contribution per unit; round up
```

18.3 · Target-profit units

```
Required units = (fixed costs + target profit) ÷ contribution per unit; round
up
```

Key definitions

Inventory

Goods held for sale, measured in units or at cost.

Direct cost

A cost traceable to the product or job being measured.

Fixed cost
A total cost unchanged within the stated activity range and period.

Worked example

Use the figure to find recorded COGS, physical ending value, and shrinkage.

1. Available units = 100 + 150 = 250; available cost = 250 × $8 = $2,000.00.
2. Recorded COGS = 180 × $8 = $1,440.00.
3. Expected ending units = 250 − 180 = 70; physical ending cost = 65 × $8 = $520.00.
4. Shrinkage = (70 − 65) × $8 = $40.00.
5. Check: $1,440 + $520 + $40 = $2,000.

Answer: Recorded COGS is $1,440.00, ending inventory is $520.00, and shrinkage is $40.00.

19. Sales Taxes, Property Taxes, and Business Taxation

Formulas & rules

19.1 · Sales tax

```
Tax = taxable subtotal × decimal tax rate
```

19.1 · Customer total

```
Total = taxable subtotal + exempt subtotal + rounded tax
```

19.1 · Reverse tax

```
Pretax price = tax-inclusive price ÷ (1 + decimal tax rate)
```

19.2 · Assessment

```
Assessed value = market value × assessment ratio
```

19.2 · Tax base

```
Taxable assessed value = maximum of 0 and (assessed value − exemption)
```

19.2 · Millage tax

```
Annual tax = taxable assessed value × mills ÷ 1,000
```

19.2 · Budget allowance

```
Monthly allowance = annual tax ÷ 12
```

19.3 · Taxable profit

```
Taxable profit = revenue − allowed expenses
```

19.3 · Bracket tax

```
Tax before credits = sum of (income in each bracket × bracket rate)
```

19.3 · Final tax

```
Final tax = maximum of 0 and (tax before credits − nonrefundable credits)
```

19.3 · Balance

```
Balance due = final tax − prepayments
```

Key definitions

Taxable subtotal

The portion of a purchase subject to the stated tax.

Market value

An estimated exchange price for property.

Taxable profit

Revenue minus expenses and deductions allowed in the stated model.

Worked example

Find the club's total using the figure.

1. Taxable subtotal = $30 + $20 = $50.00.
2. Exempt subtotal = $15.00; pretax total = $50 + $15 = $65.00.
3. Tax = $50 × 0.08 = $4.00.
4. Total due = $65 + $4 = $69.00.

Answer: The club owes $69.00, including $4.00 tax.

20. Insurance, Risk, and Financial Protection

Formulas & rules

20.1 · Annual premium

```
Annual premium = monthly premium × 12
```

20.1 · Claim payment

```
Payment = minimum of limit and maximum of 0 and (covered loss − deductible)
```

20.1 · Retained loss

```
Retained loss = covered loss − payment
```

20.1 · Annual cash cost

```
Cash cost = annual premiums + retained losses
```

20.2 · Deductible applied

```
Applied deductible = minimum of allowed amount and remaining deductible
```

20.2 · Ordinary patient share

```
Ordinary share = applied deductible + (allowed amount − applied deductible) × patient coinsurance
```

20.2 · Capped share

```
Patient share = minimum of ordinary share and remaining out-of-pocket maximum
```

20.2 · Plan payment

```
Plan payment = allowed amount − patient share
```

20.3 · Expected loss

```
Expected loss = sum of (outcome probability × outcome loss)
```

20.3 · Expected insured cost

```
Expected cost = annual premium + expected retained loss
```

20.3 · Reserve saving

```
Monthly saving = reserve target ÷ months available
```

Key definitions

Premium

Price paid to maintain coverage.

Allowed amount

The claim amount used for the covered-service calculation.

Expected loss
Probability-weighted average of possible losses.

Worked example

Use the figure to find payment and retained loss.

1. Loss above deductible = $1,800 − $200 = $1,600.00.
2. Payment = minimum of $1,600 and $1,500 = $1,500.00.
3. Retained loss = $1,800 − $1,500 = $300.00.

Answer: The insurer pays $1,500.00; the rider retains $300.00.

21. Stocks, Bonds, Mutual Funds, and Investment Mathematics

Formulas & rules

21.1 · Cost of purchase

```
Cost = shares × price per share + fees
```

21.1 · Dividend yield

```
Yield = annual dividend per share ÷ price × 100%
```

21.1 · Total return

```
Total return = (ending price − starting price + dividends) ÷ starting price × 100%
```

21.1 · P/E ratio

```
P/E = price per share ÷ earnings per share
```

21.2 · Annual coupon

```
Coupon = par × coupon rate
```

21.2 · Price from quote

```
Price = quote ÷ 100 × par
```

21.2 · Current yield

```
Current yield = annual coupon ÷ price × 100%
```

21.3 · NAV

```
NAV = (total assets − liabilities) ÷ shares outstanding
```

21.3 · Expense cost

```
Yearly cost = amount invested × expense ratio
```

21.3 · Weighted return

$$\text{Portfolio return} = w_1 r_1 + w_2 r_2 + \dots$$

Weights are decimals that add to 1.

Key definitions

Share

One unit of ownership in a company.

Bond

A loan to an issuer that pays interest and returns the face value at maturity.

Mutual fund / ETF
A pooled investment that holds many securities.

Worked example

Using Figure 21.1, find the purchase cost (no fee), the year's dividend income, and the dividend yield at the $40 purchase price.

1. Cost: 50 × $40 = $2,000.
2. Annual dividend per share: 4 × $0.30 = $1.20.
3. Dividend income: 50 × $1.20 = $60.
4. Yield: $1.20 ÷ $40 = 0.03 = 3%.

Answer: Cost $2,000; dividends $60; yield 3%

22. Business Statistics, Probability, and Data Interpretation

Formulas & rules

22.1 · Mean

```
Mean = (sum of values) ÷ (number of values)
```

22.1 · Range

```
Range = maximum − minimum
```

22.1 · Median (even count)

```
Median = (middle value 1 + middle value 2) ÷ 2
```

Always sort the data first.

22.2 · Probability

```
P(A) = favorable outcomes ÷ total outcomes
```

$0 \le P(A) \le 1$

22.2 · Complement

```
P(not A) = 1 − P(A)
```

22.2 · Independent events

```
P(A and B) = P(A) × P(B)
```

22.3 · Positive odds

```
Implied P = 100 ÷ (odds + 100)
```

22.3 · Negative odds

```
Implied P = |odds| ÷ (|odds| + 100)
```

22.3 · Expected value

```
EV = P(win) × gain − P(lose) × loss
```

22.4 · Percent change

```
% change = (new − old) ÷ old × 100%
```

22.4 · Compound growth

$$A = P(1 + r)^n$$

P = start, r = rate per period as a decimal, n = periods.

22.5 · Item revenue

```
Revenue = units sold × price per unit
```

22.5 · Weighted average price

```
Average price = sum(price × units) ÷ sum(units)
```

22.5 · Percent of total

```
Share = category amount ÷ matching total × 100%
```

Key definitions

Data set
A collection of numbers about the same thing, such as points per game.

Outcome
One possible result of an experiment, like rolling a 4.

Odds
A way of stating the payout of a bet, which also implies a probability.

Percent change
The change in a value as a percent of its starting value.

Weight
The count or importance attached to a value in an average.

Worked example

Find the mean, median, mode, and range of 18, 24, 12, 30, 24, 15, 31.

1. Sum: 18 + 24 + 12 + 30 + 24 + 15 + 31 = 154.
2. Mean: 154 ÷ 7 = 22 points.
3. Order the data: 12, 15, 18, 24, 24, 30, 31. The 4th of 7 values is the middle, so the median is 24.
4. Mode: 24 appears twice; every other value once. Mode = 24.
5. Range: 31 − 12 = 19 points.

Answer: Mean 22, median 24, mode 24, range 19

23. Linear Equations, Graphs, and Business Relationships

Formulas & rules

23.1 · Fixed-plus-variable charge

$$C = F + r \times x$$

F is the fixed charge and r is the charge per unit.

23.1 · Solve a linear equation

$$x = (c - b) \div a$$

For $ax + b = c$, with $a \neq 0$.

23.1 · Distributive property

$$a \times (x + b) = ax + ab$$

23.2 · Slope

$$m = (y_2 - y_1) \div (x_2 - x_1)$$

The two x-values must differ.

23.2 · Slope-intercept form

$$y = mx + b$$

23.2 · Find the intercept

$$b = y - mx$$

23.3 · Revenue

$$R = p \times q$$

23.3 · Total cost

$$C = F + v \times q$$

23.3 · Profit

$$P = R - C = (p - v) \times q - F$$

23.3 · Break-even quantity

$$q = F \div (p - v)$$

Requires $p > v$ for a positive break-even quantity when $F > 0$.

Key definitions

Variable

A symbol representing an unknown or changing quantity.

Ordered pair

A point written (x, y), with horizontal coordinate first.

Revenue
Sales income before subtracting expenses.

Worked example

The rental bill is $88. At $18 plus $14 per hour, how many hours were charged?

1. Define h as the number of rental hours.
2. Write 18 + 14h = 88.
3. Subtract 18: 14h = 70.
4. Divide by 14: h = 5.
5. Check: 18 + 14 × 5 = 18 + 70 = 88.

Answer: The bill covers 5 hours.

24. Inequalities, Systems of Equations, and Decision-Making

Formulas & rules

24.1 · Budget constraint

$F + v \times q \leq B$

B is the maximum budget.

24.1 · Positive per-unit cost

$q \leq (B - F) \div v$

For $v > 0$; also apply nonnegative and whole-number restrictions.

24.1 · Negative multiplication rule

$\text{If } a < b \text{ and } c < 0, \text{ then } ac > bc$

24.2 · Equal-cost comparison

$F_1 + r_1x = F_2 + r_2x$

24.2 · Comparison input

$x = (F_2 - F_1) \div (r_1 - r_2)$

Requires different rates.

24.2 · Two-category total

$x + y = n$

24.3 · Pack limit

$x + y \leq 8$

24.3 · Machine-time limit

$2x + y \leq 12$

24.3 · Contribution objective

$Z = 15x + 10y$

Key definitions

Inequality

A statement comparing quantities using $<$, $>$, $\leq$, or $\geq$.

System of equations

Equations required to hold at the same time.

Feasible region

All points satisfying every constraint.

Worked example

How many participants can attend if setup costs $20, supplies cost $8 each, and the budget is $100?

1. Define q as a nonnegative whole-number participant count.
2. Write $20 + 8q \le 100$.
3. Subtract 20: $8q \le 80$.
4. Divide by positive 8: $q \le 10$.
5. Check the endpoint: $20 + 8 \times 10 = 100$.

Answer: The feasible counts are 0 through 10 participants.

25. Functions, Quadratic Equations, and Maximum Profit

Formulas & rules

25.1 · Workshop charge

$$f(n) = 25 + 12n$$

25.1 · Evaluation pattern

$$\text{If } f(x) = ax + b, \text{ then } f(k) = a \times k + b$$

25.1 · A nonlinear rule

$$g(x) = 2x^2 + 5$$

Square the input before multiplying by 2.

25.2 · Standard quadratic equation

$$ax^2 + bx + c = 0$$

$a \neq 0$.

25.2 · Quadratic formula

$$x = (-b \pm \sqrt{(b^2 - 4ac)}) \div (2a)$$

25.2 · Zero-product property

$$\text{If } A \times B = 0, \text{ then } A = 0 \text{ or } B = 0$$

25.2 · Discriminant

$$D = b^2 - 4ac$$

25.3 · Revenue with changing price

$$R(q) = q \times p(q)$$

25.3 · Profit

$$P(q) = R(q) - C(q)$$

25.3 · Vertex input

$$x_v = -b \div (2a)$$

For $ax^2 + bx + c$, $a \neq 0$.

25.3 · Vertex output

$$y_v = a \times x_v^2 + b \times x_v + c$$

Key definitions

Function

A rule assigning exactly one output to each allowed input.

Quadratic
An expression or function whose highest variable power is 2.

Vertex
The turning point of a parabola.

Worked example

For $f(n) = 25 + 12n$, find $f(4)$, then find the participant count producing a charge of $85.

1. Substitute 4: $f(4) = 25 + 12 \times 4$.
2. Calculate: $f(4) = 25 + 48 = 73$.
3. For an $85 charge, write $25 + 12n = 85$.
4. Subtract 25 and divide by 12: $n = 60 \div 12 = 5$.
5. Check: $f(5) = 25 + 60 = 85$.

Answer: Four participants cost $73.00; an $85.00 charge corresponds to 5 participants.

26. Exponential Growth, Logarithms, and Financial Modeling

Formulas & rules

26.1 · Growth

$A = P(1 + r)^n$

26.1 · Decay

$A = P(1 - d)^n$

26.1 · Linear comparison

$A = P + kn$

26.2 · Inverse relationship

$\log_b x = n \text{ means } b^n = x$

26.2 · Time to target

$n = \ln(A \div P) \div \ln(b)$

26.2 · Doubling time

$n = \ln(2) \div \ln(1 + r)$

26.3 · Compound balance

$A = P(1 + r \div m)^{(mt)}$

26.3 · Effective annual rate

$EAR = (1 + r \div m)^m - 1$

26.3 · Loan payment

$M = P \times i \div [1 - (1 + i)^{(-n)}]$

26.3 · Borrowing cost without fees

$\text{Cost} = \text{total payments} - P$

Key definitions

Exponential model

A model with a constant multiplicative change per period.

Logarithm

The exponent needed to obtain a positive number from a valid base.

Nominal annual rate

An annual quoted rate before the effect of within-year compounding.

Worked example

Model $500 growing by 10% per year for 3 years.

1. Convert 10% to 0.10.
2. Factor = 1 + 0.10 = 1.10.
3. Compute $1.10^{3} = 1.331$.
4. Multiply $500 × 1.331 = $665.50.

Answer: The modeled balance is $665.50.

27. Ratios, Geometry, Measurement, and Business Applications

Formulas & rules

27.1 · Unit rate

```
Unit rate = total quantity ÷ number of units
```

27.1 · Proportion

```
a ÷ b = c ÷ d means a × d = b × c
```

27.1 · Scale conversion

```
Actual length = drawing length × actual units per drawing unit
```

27.2 · Rectangle

```
A = l × w; P = 2(l + w)
```

27.2 · Triangle

```
A = b × h ÷ 2
```

27.2 · Circle

```
A = πr²; C = 2πr
```

27.2 · Material planning

```
Packages = round up [area × (1 + waste rate) ÷ coverage per package]
```

27.3 · Rectangular volume

```
V = l × w × h
```

27.3 · Cylinder volume

```
V = πr²h
```

27.3 · Mass from density

```
Mass = density × volume
```

27.3 · Serving count

```
Complete servings = round down (usable volume ÷ serving volume)
```

Key definitions

Ratio

An ordered comparison of two quantities by division.

Perimeter

Total distance around a polygon.

Volume

Space occupied or enclosed, expressed in cubic units.

Worked example

Make 21 liters of drink using syrup:water = 2:5.

1. Total parts = 2 + 5 = 7.
2. Each part = 21 ÷ 7 = 3 liters.
3. Syrup = 2 × 3 = 6 liters.
4. Water = 5 × 3 = 15 liters; 6 + 15 = 21.

Answer: Use 6 liters syrup and 15 liters water.

28. Mathematical Modeling, Entrepreneurship, and Economics

Formulas & rules

28.1 · Revenue

$$R = p \times q$$

28.1 · Total cost

$$C = F + v \times q$$

28.1 · Profit

$$\text{Profit} = (p - v) \times q - F$$

28.1 · Break-even quantity

$$q = F \div (p - v), \text{ when } p > v$$

28.2 · Demand model

$$q = a - b \times p$$

28.2 · Revenue with demand

$$R = p \times (a - b \times p)$$

28.2 · Profit with demand

$$\text{Profit} = (p - v) \times (a - b \times p) - F$$

28.3 · Ending cash

$$\text{Ending cash} = \text{opening cash} + \text{collections} - \text{payments}$$

28.3 · Simplified profit

$$\text{Profit} = \text{earned revenue} - \text{associated expenses}$$

28.3 · Forecast error

$$\text{Error} = \text{actual} - \text{forecast}$$

28.3 · Percentage forecast error

$$\text{Error percent} = (\text{actual} - \text{forecast}) \div \text{forecast} \times 100\%, \text{ for positive forecast}$$

Key definitions

Variable

A symbol representing a quantity that may change.

Demand

Quantity buyers are willing and able to buy under stated conditions.

Cash flow

Cash receipts and cash payments over a period.

Worked example

Find break-even for price $8, variable cost $3, and fixed cost $120.

1. Margin = 8 − 3 = $5 per program.
2. Set profit to zero: 5q − 120 = 0.
3. Solve q = 120 ÷ 5 = 24.
4. Check R = 8 × 24 = $192 and C = 120 + 3 × 24 = $192.

Answer: Break-even is 24 programs.